Computer algebra independent integration tests

3-Logarithms/3.2.3-u-log-e-f-a+b-x-^p-c+d-x-^q-^r-^s

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [108]. This is test number [61].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 98.15 (106)	% 1.85 (2)
Mathematica	% 100. (108)	% 0. (0)
Maple	% 23.15 (25)	% 76.85 (83)
Maxima	% 60.19 (65)	% 39.81 (43)
Fricas	% 36.11 (39)	% 63.89 (69)
Sympy	% 13.89 (15)	% 86.11 (93)
Giac	% 35.19 (38)	% 64.81 (70)

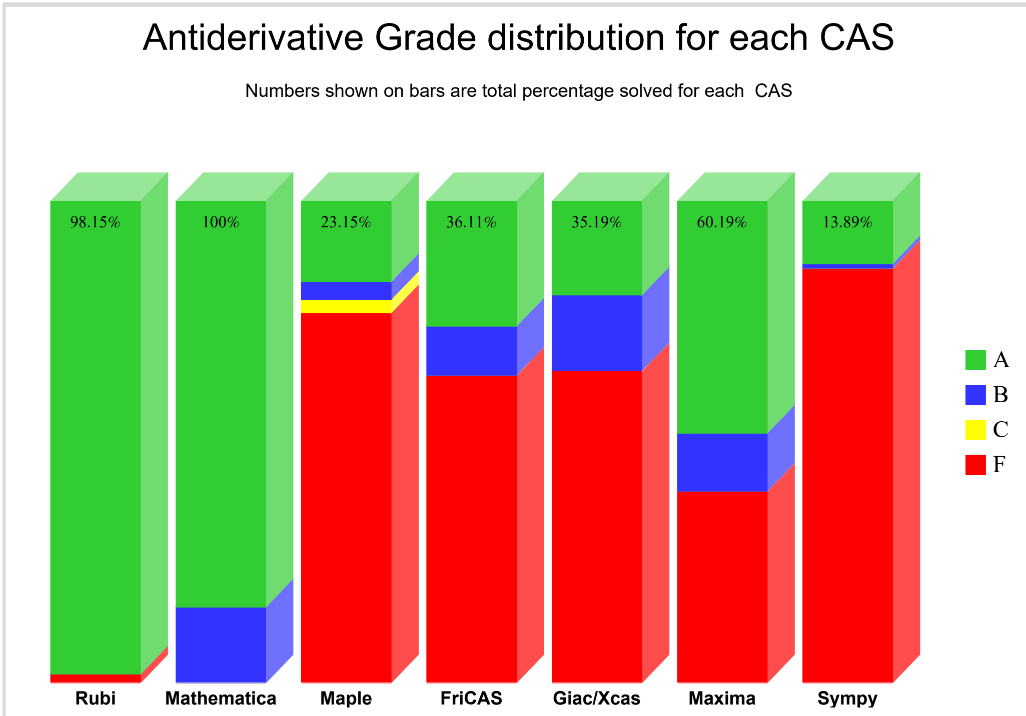
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

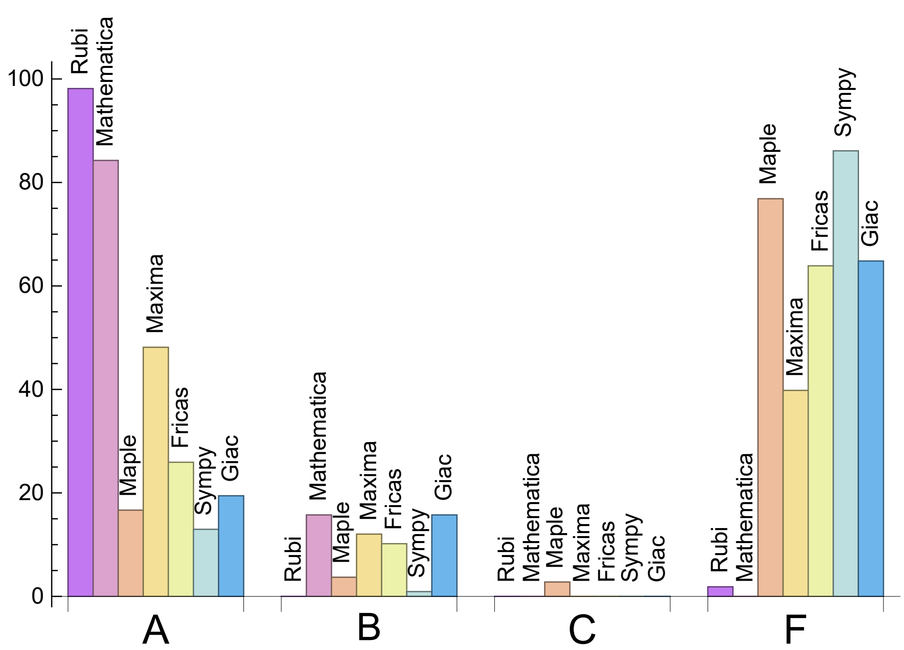
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	98.15	0.	0.	1.85
Mathematica	84.26	15.74	0.	0.
Maple	16.67	3.7	2.78	76.85
Maxima	48.15	12.04	0.	39.81
Fricas	25.93	10.19	0.	63.89
Sympy	12.96	0.93	0.	86.11
Giac	19.44	15.74	0.	64.81

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.42	350.24	0.9	182.5	1.
Mathematica	1.66	1551.08	2.54	165.	0.96
Maple	3.93	318.8	2.63	29.	1.
Maxima	1.22	558.37	2.09	193.	1.65
Fricas	1.06	353.05	2.82	103.	2.81
Sympy	19.15	32.2	0.95	26.	0.93
Giac	0.94	548.61	2.82	96.5	1.8

1.4 list of integrals that has no closed form antiderivative

{54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {67, 68}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

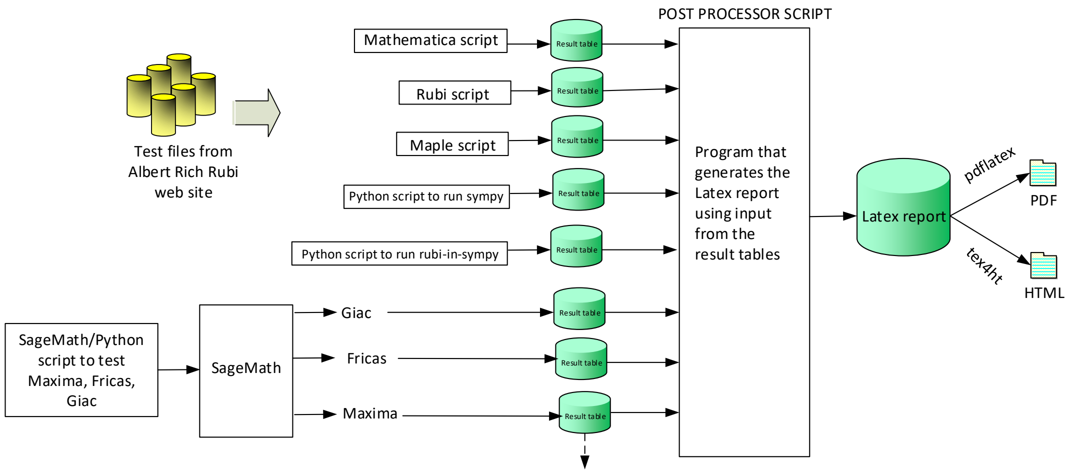
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

B grade: { }

C grade: { }

F grade: { 74, 75}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 66, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105 }

B grade: { 16, 17, 24, 40, 41, 42, 51, 56, 57, 58, 67, 68, 69, 92, 106, 107, 108 }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 29, 54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73, 89, 93, 96, 102, 106 }

B grade: { 88, 99, 104, 107 }

C grade: { 64, 74, 75 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 94, 95, 97, 98, 100, 101, 103, 105, 108 }

2.1.4 Maxima

A grade: { 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 40, 41, 47, 48, 53, 54, 55, 59, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73, 74, 75, 93, 94, 96, 97, 99, 100

B grade: { 7, 15, 24, 25, 34, 42, 44, 45, 46, 49, 50, 88, 89 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 20, 39, 43, 51, 52, 56, 57, 58, 64, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 95, 98, 101, 102, 103, 104, 105, 106, 107, 108 }

2.1.5 FriCAS

A grade: { 10, 12, 28, 29, 43, 46, 47, 48, 49, 50, 54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73, 74, 75, 89, 93, 96, 99 }

B grade: { 7, 8, 9, 13, 14, 15, 25, 26, 27, 44, 45 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

2.1.6 Sympy

A grade: { 44, 45, 46, 47, 62, 63, 65, 66, 74, 75, 89, 93, 96, 99 }

B grade: { 50 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 64, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

2.1.7 Giac

A grade: { 12, 13, 31, 47, 48, 54, 55, 60, 61, 62, 63, 65, 66, 70, 71, 72, 73, 89, 93, 96, 99 }

B grade: { 7, 8, 9, 10, 14, 15, 25, 26, 27, 28, 29, 32, 33, 34, 46, 49, 50 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 30, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 51, 52, 53, 56, 57, 58, 59, 64, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	385	336	0	0	0	0	0
normalized size	1	0.95	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.493	0.413	0.154	0.	0.	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	242	217	0	0	0	0	0
normalized size	1	0.92	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.337	0.23	0.086	0.	0.	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	135	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.225	0.096	0.089	0.	0.	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	185	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.332	0.148	0.199	0.	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	352	295	0	0	0	0	0
normalized size	1	1.09	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.494	0.594	0.084	0.	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	531	562	470	0	0	0	0	0
normalized size	1	1.06	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.797	1.653	0.084	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	185	0	533	1315	0	1002
normalized size	1	1.	0.92	0.	2.65	6.54	0.	4.99
time (sec)	N/A	0.095	0.308	0.734	1.307	0.864	0.	1.26

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	154	0	385	980	0	774
normalized size	1	1.	0.9	0.	2.24	5.7	0.	4.5
time (sec)	N/A	0.072	0.211	0.422	1.336	0.957	0.	1.406

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	127	0	262	693	0	566
normalized size	1	1.	0.89	0.	1.83	4.85	0.	3.96
time (sec)	N/A	0.061	0.135	0.386	1.288	0.88	0.	1.308

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	105	0	159	435	0	381
normalized size	1	1.	0.91	0.	1.37	3.75	0.	3.28
time (sec)	N/A	0.043	0.193	0.174	1.354	0.82	0.	1.35

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	93	0	221	0	0	0
normalized size	1	1.	0.87	0.	2.07	0.	0.	0.
time (sec)	N/A	0.083	0.109	0.436	3.137	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	89	0	134	267	0	151
normalized size	1	1.	0.94	0.	1.41	2.81	0.	1.59
time (sec)	N/A	0.037	0.058	0.421	1.265	0.902	0.	1.197

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	116	0	223	689	0	332
normalized size	1	1.	0.86	0.	1.65	5.1	0.	2.46
time (sec)	N/A	0.056	0.273	0.424	1.238	0.871	0.	1.192

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	141	0	390	1177	0	633
normalized size	1	1.	0.86	0.	2.38	7.18	0.	3.86
time (sec)	N/A	0.069	0.392	0.425	1.278	0.854	0.	1.211

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	164	0	620	1758	0	1010
normalized size	1	1.	0.85	0.	3.21	9.11	0.	5.23
time (sec)	N/A	0.085	0.407	0.424	1.279	0.849	0.	1.35

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	920	920	2508	0	1918	0	0	0
normalized size	1	1.	2.73	0.	2.08	0.	0.	0.
time (sec)	N/A	0.845	2.686	0.414	1.579	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	805	805	1853	0	1446	0	0	0
normalized size	1	1.	2.3	0.	1.8	0.	0.	0.
time (sec)	N/A	0.665	1.859	0.402	1.471	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	686	686	1211	0	1038	0	0	0
normalized size	1	1.	1.77	0.	1.51	0.	0.	0.
time (sec)	N/A	0.533	1.127	0.404	1.428	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	540	540	781	0	680	0	0	0
normalized size	1	1.	1.45	0.	1.26	0.	0.	0.
time (sec)	N/A	0.391	0.57	0.166	1.504	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	431	460	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.492	0.171	0.416	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	465	465	411	0	529	0	0	0
normalized size	1	1.	0.88	0.	1.14	0.	0.	0.
time (sec)	N/A	0.387	0.836	0.41	1.342	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	632	632	872	0	1019	0	0	0
normalized size	1	1.	1.38	0.	1.61	0.	0.	0.
time (sec)	N/A	0.49	1.605	0.412	1.481	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	764	764	1407	0	1690	0	0	0
normalized size	1	1.	1.84	0.	2.21	0.	0.	0.
time (sec)	N/A	0.606	2.731	0.414	1.761	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	884	884	14573	0	2452	0	0	0
normalized size	1	1.	16.49	0.	2.77	0.	0.	0.
time (sec)	N/A	0.738	7.025	0.418	2.175	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	275	0	842	1932	0	1715
normalized size	1	1.	0.82	0.	2.52	5.78	0.	5.13
time (sec)	N/A	0.186	0.352	0.393	1.203	1.542	0.	1.522

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	231	0	582	1377	0	1254
normalized size	1	1.	0.84	0.	2.11	4.99	0.	4.54
time (sec)	N/A	0.127	0.31	0.397	1.216	1.249	0.	1.328

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	209	0	363	917	0	837
normalized size	1	1.	0.96	0.	1.67	4.21	0.	3.84
time (sec)	N/A	0.099	0.254	0.431	1.258	1.161	0.	1.305

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	120	0	193	524	0	479
normalized size	1	1.	0.75	0.	1.21	3.28	0.	2.99
time (sec)	N/A	0.069	0.196	0.174	1.202	1.208	0.	1.31

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	57	61	101	182	0	242
normalized size	1	1.	0.93	1.	1.66	2.98	0.	3.97
time (sec)	N/A	0.015	0.063	0.071	1.157	1.089	0.	1.238

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	163	0	251	0	0	0
normalized size	1	1.	1.1	0.	1.7	0.	0.	0.
time (sec)	N/A	0.12	0.083	0.622	1.22	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	93	0	166	0	0	257
normalized size	1	1.	0.73	0.	1.3	0.	0.	2.01
time (sec)	N/A	0.052	0.233	0.493	1.253	0.	0.	1.469

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	206	0	313	0	0	1413
normalized size	1	1.	1.02	0.	1.55	0.	0.	7.
time (sec)	N/A	0.112	0.55	0.496	1.259	0.	0.	1.579

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	254	0	616	0	0	2383
normalized size	1	1.	0.98	0.	2.37	0.	0.	9.17
time (sec)	N/A	0.152	0.96	0.519	1.298	0.	0.	1.864

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	480	0	1048	0	0	6884
normalized size	1	1.	1.51	0.	3.3	0.	0.	21.65
time (sec)	N/A	0.195	1.822	0.509	1.412	0.	0.	2.777

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2240	2220	1386	0	2429	0	0	0
normalized size	1	0.99	0.62	0.	1.08	0.	0.	0.
time (sec)	N/A	2.436	3.013	0.392	1.452	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1645	1657	899	0	1516	0	0	0
normalized size	1	1.01	0.55	0.	0.92	0.	0.	0.
time (sec)	N/A	1.758	1.89	0.385	1.379	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1063	1097	480	0	841	0	0	0
normalized size	1	1.03	0.45	0.	0.79	0.	0.	0.
time (sec)	N/A	1.163	1.127	0.176	1.376	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	437	0	402	0	0	0
normalized size	1	1.	1.62	0.	1.49	0.	0.	0.
time (sec)	N/A	0.153	0.216	0.131	1.366	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1471	2096	1370	0	0	0	0	0
normalized size	1	1.42	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	1.931	0.281	0.578	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	832	878	2930	0	1006	0	0	0
normalized size	1	1.06	3.52	0.	1.21	0.	0.	0.
time (sec)	N/A	0.932	2.671	0.498	1.577	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1304	1362	15976	0	2507	0	0	0
normalized size	1	1.04	12.25	0.	1.92	0.	0.	0.
time (sec)	N/A	1.408	6.322	0.5	2.45	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1957	2013	47110	0	6388	0	0	0
normalized size	1	1.03	24.07	0.	3.26	0.	0.	0.
time (sec)	N/A	2.102	6.541	0.52	5.212	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	138	0	0
normalized size	1	1.	1.	0.	0.	3.29	0.	0.
time (sec)	N/A	0.08	0.029	0.485	0.	2.193	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	710	248	65	0
normalized size	1	1.	1.	0.	19.19	6.7	1.76	0.
time (sec)	N/A	0.061	0.009	0.411	1.352	2.012	47.02	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	362	184	65	0
normalized size	1	1.	1.	0.	9.78	4.97	1.76	0.
time (sec)	N/A	0.063	0.01	0.342	1.277	2.129	23.341	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	142	119	61	116
normalized size	1	1.	1.	0.	3.84	3.22	1.65	3.14
time (sec)	N/A	0.038	0.006	0.364	1.207	1.995	30.809	1.263

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	0	49	72	53	42
normalized size	1	1.	1.	0.	1.44	2.12	1.56	1.24
time (sec)	N/A	0.068	0.036	0.351	1.482	2.005	164.757	1.161

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	0	46	72	0	46
normalized size	1	1.	1.	0.	1.35	2.12	0.	1.35
time (sec)	N/A	0.065	0.01	0.356	1.583	1.991	0.	1.259

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	108	142	0	115
normalized size	1	1.	1.	0.	2.92	3.84	0.	3.11
time (sec)	N/A	0.065	0.011	0.349	1.609	1.847	0.	1.304

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	0	112	59	65	78
normalized size	1	1.	1.	0.	3.73	1.97	2.17	2.6
time (sec)	N/A	0.023	0.009	0.389	1.924	1.947	13.519	1.298

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	22595	0	0	0	0	0
normalized size	1	1.	55.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.471	7.545	0.982	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	436	0	0	0	0	0
normalized size	1	1.	1.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.288	2.552	0.75	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	166	0	275	0	0	0
normalized size	1	1.	0.97	0.	1.6	0.	0.	0.
time (sec)	N/A	0.136	0.089	0.491	1.262	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	50	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.362	0.723	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	50	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	2.743	0.739	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	1241	0	0	0	0	0
normalized size	1	1.	3.78	0.	0.	0.	0.	0.
time (sec)	N/A	1.252	1.827	2.213	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	839	0	0	0	0	0
normalized size	1	1.	3.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.91	0.913	1.8	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	451	0	0	0	0	0
normalized size	1	1.	2.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.603	0.448	1.784	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	0	170	0	0	0
normalized size	1	1.	0.96	0.	2.1	0.	0.	0.
time (sec)	N/A	0.066	0.063	0.123	1.208	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.545	0.445	1.772	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.425	2.462	1.78	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	5.094	0.385	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	5.084	0.209	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	66	450	0	0	0	0
normalized size	1	1.	0.8	5.49	0.	0.	0.	0.
time (sec)	N/A	0.177	5.027	0.121	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	5.098	0.089	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	33.103	0.327	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	620	649	18164	0	0	0	0	0
normalized size	1	1.05	29.3	0.	0.	0.	0.	0.
time (sec)	N/A	1.133	21.544	7.167	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	496	517	9211	0	0	0	0	0
normalized size	1	1.04	18.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.819	10.071	5.039	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	384	1408	0	0	0	0	0
normalized size	1	1.04	3.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.558	6.028	4.115	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	106	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.5	1.908	38.288	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	92	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	1.287	48.165	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	112	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.519	0.632	1.07	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	112	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.484	0.575	1.008	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	A	A	A	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	45	0	44	662	80	104	44	0
normalized size	1	0.	0.98	14.71	1.78	2.31	0.98	0.
time (sec)	N/A	0.516	0.312	1.322	1.497	2.068	3.06	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	A	A	A	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	45	0	45	503	78	103	44	0
normalized size	1	0.	1.	11.18	1.73	2.29	0.98	0.
time (sec)	N/A	0.512	0.085	1.214	1.68	1.992	3.329	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	560	461	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.735	0.387	0.493	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	550	550	467	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.565	0.24	0.418	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	413	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.352	0.121	0.44	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	468	421	0	0	0	0	0
normalized size	1	1.61	1.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.317	0.102	0.474	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	518	518	487	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.602	0.187	0.457	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	596	596	479	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.583	0.309	0.455	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1046	1046	1240	0	0	0	0	0
normalized size	1	1.	1.19	0.	0.	0.	0.	0.
time (sec)	N/A	1.717	1.377	1.511	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	831	831	1105	0	0	0	0	0
normalized size	1	1.	1.33	0.	0.	0.	0.	0.
time (sec)	N/A	1.068	5.472	1.393	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	685	685	539	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.636	0.685	1.349	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	545	515	0	0	0	0	0
normalized size	1	1.36	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.518	0.308	1.362	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	800	800	625	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.98	0.94	1.414	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	995	995	721	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	1.291	0.825	1.398	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	84	97	128	0	0	0
normalized size	1	1.	1.83	2.11	2.78	0.	0.	0.
time (sec)	N/A	0.159	0.017	0.098	1.129	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	29	190	38	14	24
normalized size	1	1.	1.	1.45	9.5	1.9	0.7	1.2
time (sec)	N/A	0.058	0.096	0.059	1.195	1.982	0.391	1.154

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	76	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.168	0.014	0.531	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	110	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.248	0.038	1.102	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	559	0	0	0	0	0
normalized size	1	1.	3.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.25	0.456	5.443	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	28	44	34	63	20	35
normalized size	1	1.	1.12	1.76	1.36	2.52	0.8	1.4
time (sec)	N/A	0.012	0.005	0.188	1.177	1.937	0.342	1.257

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	63	0	153	0	0	0
normalized size	1	1.	0.94	0.	2.28	0.	0.	0.
time (sec)	N/A	0.074	0.014	0.655	1.179	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	91	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.025	0.549	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	40	38	93	26	39
normalized size	1	1.	1.	1.43	1.36	3.32	0.93	1.39
time (sec)	N/A	0.007	0.003	0.125	1.192	1.877	0.312	1.147

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	106	0	159	0	0	0
normalized size	1	1.	1.58	0.	2.37	0.	0.	0.
time (sec)	N/A	0.157	0.016	0.526	1.224	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	98	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	0.025	0.504	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	79	38	88	26	39
normalized size	1	1.	1.	2.82	1.36	3.14	0.93	1.39
time (sec)	N/A	0.007	0.003	0.224	1.241	1.962	0.364	1.263

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	106	0	159	0	0	0
normalized size	1	1.	1.58	0.	2.37	0.	0.	0.
time (sec)	N/A	0.155	0.014	0.94	1.145	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	102	98	0	0	0	0	0
normalized size	1	1.04	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	0.025	0.786	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	36	0	0	0	0
normalized size	1	1.	0.86	1.03	0.	0.	0.	0.
time (sec)	N/A	0.064	0.012	0.062	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	68	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.134	0.024	1.318	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	149	135	879	0	0	0	0
normalized size	1	1.06	0.96	6.28	0.	0.	0.	0.
time (sec)	N/A	0.18	0.042	0.065	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	96	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.164	0.027	2.268	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	206	1636	357	0	0	0	0
normalized size	1	1.01	8.02	1.75	0.	0.	0.	0.
time (sec)	N/A	0.254	0.469	0.063	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	334	1080	4733	0	0	0	0
normalized size	1	1.04	3.35	14.7	0.	0.	0.	0.
time (sec)	N/A	0.507	0.234	0.072	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	445	908	0	0	0	0	0
normalized size	1	1.03	2.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.594	0.799	2.336	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [42] had the largest ratio of [0.5484]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	20	10	0.95	32	0.312
2	A	16	10	0.92	32	0.312
3	A	12	7	1.	30	0.233
4	A	14	8	1.	32	0.25
5	A	18	11	1.09	32	0.344
6	A	22	11	1.06	32	0.344
7	A	4	3	1.	29	0.103
8	A	4	3	1.	29	0.103
9	A	4	3	1.	29	0.103
10	A	4	2	1.	27	0.074
11	A	6	6	1.	29	0.207
12	A	5	4	1.	29	0.138
13	A	4	3	1.	29	0.103

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
14	A	4	3	1.	29	0.103
15	A	4	3	1.	29	0.103
16	A	32	14	1.	31	0.452
17	A	28	14	1.	31	0.452
18	A	24	14	1.	31	0.452
19	A	20	13	1.	29	0.448
20	A	19	15	1.	31	0.484
21	A	20	12	1.	31	0.387
22	A	24	13	1.	31	0.419
23	A	28	13	1.	31	0.419
24	A	32	13	1.	31	0.419
25	A	5	2	1.	29	0.069
26	A	5	2	1.	29	0.069
27	A	5	2	1.	29	0.069
28	A	5	2	1.	27	0.074
29	A	3	3	1.	21	0.143
30	A	7	4	1.	29	0.138
31	A	7	3	1.	29	0.103
32	A	5	2	1.	29	0.069
33	A	5	2	1.	29	0.069
34	A	5	2	1.	29	0.069
35	A	49	14	0.99	31	0.452
36	A	47	15	1.01	31	0.484
37	A	39	15	1.03	29	0.517
38	A	10	9	1.	23	0.391
39	A	29	14	1.42	31	0.452
40	A	35	12	1.06	31	0.387
41	A	47	16	1.04	31	0.516
42	A	61	17	1.03	31	0.548
43	A	3	3	1.	40	0.075
44	A	3	3	1.	40	0.075
45	A	3	3	1.	40	0.075
46	A	2	2	1.	38	0.053
47	A	3	3	1.	40	0.075
48	A	3	3	1.	40	0.075
49	A	3	3	1.	40	0.075
50	A	1	1	1.	34	0.029
51	A	11	6	1.	48	0.125
52	A	9	5	1.	46	0.109
53	A	7	4	1.	32	0.125
54	A	0	0	0.	0	0.
55	A	0	0	0.	0	0.
56	A	13	6	1.	39	0.154
57	A	11	6	1.	39	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
58	A	9	5	1.	37	0.135
59	A	5	3	1.	25	0.12
60	A	0	0	0.	0	0.
61	A	0	0	0.	0	0.
62	A	0	0	0.	0	0.
63	A	0	0	0.	0	0.
64	A	5	5	1.	26	0.192
65	A	0	0	0.	0	0.
66	A	0	0	0.	0	0.
67	A	12	7	1.05	45	0.156
68	A	10	7	1.04	45	0.156
69	A	8	6	1.04	43	0.14
70	A	0	0	0.	0	0.
71	A	0	0	0.	0	0.
72	A	0	0	0.	0	0.
73	A	0	0	0.	0	0.
74	F	0	0	N/A	0	N/A
75	F	0	0	N/A	0	N/A
76	A	30	9	1.	32	0.281
77	A	27	10	1.	32	0.312
78	A	18	6	1.	30	0.2
79	A	18	6	1.61	29	0.207
80	A	29	11	1.	32	0.344
81	A	31	12	1.	32	0.375
82	A	37	14	1.	34	0.412
83	A	30	12	1.	34	0.353
84	A	21	9	1.	32	0.281
85	A	19	7	1.36	31	0.226
86	A	31	12	1.	34	0.353
87	A	40	16	1.	34	0.471
88	A	5	5	1.	19	0.263
89	A	1	1	1.	24	0.042
90	A	3	3	1.	34	0.088
91	A	3	3	1.	55	0.055
92	A	3	3	1.	58	0.052
93	A	4	4	1.	11	0.364
94	A	5	5	1.	13	0.385
95	A	7	7	1.	13	0.538
96	A	2	2	1.	13	0.154
97	A	6	6	1.	15	0.4
98	A	4	4	1.	15	0.267
99	A	2	2	1.	13	0.154
100	A	6	6	1.	15	0.4
101	A	4	4	1.04	15	0.267

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
102	A	1	1	1.	38	0.026
103	A	2	2	1.	50	0.04
104	A	3	3	1.06	42	0.071
105	A	2	2	1.	62	0.032
106	A	3	3	1.01	49	0.061
107	A	7	5	1.04	42	0.119
108	A	8	6	1.03	65	0.092

Chapter 3

Listing of integrals

3.1 $\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

Optimal. Leaf size=404

$$-3Bf^2gn\text{PolyLog}\left(2,-\frac{bx}{a}\right)+3Bf^2gn\text{PolyLog}\left(2,-\frac{dx}{c}\right)-\frac{1}{2}Bg^3n\log(x)\left(\frac{b^2}{a^2}-\frac{d^2}{c^2}\right)+\frac{b^2Bg^3n\log(a+bx)}{2a^2}+3f^2g\log$$

```
[Out] -(B*(b*c - a*d)*g^3*n)/(2*a*c*x) + A*f^3*x - (B*(b^2/a^2 - d^2/c^2)*g^3*n*L
og[x])/2 + (b^2*B*g^3*n*Log[a + b*x])/(2*a^2) - 3*B*f^2*g*n*Log[x]*Log[1 +
(b*x)/a] + (B*f^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^3*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/(2*x^2) + (3*(b*c - a*d)*f*g^2*(a + b*x)*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*(c + d*x)*(a - (c*(a + b*x))/(c
+ d*x))) + 3*f^2*g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c
- a*d)*f^3*n*Log[c + d*x])/(b*d) - (B*d^2*g^3*n*Log[c + d*x])/(2*c^2) + 3*B
*f^2*g*n*Log[x]*Log[1 + (d*x)/c] + (3*B*(b*c - a*d)*f*g^2*n*Log[a - (c*(a +
 b*x))/(c + d*x)])/(a*c) - 3*B*f^2*g*n*PolyLog[2, -((b*x)/a)] + 3*B*f^2*g*n
*PolyLog[2, -((d*x)/c)]
```

Rubi [A] time = 0.493055, antiderivative size = 385, normalized size of antiderivative = 0.95, number of steps used = 20, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2528, 2486, 31, 2525, 12, 72, 2524, 2357, 2317, 2391}

$$-3Bf^2gn\text{PolyLog}\left(2,-\frac{bx}{a}\right)+3Bf^2gn\text{PolyLog}\left(2,-\frac{dx}{c}\right)-\frac{1}{2}Bg^3n\log(x)\left(\frac{b^2}{a^2}-\frac{d^2}{c^2}\right)+\frac{b^2Bg^3n\log(a+bx)}{2a^2}+3f^2g\log$$

Antiderivative was successfully verified.

```
[In] Int[(f + g/x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -(B*(b*c - a*d)*g^3*n)/(2*a*c*x) + A*f^3*x + (3*B*(b*c - a*d)*f*g^2*n*Log[x
])/(a*c) - (B*(b^2/a^2 - d^2/c^2)*g^3*n*Log[x])/2 - (3*b*B*f*g^2*n*Log[a +
b*x])/a + (b^2*B*g^3*n*Log[a + b*x])/(2*a^2) - 3*B*f^2*g*n*Log[x]*Log[1 + (
b*x)/a] + (B*f^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^3*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(2*x^2) - (3*f*g^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/x + 3*f^2*g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
- (B*(b*c - a*d)*f^3*n*Log[c + d*x])/(b*d) + (3*B*d*f*g^2*n*Log[c + d*x])/c
 - (B*d^2*g^3*n*Log[c + d*x])/(2*c^2) + 3*B*f^2*g*n*Log[x]*Log[1 + (d*x)/c]
```

```
- 3*B*f^2*g*n*PolyLog[2, -((b*x)/a)] + 3*B*f^2*g*n*PolyLog[2, -((d*x)/c)]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
```

```
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \left(f+\frac{g}{x}\right)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \int \left(f^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x^3}+\frac{3fg^2\left(\right.}{}\right.$$

$$=f^3\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx+\left(3f^2g\right)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x}\, d$$

$$=Af^3x-\frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2}-\frac{3fg^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x}+3f$$

$$=Af^3x+\frac{Bf^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2}-\frac{3fg}{}$$

$$=Af^3x+\frac{Bf^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2x^2}-\frac{3fg}{}$$

$$=-\frac{B(bc-ad)g^3n}{2acx}+Af^3x+\frac{3B(bc-ad)fg^2n\log(x)}{ac}-\frac{B(bc-ad)(bc+ad}{2a^2c^2}$$

$$=-\frac{B(bc-ad)g^3n}{2acx}+Af^3x+\frac{3B(bc-ad)fg^2n\log(x)}{ac}-\frac{B(bc-ad)(bc+ad}{2a^2c^2}$$

Mathematica [A] time = 0.413278, size = 336, normalized size = 0.83

$$-3Bf^2gn\left(\text{PolyLog}\left(2,-\frac{bx}{a}\right)-\text{PolyLog}\left(2,-\frac{dx}{c}\right)+\log(x)\left(\log\left(\frac{bx}{a}+1\right)-\log\left(\frac{dx}{c}+1\right)\right)\right)+\frac{Bg^3n\left(\log(x)\left(a^2d^2x-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g/x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f^3*x + (B*f^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^3*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(2*x^2) - (3*f*g^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/x + 3*f^2*g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
- (B*(b*c - a*d)*f^3*n*Log[c + d*x])/(b*d) + (3*B*f*g^2*n*((b*c - a*d)*Log[
x] - b*c*Log[a + b*x] + a*d*Log[c + d*x]))/(a*c) + (B*g^3*n*((-(b^2*c^2*x)
+ a^2*d^2*x)*Log[x] + b^2*c^2*x*Log[a + b*x] + a*(-(b*c^2) + a*c*d - a*d^2*
x*Log[c + d*x])))/(2*a^2*c^2*x) - 3*B*f^2*g*n*(Log[x]*(Log[1 + (b*x)/a] - L
og[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)] - PolyLog[2, -((d*x)/c)])
```

Maple [F] time = 0.154, size = 0, normalized size = 0.

$$\int \left(f+\frac{g}{x}\right)^{3} \left(A+B\ln \left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/x)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((f+g/x)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$Bf^{3}n\left(\frac{a\log\left(bx+a\right)}{b}-\frac{c\log\left(dx+c\right)}{d}\right)-3\,Bfg^{2}n\left(\frac{b\log\left(bx+a\right)}{a}-\frac{d\log\left(dx+c\right)}{c}-\frac{\left(bc-ad\right)\log\left(x\right)}{ac}\right)+\frac{1}{2}\,Bg^{3}n\left(\frac{b^{2}\log\left(bx\right.}{a^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] B*f^3*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) - 3*B*f*g^2*n*(b*log(b*x + a)
/a - d*log(d*x + c)/c - (b*c - a*d)*log(x)/(a*c)) + 1/2*B*g^3*n*(b^2*log(b*
x + a)/a^2 - d^2*log(d*x + c)/c^2 - (b*c - a*d)/(a*c*x) - (b^2*c^2 - a^2*d^
2)*log(x)/(a^2*c^2)) + B*f^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f
^3*x - 3*B*f^2*g*integrate(-(log((b*x + a)^n) - log((d*x + c)^n) + log(e))/
x, x) + 3*A*f^2*g*log(x) - 3*B*f*g^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
/x - 3*A*f*g^2/x - 1/2*B*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/x^2 - 1
/2*A*g^3/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{Af^{3}x^{3}+3\,Af^{2}gx^{2}+3\,Afg^{2}x+Ag^{3}+\left(Bf^{3}x^{3}+3\,Bf^{2}gx^{2}+3\,Bfg^{2}x+Bg^{3}\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] integral((A*f^3*x^3 + 3*A*f^2*g*x^2 + 3*A*f*g^2*x + A*g^3 + (B*f^3*x^3 + 3*
B*f^2*g*x^2 + 3*B*f*g^2*x + B*g^3)*log(e*((b*x + a)/(d*x + c))^n))/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)\left(f+\frac{g}{x}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(f + g/x)^3, x)
```

3.2 $\int\left(f+\frac{g}{x}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

Optimal. Leaf size=263

$$-2Bfgn\,\text{PolyLog}\left(2,-\frac{bx}{a}\right)+2Bfgn\,\text{PolyLog}\left(2,-\frac{dx}{c}\right)+2fg\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{g^2(a+bx)(bc-ad)\left(B\right.}{a(c+dx)\left(a\right.}$$

```
[Out] A*f^2*x - 2*B*f*g*n*Log[x]*Log[1 + (b*x)/a] + (B*f^2*(a + b*x)*Log[e*((a +
b*x)/(c + d*x))^n])/b + ((b*c - a*d)*g^2*(a + b*x)*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(a*(c + d*x)*(a - (c*(a + b*x))/(c + d*x))) + 2*f*g*Log[x]*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*f^2*n*Log[c + d*x])
/(b*d) + 2*B*f*g*n*Log[x]*Log[1 + (d*x)/c] + (B*(b*c - a*d)*g^2*n*Log[a - (
c*(a + b*x))/(c + d*x)])/(a*c) - 2*B*f*g*n*PolyLog[2, -((b*x)/a)] + 2*B*f*g
*n*PolyLog[2, -((d*x)/c)]
```

Rubi [A] time = 0.337362, antiderivative size = 242, normalized size of antiderivative = 0.92, number of steps used = 16, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2528, 2486, 31, 2525, 12, 72, 2524, 2357, 2317, 2391}

$$-2Bfgn\,\text{PolyLog}\left(2,-\frac{bx}{a}\right)+2Bfgn\,\text{PolyLog}\left(2,-\frac{dx}{c}\right)+2fg\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{g^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g/x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f^2*x + (B*(b*c - a*d)*g^2*n*Log[x])/(a*c) - (b*B*g^2*n*Log[a + b*x])/a -
 2*B*f*g*n*Log[x]*Log[1 + (b*x)/a] + (B*f^2*(a + b*x)*Log[e*((a + b*x)/(c +
 d*x))^n])/b - (g^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/x + 2*f*g*Log[x
]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*f^2*n*Log[c + d*x
])/(b*d) + (B*d*g^2*n*Log[c + d*x])/c + 2*B*f*g*n*Log[x]*Log[1 + (d*x)/c] -
 2*B*f*g*n*PolyLog[2, -((b*x)/a)] + 2*B*f*g*n*PolyLog[2, -((d*x)/c)]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \left(f+\frac{g}{x}\right)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \int \left(f^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) + \frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x^2} + \frac{2fg\left(A+\right.}{}\right.$$

$$= f^2\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx + (2fg)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x} dx + g$$

$$= Af^2x - \frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x} + 2fg\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)$$

$$= Af^2x + \frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x} + 2fg\log$$

$$= Af^2x + \frac{Bf^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b} - \frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x} + 2fg\log$$

$$= Af^2x + \frac{B(bc-ad)g^2n\log(x)}{ac} - \frac{bBg^2n\log(a+bx)}{a} - 2Bfgn\log(x)\log\left(1+\right.$$

$$= Af^2x + \frac{B(bc-ad)g^2n\log(x)}{ac} - \frac{bBg^2n\log(a+bx)}{a} - 2Bfgn\log(x)\log\left(1+\right.$$

Mathematica [A] time = 0.230476, size = 217, normalized size = 0.83

$$-2Bfgn\left(\text{PolyLog}\left(2,-\frac{bx}{a}\right)-\text{PolyLog}\left(2,-\frac{dx}{c}\right)+\log(x)\left(\log\left(\frac{bx}{a}+1\right)-\log\left(\frac{dx}{c}+1\right)\right)\right)+2fg\log(x)\left(B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g/x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f^2*x + (B*f^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/x + 2*f*g*Log[x]*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]) - (B*(b*c - a*d)*f^2*n*Log[c + d*x])/(b*d) + (B*g^2*n*((b*c -
 a*d)*Log[x] - b*c*Log[a + b*x] + a*d*Log[c + d*x]))/(a*c) - 2*B*f*g*n*(Log
[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)] - PolyLo
g[2, -((d*x)/c)])
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \left(f+\frac{g}{x}\right)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/x)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((f+g/x)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maximu

Maxima [F] time = 0., size = 0, normalized size = 0.

$$Bf^2n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)-Bg^2n\left(\frac{b\log(bx+a)}{a}-\frac{d\log(dx+c)}{c}-\frac{(bc-ad)\log(x)}{ac}\right)+Bf^2x\log\left(e\left(\frac{bx}{dx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] B*f^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) - B*g^2*n*(b*log(b*x + a)/a -
 d*log(d*x + c)/c - (b*c - a*d)*log(x)/(a*c)) + B*f^2*x*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n) + A*f^2*x - 2*B*f*g*integrate(-(log((b*x + a)^n) - log(
(d*x + c)^n) + log(e))/x, x) + 2*A*f*g*log(x) - B*g^2*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/x - A*g^2/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Af^2x^2+2\,Afgx+Ag^2+\left(Bf^2x^2+2\,Bfgx+Bg^2\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] integral((A*f^2*x^2 + 2*A*f*g*x + A*g^2 + (B*f^2*x^2 + 2*B*f*g*x + B*g^2)*l
og(e*((b*x + a)/(d*x + c))^n))/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)\left(f+\frac{g}{x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(f + g/x)^2, x)
```

3.3 $\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$

Optimal. Leaf size=143

$$-Bgn\text{PolyLog}\left(2,-\frac{bx}{a}\right)+Bgn\text{PolyLog}\left(2,-\frac{dx}{c}\right)+g\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\ldots$$

```
[Out] A*f*x - B*g*n*Log[x]*Log[1 + (b*x)/a] + (B*f*(a + b*x)*Log[e*((a + b*x)/(c
+ d*x))^n])/b + g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c -
 a*d)*f*n*Log[c + d*x])/(b*d) + B*g*n*Log[x]*Log[1 + (d*x)/c] - B*g*n*PolyL
og[2, -((b*x)/a)] + B*g*n*PolyLog[2, -((d*x)/c)]
```

Rubi [A] time = 0.224888, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2528, 2486, 31, 2524, 2357, 2317, 2391}

$$-Bgn\text{PolyLog}\left(2,-\frac{bx}{a}\right)+Bgn\text{PolyLog}\left(2,-\frac{dx}{c}\right)+g\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(f + g/x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f*x - B*g*n*Log[x]*Log[1 + (b*x)/a] + (B*f*(a + b*x)*Log[e*((a + b*x)/(c
+ d*x))^n])/b + g*Log[x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c -
 a*d)*f*n*Log[c + d*x])/(b*d) + B*g*n*Log[x]*Log[1 + (d*x)/c] - B*g*n*PolyL
og[2, -((b*x)/a)] + B*g*n*PolyLog[2, -((d*x)/c)]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
```

```
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \left(f+\frac{g}{x}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx=\int \left(f\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{x}\right)dx$$

$$=f\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx+g\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x}dx$$

$$=Afx+g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+(Bf)\int \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)dx$$

$$=Afx+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}+g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-$$

$$=Afx+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}+g\log(x)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-$$

$$=Afx-Bgn\log(x)\log\left(1+\frac{bx}{a}\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}+g\log(x)($$

$$=Afx-Bgn\log(x)\log\left(1+\frac{bx}{a}\right)+\frac{Bf(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}+g\log(x)($$

Mathematica [A] time = 0.0960526, size = 135, normalized size = 0.94

$$-Bgn\left(\text{PolyLog}\left(2,-\frac{bx}{a}\right)-\text{PolyLog}\left(2,-\frac{dx}{c}\right)+\log(x)\left(\log\left(\frac{bx}{a}+1\right)-\log\left(\frac{dx}{c}+1\right)\right)\right)+g\log(x)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g/x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] A*f*x + (B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + g*Log[x]*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*f*n*Log[c + d*x])/(b*d) -
B*g*n*(Log[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)
] - PolyLog[2, -((d*x)/c)])
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int \left(f+\frac{g}{x}\right)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f+g/x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

```
[Out] int((f+g/x)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$Bfn\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)+Bfx\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^{n}\right)+Afx-Bg\int-\frac{\log\left((bx+a)^{n}\right)-\log\left((dx+c)^{n}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")
```

```
[Out] B*f*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*f*x*log(e*(b*x/(d*x + c) +
a/(d*x + c))^n) + A*f*x - B*g*integrate(-(log((b*x + a)^n) - log((d*x + c)^
n) + log(e))/x, x) + A*g*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Afx+Ag+\left(Bfx+Bg\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] integral((A*f*x + A*g + (B*f*x + B*g)*log(e*((b*x + a)/(d*x + c))^n))/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)\left(f+\frac{g}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f+g/x)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(f + g/x), x)
```

3.4 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}}\,dx$

Optimal. Leaf size=217

$$\frac{BgnPolyLog\left(2,-\frac{b(fx+g)}{af-bg}\right)}{f^2}-\frac{BgnPolyLog\left(2,-\frac{d(fx+g)}{cf-dg}\right)}{f^2}-\frac{g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^2}+\frac{B(a+bx)\log\left(e\left(\frac{a+b}{c+d}\right.\right.}{bf}$$

```
[Out] (A*x)/f + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(b*f) - (B*(b*c - a*
d)*n*Log[c + d*x])/(b*d*f) + (B*g*n*Log[(f*(a + b*x))/(a*f - b*g)]*Log[g +
f*x])/f^2 - (g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x])/f^2 - (
B*g*n*Log[(f*(c + d*x))/(c*f - d*g)]*Log[g + f*x])/f^2 + (B*g*n*PolyLog[2,
-((b*(g + f*x))/(a*f - b*g))])/f^2 - (B*g*n*PolyLog[2, -((d*(g + f*x))/(c*f
 - d*g))])/f^2
```

Rubi [A] time = 0.33225, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2528, 2486, 31, 2524, 2418, 2394, 2393, 2391}

$$\frac{BgnPolyLog\left(2,-\frac{b(fx+g)}{af-bg}\right)}{f^2}-\frac{BgnPolyLog\left(2,-\frac{d(fx+g)}{cf-dg}\right)}{f^2}-\frac{g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^2}+\frac{B(a+bx)\log\left(e\left(\frac{a+b}{c+d}\right.\right.}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x),x]
```

```
[Out] (A*x)/f + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(b*f) - (B*(b*c - a*
d)*n*Log[c + d*x])/(b*d*f) + (B*g*n*Log[(f*(a + b*x))/(a*f - b*g)]*Log[g +
f*x])/f^2 - (g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x])/f^2 - (
B*g*n*Log[(f*(c + d*x))/(c*f - d*g)]*Log[g + f*x])/f^2 + (B*g*n*PolyLog[2,
-((b*(g + f*x))/(a*f - b*g))])/f^2 - (B*g*n*PolyLog[2, -((d*(g + f*x))/(c*f
 - d*g))])/f^2
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+\frac{g}{x}}\,dx = \int \left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f} - \frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f(g+fx)}\right)dx$$

$$= \frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{f} - \frac{g\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g+fx}dx}{f}$$

$$= \frac{Ax}{f} - \frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^2} + \frac{B\int \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)dx}{f} + \frac{(Bgn)\int \frac{(c+dx)\left(-\frac{d(\ldots}{(c\ldots}\right.}{\ldots}}{\ldots}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^2} - \frac{(B(bc-ad)n)\ldots}{bf}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n\log(c+dx)}{bdf} - \frac{g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\ldots}{f^2}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n\log(c+dx)}{bdf} + \frac{Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx\ldots}{f^2}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n\log(c+dx)}{bdf} + \frac{Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx\ldots}{f^2}$$

$$= \frac{Ax}{f} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf} - \frac{B(bc-ad)n\log(c+dx)}{bdf} + \frac{Bgn\log\left(\frac{f(a+bx)}{af-bg}\right)\log(g+fx\ldots}{f^2}$$

Mathematica [A] time = 0.148163, size = 185, normalized size = 0.85

$$\frac{Bgn\left(\text{PolyLog}\left(2,\frac{b(fx+g)}{bg-af}\right)-\text{PolyLog}\left(2,\frac{d(fx+g)}{dg-cf}\right)+\log(fx+g)\left(\log\left(\frac{f(a+bx)}{af-bg}\right)-\log\left(\frac{f(c+dx)}{cf-dg}\right)\right)\right)-g\log(fx+g)\left(B\log\left(e\ldots\right.\right.}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x),x]
```

```
[Out] (A*f*x + (B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (B*(b*c - a*d)*
f*n*Log[c + d*x])/(b*d) - g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g +
f*x] + B*g*n*((Log[(f*(a + b*x))/(a*f - b*g)] - Log[(f*(c + d*x))/(c*f - d*
g)])*Log[g + f*x] + PolyLog[2, (b*(g + f*x))/(-(a*f) + b*g)] - PolyLog[2, (
d*(g + f*x))/(-(c*f) + d*g)]))/f^2
```

Maple [F] time = 0.199, size = 0, normalized size = 0.

$$\int \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)\left(f+\frac{g}{x}\right)^{-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$A\left(\frac{x}{f} - \frac{g\log\left(fx+g\right)}{f^2}\right) - B\int -\frac{x\log\left((bx+a)^n\right) - x\log\left((dx+c)^n\right) + x\log\left(e\right)}{fx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x, algorithm="maxima")
```

```
[Out] A*(x/f - g*log(f*x + g)/f^2) - B*integrate(-(x*log((b*x + a)^n) - x*log((d*
x + c)^n) + x*log(e))/(f*x + g), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Bx\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + Ax}{fx+g}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x, algorithm="fricas")
```

```
[Out] integral((B*x*log(e*((b*x + a)/(d*x + c))^n) + A*x)/(f*x + g), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(f+g/x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{f + \frac{g}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x),x, algorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(f + g/x), x)
```

3.5 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2}\,dx$

Optimal. Leaf size=322

$$\frac{2BgnPolyLog\left(2,-\frac{b(fx+g)}{af-bg}\right)}{f^3}-\frac{2BgnPolyLog\left(2,-\frac{d(fx+g)}{cf-dg}\right)}{f^3}-\frac{g^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^2(fx+g)(af-bg)}-\frac{2g\log(fx+g)\left(B\log\right.}{f^3}$$

```
[Out] (A*x)/f^2 + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(b*f^2) - (g^2*(a
+ b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(f^2*(a*f - b*g)*(g + f*x))
- (B*(b*c - a*d)*n*Log[c + d*x])/(b*d*f^2) + (2*B*g*n*Log[(f*(a + b*x))/(a*
f - b*g)]*Log[g + f*x])/f^3 - (2*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*L
og[g + f*x])/f^3 - (2*B*g*n*Log[(f*(c + d*x))/(c*f - d*g)]*Log[g + f*x])/f^
3 + (B*(b*c - a*d)*g^2*n*Log[(g + f*x)/(c + d*x)])/(f^2*(a*f - b*g)*(c*f -
d*g)) + (2*B*g*n*PolyLog[2, -((b*(g + f*x))/(a*f - b*g))])/f^3 - (2*B*g*n*P
olyLog[2, -((d*(g + f*x))/(c*f - d*g))])/f^3
```

Rubi [A] time = 0.493759, antiderivative size = 352, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2528, 2486, 31, 2525, 12, 72, 2524, 2418, 2394, 2393, 2391}

$$\frac{2BgnPolyLog\left(2,-\frac{b(fx+g)}{af-bg}\right)}{f^3}-\frac{2BgnPolyLog\left(2,-\frac{d(fx+g)}{cf-dg}\right)}{f^3}-\frac{g^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{f^3(fx+g)}-\frac{2g\log(fx+g)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^2,x]
```

```
[Out] (A*x)/f^2 - (b*B*g^2*n*Log[a + b*x])/(f^3*(a*f - b*g)) + (B*(a + b*x)*Log[e
*((a + b*x)/(c + d*x))^n])/(b*f^2) - (g^2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(f^3*(g + f*x)) - (B*(b*c - a*d)*n*Log[c + d*x])/(b*d*f^2) + (B*d*g^
2*n*Log[c + d*x])/(f^3*(c*f - d*g)) + (B*(b*c - a*d)*g^2*n*Log[g + f*x])/(f
^2*(a*f - b*g)*(c*f - d*g)) + (2*B*g*n*Log[(f*(a + b*x))/(a*f - b*g)]*Log[g
 + f*x])/f^3 - (2*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x])/f^
3 - (2*B*g*n*Log[(f*(c + d*x))/(c*f - d*g)]*Log[g + f*x])/f^3 + (2*B*g*n*Po
lyLog[2, -((b*(g + f*x))/(a*f - b*g))])/f^3 - (2*B*g*n*PolyLog[2, -((d*(g +
 f*x))/(c*f - d*g))])/f^3
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^2}dx = \int \left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f^2} + \frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^2(g+fx)^2} - \frac{2g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^2(g+fx)}\right)dx$$

$$= \frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{f^2} - \frac{(2g)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g+fx}dx}{f^2} + \frac{g^2\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(g+fx)^2}dx}{f^2}$$

$$= \frac{Ax}{f^2} - \frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)} - \frac{2g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(g+fx)}{f^3} + \frac{B\int\log\left(e\left(\right.\right.}{f^{\cdots}}$$

$$= \frac{Ax}{f^2} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)} - \frac{2g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{f^3}$$

$$= \frac{Ax}{f^2} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)} - \frac{B(bc-ad)n\log(c+dx)}{bdf^2} - $$

$$= \frac{Ax}{f^2} - \frac{bBg^2n\log(a+bx)}{f^3(af-bg)} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)} - B(bc$$

$$= \frac{Ax}{f^2} - \frac{bBg^2n\log(a+bx)}{f^3(af-bg)} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)} - B(bc$$

$$= \frac{Ax}{f^2} - \frac{bBg^2n\log(a+bx)}{f^3(af-bg)} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^2} - \frac{g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)} - B(bc$$

Mathematica [A] time = 0.593835, size = 295, normalized size = 0.92

$$\frac{2Bgn\left(\text{PolyLog}\left(2,\frac{b(fx+g)}{bg-af}\right) - \text{PolyLog}\left(2,\frac{d(fx+g)}{dg-cf}\right) + \log(fx+g)\left(\log\left(\frac{f(a+bx)}{af-bg}\right) - \log\left(\frac{f(c+dx)}{cf-dg}\right)\right)\right) - \frac{g^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{fx+g} - }{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^2,x]
```

```
[Out] (A*f*x + (B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b - (g^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]))/(g + f*x) - (B*(b*c - a*d)*f*n*Log[c + d*x])/
(b*d) - 2*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x] + (B*g^2*n*
(b*(-(c*f) + d*g)*Log[a + b*x] + d*(a*f - b*g)*Log[c + d*x] + (b*c - a*d)*f
*Log[g + f*x]))/((a*f - b*g)*(c*f - d*g)) + 2*B*g*n*((Log[(f*(a + b*x))/(a*
f - b*g)] - Log[(f*(c + d*x))/(c*f - d*g)])*Log[g + f*x] + PolyLog[2, (b*(g
 + f*x))/(-(a*f) + b*g)] - PolyLog[2, (d*(g + f*x))/(-(c*f) + d*g)]))/f^3
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)\left(f+\frac{g}{x}\right)^{-2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-A\left(\frac{g^2}{f^4x+f^3g}-\frac{x}{f^2}+\frac{2g\log\left(fx+g\right)}{f^3}\right)-B\int -\frac{x^2\log\left((bx+a)^n\right)-x^2\log\left((dx+c)^n\right)+x^2\log\left(e\right)}{f^2x^2+2fgx+g^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x, algorithm="maxima")
```

```
[Out] -A*(g^2/(f^4*x + f^3*g) - x/f^2 + 2*g*log(f*x + g)/f^3) - B*integrate(-(x^2
*log((b*x + a)^n) - x^2*log((d*x + c)^n) + x^2*log(e))/(f^2*x^2 + 2*f*g*x +
 g^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^2\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+Ax^2}{f^2x^2+2fgx+g^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x, algorithm="fricas")
```

```
[Out] integral((B*x^2*log(e*((b*x + a)/(d*x + c))^n) + A*x^2)/(f^2*x^2 + 2*f*g*x
+ g^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(f+g/x)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{\left(f+\frac{g}{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^2,x, algorithm="giac")

[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(f + g/x)^2, x)
```

3.6 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3} dx$

Optimal. Leaf size=531

$$\frac{3BgnPolyLog\left(2,-\frac{b(fx+g)}{af-bg}\right)}{f^4}-\frac{3BgnPolyLog\left(2,-\frac{d(fx+g)}{cf-dg}\right)}{f^4}+\frac{g^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2f^4(fx+g)^2}-\frac{3g^2(a+bx)\left(B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{f^3(fx+g)(af-b}$$

```
[Out] (A*x)/f^3 + (B*(b*c - a*d)*g^3*n)/(2*f^3*(a*f - b*g)*(c*f - d*g)*(g + f*x))
 - (b^2*B*g^3*n*Log[a + b*x])/(2*f^4*(a*f - b*g)^2) + (B*(a + b*x)*Log[e*((
a + b*x)/(c + d*x))^n])/(b*f^3) + (g^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]))/(2*f^4*(g + f*x)^2) - (3*g^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(f^3*(a*f - b*g)*(g + f*x)) - (B*(b*c - a*d)*n*Log[c + d*x])/(b*d*f
^3) + (B*d^2*g^3*n*Log[c + d*x])/(2*f^4*(c*f - d*g)^2) + (B*(b*c - a*d)*g^3
*(b*c*f + a*d*f - 2*b*d*g)*n*Log[g + f*x])/(2*f^3*(a*f - b*g)^2*(c*f - d*g)
^2) + (3*B*g*n*Log[(f*(a + b*x))/(a*f - b*g)]*Log[g + f*x])/f^4 - (3*g*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x])/f^4 - (3*B*g*n*Log[(f*(c +
 d*x))/(c*f - d*g)]*Log[g + f*x])/f^4 + (3*B*(b*c - a*d)*g^2*n*Log[(g + f*x
)/(c + d*x)])/(f^3*(a*f - b*g)*(c*f - d*g)) + (3*B*g*n*PolyLog[2, -((b*(g +
 f*x))/(a*f - b*g))])/f^4 - (3*B*g*n*PolyLog[2, -((d*(g + f*x))/(c*f - d*g)
)])/f^4
```

Rubi [A] time = 0.796941, antiderivative size = 562, normalized size of antiderivative = 1.06, number of steps used = 22, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2528, 2486, 31, 2525, 12, 72, 2524, 2418, 2394, 2393, 2391}

$$\frac{3BgnPolyLog\left(2,-\frac{b(fx+g)}{af-bg}\right)}{f^4}-\frac{3BgnPolyLog\left(2,-\frac{d(fx+g)}{cf-dg}\right)}{f^4}+\frac{g^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2f^4(fx+g)^2}-\frac{3g^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right.}{f^4(fx+g)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^3,x]
```

```
[Out] (A*x)/f^3 + (B*(b*c - a*d)*g^3*n)/(2*f^3*(a*f - b*g)*(c*f - d*g)*(g + f*x))
 - (b^2*B*g^3*n*Log[a + b*x])/(2*f^4*(a*f - b*g)^2) - (3*b*B*g^2*n*Log[a +
b*x])/(f^4*(a*f - b*g)) + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(b*f
^3) + (g^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*f^4*(g + f*x)^2) - (3
*g^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(f^4*(g + f*x)) - (B*(b*c - a*
d)*n*Log[c + d*x])/(b*d*f^3) + (B*d^2*g^3*n*Log[c + d*x])/(2*f^4*(c*f - d*g
)^2) + (3*B*d*g^2*n*Log[c + d*x])/(f^4*(c*f - d*g)) + (3*B*(b*c - a*d)*g^2*
n*Log[g + f*x])/(f^3*(a*f - b*g)*(c*f - d*g)) + (B*(b*c - a*d)*g^3*(b*c*f +
 a*d*f - 2*b*d*g)*n*Log[g + f*x])/(2*f^3*(a*f - b*g)^2*(c*f - d*g)^2) + (3*
B*g*n*Log[(f*(a + b*x))/(a*f - b*g)]*Log[g + f*x])/f^4 - (3*g*(A + B*Log[e*
((a + b*x)/(c + d*x))^n])*Log[g + f*x])/f^4 - (3*B*g*n*Log[(f*(c + d*x))/(c
*f - d*g)]*Log[g + f*x])/f^4 + (3*B*g*n*PolyLog[2, -((b*(g + f*x))/(a*f - b
*g))])/f^4 - (3*B*g*n*PolyLog[2, -((d*(g + f*x))/(c*f - d*g))])/f^4
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
```

```
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(f+\frac{g}{x}\right)^3}\,dx = \int \left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f^3} - \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)^3} + \frac{3g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^3(g+fx)^2}\right.$$

$$= \frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{f^3} - \frac{(3g)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g+fx}dx}{f^3} + \frac{\left(3g^2\right)\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(g+fx)^2}d}{f^3}$$

$$= \frac{Ax}{f^3} + \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2} - \frac{3g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{f^4(g+fx)} - \frac{3g\left(A+B\log\left(e\left(\frac{a}{c}\right.\right.\right.}{f}$$

$$= \frac{Ax}{f^3} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^3} + \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2} - \frac{3g^2\left(A+B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{f^4(g+fx)}$$

$$= \frac{Ax}{f^3} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bf^3} + \frac{g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2f^4(g+fx)^2} - \frac{3g^2\left(A+B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{f^4(g+fx)}$$

$$= \frac{Ax}{f^3} + \frac{B(bc-ad)g^3n}{2f^3(af-bg)(cf-dg)(g+fx)} - \frac{b^2Bg^3n\log(a+bx)}{2f^4(af-bg)^2} - \frac{3bBg^2n\log(a+bx)}{f^4(af-bg)} + \frac{B}{}$$

$$= \frac{Ax}{f^3} + \frac{B(bc-ad)g^3n}{2f^3(af-bg)(cf-dg)(g+fx)} - \frac{b^2Bg^3n\log(a+bx)}{2f^4(af-bg)^2} - \frac{3bBg^2n\log(a+bx)}{f^4(af-bg)} + \frac{B}{}$$

$$= \frac{Ax}{f^3} + \frac{B(bc-ad)g^3n}{2f^3(af-bg)(cf-dg)(g+fx)} - \frac{b^2Bg^3n\log(a+bx)}{2f^4(af-bg)^2} - \frac{3bBg^2n\log(a+bx)}{f^4(af-bg)} + \frac{B}{}$$

Mathematica [A] time = 1.65344, size = 470, normalized size = 0.89

$$6Bgn\left(\text{PolyLog}\left(2,\frac{b(fx+g)}{bg-af}\right)-\text{PolyLog}\left(2,\frac{d(fx+g)}{dg-cf}\right)+\log(fx+g)\left(\log\left(\frac{f(a+bx)}{af-bg}\right)-\log\left(\frac{f(c+dx)}{cf-dg}\right)\right)\right)+\frac{g^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right.}{(fx+g)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g/x)^3,x]
```

```
[Out] (2*A*f*x + (2*B*f*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + (g^3*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/(g + f*x)^2 - (6*g^2*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(g + f*x) - (2*B*(b*c - a*d)*f*n*Log[c + d*x])/(b*d) -
```

```
6*g*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[g + f*x] + (6*B*g^2*n*(b*(-(
c*f) + d*g)*Log[a + b*x] + d*(a*f - b*g)*Log[c + d*x] + (b*c - a*d)*f*Log[g
 + f*x]))/((a*f - b*g)*(c*f - d*g)) + B*(b*c - a*d)*g^3*n*(-((b^2*Log[a + b
*x])/((b*c - a*d)*(a*f - b*g)^2)) + ((d^2*Log[c + d*x])/(b*c - a*d) + (f*((
(a*f - b*g)*(c*f - d*g))/(g + f*x) + (b*c*f + a*d*f - 2*b*d*g)*Log[g + f*x]
))/(a*f - b*g)^2)/(c*f - d*g)^2) + 6*B*g*n*((Log[(f*(a + b*x))/(a*f - b*g)]
 - Log[(f*(c + d*x))/(c*f - d*g)])*Log[g + f*x] + PolyLog[2, (b*(g + f*x))/
(-(a*f) + b*g)] - PolyLog[2, (d*(g + f*x))/(-(c*f) + d*g)]))/(2*f^4)
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)\left(f+\frac{g}{x}\right)^{-3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}A\left(\frac{6fg^2x+5g^3}{f^6x^2+2f^5gx+f^4g^2}-\frac{2x}{f^3}+\frac{6g\log\left(fx+g\right)}{f^4}\right)-B\int -\frac{x^3\log\left((bx+a)^n\right)-x^3\log\left((dx+c)^n\right)+x^3\log\left(e\right)}{f^3x^3+3f^2gx^2+3fg^2x+g^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*A*((6*f*g^2*x + 5*g^3)/(f^6*x^2 + 2*f^5*g*x + f^4*g^2) - 2*x/f^3 + 6*g
*log(f*x + g)/f^4) - B*integrate(-(x^3*log((b*x + a)^n) - x^3*log((d*x + c)
^n) + x^3*log(e))/(f^3*x^3 + 3*f^2*g*x^2 + 3*f*g^2*x + g^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{Bx^3\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+Ax^3}{f^3x^3+3f^2gx^2+3fg^2x+g^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x, algorithm="fricas")
```

```
[Out] integral((B*x^3*log(e*((b*x + a)/(d*x + c))^n) + A*x^3)/(f^3*x^3 + 3*f^2*g*
x^2 + 3*f*g^2*x + g^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(f+g/x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A}{\left(f+\frac{g}{x}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(f+g/x)^3,x, algorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(f + g/x)^3, x)
```

3.7 $\int (a+bx)^4 \log \left(e \left(f (a+bx)^p (c+dx)^q\right)^r\right) \, dx$

Optimal. Leaf size=201

$$-\frac{qrx(bc-ad)^4}{5d^4}+\frac{qr(a+bx)^2(bc-ad)^3}{10bd^3}-\frac{qr(a+bx)^3(bc-ad)^2}{15bd^2}+\frac{qr(bc-ad)^5\log(c+dx)}{5bd^5}+\frac{(a+bx)^5\log\left(e\left(f(a+bx)^p\right.\right.}{5b}$$

```
[Out] -((b*c - a*d)^4*q*r*x)/(5*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)^2)/(10*b*d^3)
 - ((b*c - a*d)^2*q*r*(a + b*x)^3)/(15*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)^
4)/(20*b*d) - (p*r*(a + b*x)^5)/(25*b) - (q*r*(a + b*x)^5)/(25*b) + ((b*c -
 a*d)^5*q*r*Log[c + d*x])/(5*b*d^5) + ((a + b*x)^5*Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r])/(5*b)
```

Rubi [A] time = 0.0949281, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2495, 32, 43}

$$-\frac{qrx(bc-ad)^4}{5d^4}+\frac{qr(a+bx)^2(bc-ad)^3}{10bd^3}-\frac{qr(a+bx)^3(bc-ad)^2}{15bd^2}+\frac{qr(bc-ad)^5\log(c+dx)}{5bd^5}+\frac{(a+bx)^5\log\left(e\left(f(a+bx)^p\right.\right.}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -((b*c - a*d)^4*q*r*x)/(5*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)^2)/(10*b*d^3)
 - ((b*c - a*d)^2*q*r*(a + b*x)^3)/(15*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)^
4)/(20*b*d) - (p*r*(a + b*x)^5)/(25*b) - (q*r*(a + b*x)^5)/(25*b) + ((b*c -
 a*d)^5*q*r*Log[c + d*x])/(5*b*d^5) + ((a + b*x)^5*Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r])/(5*b)
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \frac{(a+bx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5b} - \frac{1}{5}(pr)\int (a+bx)^4\,dx - \frac{(dqr)}{}$$

$$= -\frac{pr(a+bx)^5}{25b} + \frac{(a+bx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5b} - \frac{(dqr)\int\left(\frac{b(bc}{}\right.}{}$$

$$= -\frac{(bc-ad)^4 qrx}{5d^4} + \frac{(bc-ad)^3 qr(a+bx)^2}{10bd^3} - \frac{(bc-ad)^2 qr(a+bx)^3}{15bd^2} + \frac{(bc}{}$$

Mathematica [A] time = 0.308289, size = 185, normalized size = 0.92

$$\frac{(a+bx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - \frac{r\left(-60b^2(2p+5q)(c+dx)^2(bc-ad)^3+40b^3(3p+5q)(c+dx)^3(bc-ad)^2-15b^4(4p+5q)(c+dx)^4(bc-ad)+60bdx(\right.}{60d^5}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (-(r*(60*b*d*(b*c - a*d)^4*(p + 5*q)*x - 60*b^2*(b*c - a*d)^3*(2*p + 5*q)*(
c + d*x)^2 + 40*b^3*(b*c - a*d)^2*(3*p + 5*q)*(c + d*x)^3 - 15*b^4*(b*c - a
*d)*(4*p + 5*q)*(c + d*x)^4 + 12*b^5*(p + q)*(c + d*x)^5 - 60*(b*c - a*d)^5
*q*Log[c + d*x]))/(60*d^5) + (a + b*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^
r])/(5*b)
```

Maple [F] time = 0.734, size = 0, normalized size = 0.

$$\int (bx+a)^4 \ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Maxima [B] time = 1.30716, size = 533, normalized size = 2.65

$$\frac{1}{5}\left(b^4x^5 + 5ab^3x^4 + 10a^2b^2x^3 + 10a^3bx^2 + 5a^4x\right)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right) + \frac{\left(\frac{60a^5fp\log(bx+a)}{b} - \frac{12b^4d^4f(p+q)x^5+15(}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)*log((
(b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/300*(60*a^5*f*p*log(b*x + a)/b - (12*b^
4*d^4*f*(p + q)*x^5 + 15*(a*b^3*d^4*f*(4*p + 5*q) - b^4*c*d^3*f*q)*x^4 + 20
*(2*a^2*b^2*d^4*f*(3*p + 5*q) + b^4*c^2*d^2*f*q - 5*a*b^3*c*d^3*f*q)*x^3 +
30*(2*a^3*b*d^4*f*(2*p + 5*q) - b^4*c^3*d*f*q + 5*a*b^3*c^2*d^2*f*q - 10*a^
2*b^2*c*d^3*f*q)*x^2 + 60*(a^4*d^4*f*(p + 5*q) + b^4*c^4*f*q - 5*a*b^3*c^3*
```

```
d*f*q + 10*a^2*b^2*c^2*d^2*f*q - 10*a^3*b*c*d^3*f*q)*x)/d^4 + 60*(b^4*c^5*f
*q - 5*a*b^3*c^4*d*f*q + 10*a^2*b^2*c^3*d^2*f*q - 10*a^3*b*c^2*d^3*f*q + 5*
a^4*c*d^4*f*q)*log(d*x + c)/d^5)*r/f
```

Fricas [B] time = 0.864394, size = 1315, normalized size = 6.54

$$-\frac{12\left(b^5d^5p + b^5d^5q\right)rx^5 + 15\left(4\,ab^4d^5p - \left(b^5cd^4 - 5\,ab^4d^5\right)q\right)rx^4 + 20\left(6\,a^2b^3d^5p + \left(b^5c^2d^3 - 5\,ab^4cd^4 + 10\,a^2b^3d^5\right)q\right)rx^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/300*(12*(b^5*d^5*p + b^5*d^5*q)*r*x^5 + 15*(4*a*b^4*d^5*p - (b^5*c*d^4 -
 5*a*b^4*d^5)*q)*r*x^4 + 20*(6*a^2*b^3*d^5*p + (b^5*c^2*d^3 - 5*a*b^4*c*d^4
 + 10*a^2*b^3*d^5)*q)*r*x^3 + 30*(4*a^3*b^2*d^5*p - (b^5*c^3*d^2 - 5*a*b^4*
c^2*d^3 + 10*a^2*b^3*c*d^4 - 10*a^3*b^2*d^5)*q)*r*x^2 + 60*(a^4*b*d^5*p + (
b^5*c^4*d - 5*a*b^4*c^3*d^2 + 10*a^2*b^3*c^2*d^3 - 10*a^3*b^2*c*d^4 + 5*a^4
*b*d^5)*q)*r*x - 60*(b^5*d^5*p*r*x^5 + 5*a*b^4*d^5*p*r*x^4 + 10*a^2*b^3*d^5
*p*r*x^3 + 10*a^3*b^2*d^5*p*r*x^2 + 5*a^4*b*d^5*p*r*x + a^5*d^5*p*r)*log(b*
x + a) - 60*(b^5*d^5*q*r*x^5 + 5*a*b^4*d^5*q*r*x^4 + 10*a^2*b^3*d^5*q*r*x^3
 + 10*a^3*b^2*d^5*q*r*x^2 + 5*a^4*b*d^5*q*r*x + (b^5*c^5 - 5*a*b^4*c^4*d +
10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4)*q*r)*log(d*x + c)
- 60*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x
^2 + 5*a^4*b*d^5*x)*log(e) - 60*(b^5*d^5*r*x^5 + 5*a*b^4*d^5*r*x^4 + 10*a^2
*b^3*d^5*r*x^3 + 10*a^3*b^2*d^5*r*x^2 + 5*a^4*b*d^5*r*x)*log(f))/(b*d^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26035, size = 1002, normalized size = 4.99

$$-\frac{1}{25}\left(b^4pr + b^4qr - 5\,b^4r\log\left(f\right) - 5\,b^4\right)x^5 - \frac{\left(4\,ab^3dpr - b^4cqr + 5\,ab^3dqr - 20\,ab^3dr\log\left(f\right) - 20\,ab^3d\right)x^4}{20\,d} - \frac{\left(6\,a^2b^2d^2pr\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] -1/25*(b^4*p*r + b^4*q*r - 5*b^4*r*log(f) - 5*b^4)*x^5 - 1/20*(4*a*b^3*d*p*
r - b^4*c*q*r + 5*a*b^3*d*q*r - 20*a*b^3*d*r*log(f) - 20*a*b^3*d)*x^4/d - 1
/15*(6*a^2*b^2*d^2*p*r + b^4*c^2*q*r - 5*a*b^3*c*d*q*r + 10*a^2*b^2*d^2*q*r
 - 30*a^2*b^2*d^2*r*log(f) - 30*a^2*b^2*d^2)*x^3/d^2 + 1/5*(b^4*p*r*x^5 + 5
*a*b^3*p*r*x^4 + 10*a^2*b^2*p*r*x^3 + 10*a^3*b*p*r*x^2 + 5*a^4*p*r*x)*log(b
```

```
*x + a) + 1/5*(b^4*q*r*x^5 + 5*a*b^3*q*r*x^4 + 10*a^2*b^2*q*r*x^3 + 10*a^3*
b*q*r*x^2 + 5*a^4*q*r*x)*log(d*x + c) - 1/10*(4*a^3*b*d^3*p*r - b^4*c^3*q*r
 + 5*a*b^3*c^2*d*q*r - 10*a^2*b^2*c*d^2*q*r + 10*a^3*b*d^3*q*r - 20*a^3*b*d
^3*r*log(f) - 20*a^3*b*d^3)*x^2/d^3 - 1/5*(a^4*d^4*p*r + b^4*c^4*q*r - 5*a*
b^3*c^3*d*q*r + 10*a^2*b^2*c^2*d^2*q*r - 10*a^3*b*c*d^3*q*r + 5*a^4*d^4*q*r
 - 5*a^4*d^4*r*log(f) - 5*a^4*d^4)*x/d^4 + 1/10*(a^5*d^5*p*r + b^5*c^5*q*r
- 5*a*b^4*c^4*d*q*r + 10*a^2*b^3*c^3*d^2*q*r - 10*a^3*b^2*c^2*d^3*q*r + 5*a
^4*b*c*d^4*q*r)*log(abs(b*d*x^2 + b*c*x + a*d*x + a*c))/(b*d^5) + 1/10*(a^5
*b*c*d^5*p*r - a^6*d^6*p*r - b^6*c^6*q*r + 6*a*b^5*c^5*d*q*r - 15*a^2*b^4*c
^4*d^2*q*r + 20*a^3*b^3*c^3*d^3*q*r - 15*a^4*b^2*c^2*d^4*q*r + 5*a^5*b*c*d^
5*q*r)*log(abs((2*b*d*x + b*c + a*d - abs(b*c - a*d))/(2*b*d*x + b*c + a*d
+ abs(b*c - a*d))))/(b*d^5*abs(b*c - a*d))
```

3.8 $\int (a+bx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

Optimal. Leaf size=172

$$\frac{qrx(bc-ad)^3}{4d^3}-\frac{qr(a+bx)^2(bc-ad)^2}{8bd^2}-\frac{qr(bc-ad)^4\log(c+dx)}{4bd^4}+\frac{(a+bx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b}+\frac{qr(a+bx)^3}{12b}$$

```
[Out] ((b*c - a*d)^3*q*r*x)/(4*d^3) - ((b*c - a*d)^2*q*r*(a + b*x)^2)/(8*b*d^2) +
 ((b*c - a*d)*q*r*(a + b*x)^3)/(12*b*d) - (p*r*(a + b*x)^4)/(16*b) - (q*r*(
a + b*x)^4)/(16*b) - ((b*c - a*d)^4*q*r*Log[c + d*x])/(4*b*d^4) + ((a + b*x
)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*b)
```

Rubi [A] time = 0.0722046, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2495, 32, 43}

$$\frac{qrx(bc-ad)^3}{4d^3}-\frac{qr(a+bx)^2(bc-ad)^2}{8bd^2}-\frac{qr(bc-ad)^4\log(c+dx)}{4bd^4}+\frac{(a+bx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b}+\frac{qr(a+bx)^3}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] ((b*c - a*d)^3*q*r*x)/(4*d^3) - ((b*c - a*d)^2*q*r*(a + b*x)^2)/(8*b*d^2) +
 ((b*c - a*d)*q*r*(a + b*x)^3)/(12*b*d) - (p*r*(a + b*x)^4)/(16*b) - (q*r*(
a + b*x)^4)/(16*b) - ((b*c - a*d)^4*q*r*Log[c + d*x])/(4*b*d^4) + ((a + b*x
)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*b)
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \frac{(a+bx)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b} - \frac{1}{4}(pr)\int (a+bx)^3\,dx - \frac{(dqr)}{}$$

$$= -\frac{pr(a+bx)^4}{16b} + \frac{(a+bx)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b} - \frac{(dqr)\int \left(-\right.}{}$$

$$= \frac{(bc-ad)^3 qrx}{4d^3} - \frac{(bc-ad)^2 qr(a+bx)^2}{8bd^2} + \frac{(bc-ad)qr(a+bx)^3}{12bd} - \frac{pr(a+}{16}$$

Mathematica [A] time = 0.211291, size = 154, normalized size = 0.9

$$\frac{\frac{r\left(-18b^2(p+2q)(c+dx)^2(bc-ad)^2+4b^3(3p+4q)(c+dx)^3(bc-ad)+12bdx(p+4q)(bc-ad)^3-12q(bc-ad)^4\log(c+dx)-3b^4(p+q)(c+dx)^4\right)}{12d^4} + (a+bx)^4\log\left(e\left(f(\right.\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] ((r*(12*b*d*(b*c - a*d)^3*(p + 4*q)*x - 18*b^2*(b*c - a*d)^2*(p + 2*q)*(c +
 d*x)^2 + 4*b^3*(b*c - a*d)*(3*p + 4*q)*(c + d*x)^3 - 3*b^4*(p + q)*(c + d*
x)^4 - 12*(b*c - a*d)^4*q*Log[c + d*x]))/(12*d^4) + (a + b*x)^4*Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r])/(4*b)
```

Maple [F] time = 0.422, size = 0, normalized size = 0.

$$\int (bx+a)^3 \ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Maxima [A] time = 1.33621, size = 385, normalized size = 2.24

$$\frac{1}{4}\left(b^3x^4 + 4ab^2x^3 + 6a^2bx^2 + 4a^3x\right)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right) + \frac{\left(\frac{12a^4fp\log(bx+a)}{b} - \frac{3b^3d^3f(p+q)x^4+4\left(ab^2d^3f(3p+4q)-b^3\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e) + 1/48*(12*a^4*f*p*log(b*x + a)/b - (3*b^3*d^3*f*(p + q)*x^4
+ 4*(a*b^2*d^3*f*(3*p + 4*q) - b^3*c*d^2*f*q)*x^3 + 6*(3*a^2*b*d^3*f*(p + 2
*q) + b^3*c^2*d*f*q - 4*a*b^2*c*d^2*f*q)*x^2 + 12*(a^3*d^3*f*(p + 4*q) - b^
3*c^3*f*q + 4*a*b^2*c^2*d*f*q - 6*a^2*b*c*d^2*f*q)*x)/d^3 - 12*(b^3*c^4*f*q
 - 4*a*b^2*c^3*d*f*q + 6*a^2*b*c^2*d^2*f*q - 4*a^3*c*d^3*f*q)*log(d*x + c)/
```

```
d^4)*r/f
```

Fricas [B] time = 0.956594, size = 980, normalized size = 5.7

$$-\frac{3\left(b^4d^4p+b^4d^4q\right)rx^4+4\left(3\,ab^3d^4p-\left(b^4cd^3-4\,ab^3d^4\right)q\right)rx^3+6\left(3\,a^2b^2d^4p+\left(b^4c^2d^2-4\,ab^3cd^3+6\,a^2b^2d^4\right)q\right)rx^2+12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/48*(3*(b^4*d^4*p + b^4*d^4*q)*r*x^4 + 4*(3*a*b^3*d^4*p - (b^4*c*d^3 - 4*
a*b^3*d^4)*q)*r*x^3 + 6*(3*a^2*b^2*d^4*p + (b^4*c^2*d^2 - 4*a*b^3*c*d^3 + 6
*a^2*b^2*d^4)*q)*r*x^2 + 12*(a^3*b*d^4*p - (b^4*c^3*d - 4*a*b^3*c^2*d^2 + 6
*a^2*b^2*c*d^3 - 4*a^3*b*d^4)*q)*r*x - 12*(b^4*d^4*p*r*x^4 + 4*a*b^3*d^4*p*
r*x^3 + 6*a^2*b^2*d^4*p*r*x^2 + 4*a^3*b*d^4*p*r*x + a^4*d^4*p*r)*log(b*x +
a) - 12*(b^4*d^4*q*r*x^4 + 4*a*b^3*d^4*q*r*x^3 + 6*a^2*b^2*d^4*q*r*x^2 + 4*
a^3*b*d^4*q*r*x - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*
d^3)*q*r)*log(d*x + c) - 12*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*
x^2 + 4*a^3*b*d^4*x)*log(e) - 12*(b^4*d^4*r*x^4 + 4*a*b^3*d^4*r*x^3 + 6*a^2
*b^2*d^4*r*x^2 + 4*a^3*b*d^4*r*x)*log(f))/(b*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40639, size = 774, normalized size = 4.5

$$-\frac{1}{16}\left(b^3pr+b^3qr-4\,b^3r\log\left(f\right)-4\,b^3\right)x^4-\frac{\left(3\,ab^2dpr-b^3cqr+4\,ab^2dqr-12\,ab^2dr\log\left(f\right)-12\,ab^2d\right)x^3}{12\,d}+\frac{1}{4}\left(b^3prx^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] -1/16*(b^3*p*r + b^3*q*r - 4*b^3*r*log(f) - 4*b^3)*x^4 - 1/12*(3*a*b^2*d*p*
r - b^3*c*q*r + 4*a*b^2*d*q*r - 12*a*b^2*d*r*log(f) - 12*a*b^2*d)*x^3/d + 1
/4*(b^3*p*r*x^4 + 4*a*b^2*p*r*x^3 + 6*a^2*b*p*r*x^2 + 4*a^3*p*r*x)*log(b*x
+ a) + 1/4*(b^3*q*r*x^4 + 4*a*b^2*q*r*x^3 + 6*a^2*b*q*r*x^2 + 4*a^3*q*r*x)*
log(d*x + c) - 1/8*(3*a^2*b*d^2*p*r + b^3*c^2*q*r - 4*a*b^2*c*d*q*r + 6*a^2
*b*d^2*q*r - 12*a^2*b*d^2*r*log(f) - 12*a^2*b*d^2)*x^2/d^2 - 1/4*(a^3*d^3*p
*r - b^3*c^3*q*r + 4*a*b^2*c^2*d*q*r - 6*a^2*b*c*d^2*q*r + 4*a^3*d^3*q*r -
4*a^3*d^3*r*log(f) - 4*a^3*d^3)*x/d^3 + 1/8*(a^4*d^4*p*r - b^4*c^4*q*r + 4*
a*b^3*c^3*d*q*r - 6*a^2*b^2*c^2*d^2*q*r + 4*a^3*b*c*d^3*q*r)*log(abs(b*d*x^
2 + b*c*x + a*d*x + a*c))/(b*d^4) + 1/8*(a^4*b*c*d^4*p*r - a^5*d^5*p*r + b^
```

```
5*c^5*q*r - 5*a*b^4*c^4*d*q*r + 10*a^2*b^3*c^3*d^2*q*r - 10*a^3*b^2*c^2*d^3
*q*r + 4*a^4*b*c*d^4*q*r)*log(abs((2*b*d*x + b*c + a*d - abs(b*c - a*d))/(2
*b*d*x + b*c + a*d + abs(b*c - a*d))))/(b*d^4*abs(b*c - a*d))
```

3.9 $\int (a+bx)^2 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$

Optimal. Leaf size=143

$$-\frac{qrx(bc-ad)^2}{3d^2}+\frac{qr(bc-ad)^3\log(c+dx)}{3bd^3}+\frac{(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b}+\frac{qr(a+bx)^2(bc-ad)}{6bd}-\frac{pr(a+bx)}{9b}$$

```
[Out] -((b*c - a*d)^2*q*r*x)/(3*d^2) + ((b*c - a*d)*q*r*(a + b*x)^2)/(6*b*d) - (p
*r*(a + b*x)^3)/(9*b) - (q*r*(a + b*x)^3)/(9*b) + ((b*c - a*d)^3*q*r*Log[c
+ d*x])/(3*b*d^3) + ((a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*b
)
```

Rubi [A] time = 0.0608917, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2495, 32, 43}

$$-\frac{qrx(bc-ad)^2}{3d^2}+\frac{qr(bc-ad)^3\log(c+dx)}{3bd^3}+\frac{(a+bx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b}+\frac{qr(a+bx)^2(bc-ad)}{6bd}-\frac{pr(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```


```
[Out] -((b*c - a*d)^2*q*r*x)/(3*d^2) + ((b*c - a*d)*q*r*(a + b*x)^2)/(6*b*d) - (p
*r*(a + b*x)^3)/(9*b) - (q*r*(a + b*x)^3)/(9*b) + ((b*c - a*d)^3*q*r*Log[c
+ d*x])/(3*b*d^3) + ((a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*b
)
```


Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^2 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \frac{(a+bx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b} - \frac{1}{3}(pr)\int (a+bx)^2\,dx - \frac{(dqr)}{}$$

$$= -\frac{pr(a+bx)^3}{9b} + \frac{(a+bx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b} - \frac{(dqr)\int\left(\frac{b(bc}{}\right.}{}$$

$$= -\frac{(bc-ad)^2 qrx}{3d^2} + \frac{(bc-ad)qr(a+bx)^2}{6bd} - \frac{pr(a+bx)^3}{9b} - \frac{qr(a+bx)^3}{9b} +$$

Mathematica [A] time = 0.134685, size = 127, normalized size = 0.89

$$\frac{(a+bx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - \frac{r\left(-3b^2(2p+3q)(c+dx)^2(bc-ad)+6bdx(p+3q)(bc-ad)^2-6q(bc-ad)^3\log(c+dx)+2b^3(p+q)(c+dx)^3\right)}{6d^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (-(r*(6*b*d*(b*c - a*d)^2*(p + 3*q)*x - 3*b^2*(b*c - a*d)*(2*p + 3*q)*(c +
d*x)^2 + 2*b^3*(p + q)*(c + d*x)^3 - 6*(b*c - a*d)^3*q*Log[c + d*x]))/(6*d^
3) + (a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*b)
```

Maple [F] time = 0.386, size = 0, normalized size = 0.

$$\int (bx+a)^2 \ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Maxima [A] time = 1.28831, size = 262, normalized size = 1.83

$$\frac{1}{3}\left(b^2x^3 + 3\,abx^2 + 3\,a^2x\right)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right) + \frac{\left(\frac{6\,a^3 fp\log(bx+a)}{b} - \frac{2\,b^2d^2 f(p+q)x^3 + 3\left(abd^2 f(2\,p+3\,q) - b^2 cdfq\right)x^2 + 6\left(a^2d^2\right.}{d^2}\right.}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) +
1/18*(6*a^3*f*p*log(b*x + a)/b - (2*b^2*d^2*f*(p + q)*x^3 + 3*(a*b*d^2*f*(2
*p + 3*q) - b^2*c*d*f*q)*x^2 + 6*(a^2*d^2*f*(p + 3*q) + b^2*c^2*f*q - 3*a*b
*c*d*f*q)*x)/d^2 + 6*(b^2*c^3*f*q - 3*a*b*c^2*d*f*q + 3*a^2*c*d^2*f*q)*log(
d*x + c)/d^3)*r/f
```

Fricas [B] time = 0.880294, size = 693, normalized size = 4.85

$$-\frac{2\left(b^3d^3p + b^3d^3q\right)rx^3 + 3\left(2\,ab^2d^3p - \left(b^3cd^2 - 3\,ab^2d^3\right)q\right)rx^2 + 6\left(a^2bd^3p + \left(b^3c^2d - 3\,ab^2cd^2 + 3\,a^2bd^3\right)q\right)rx - 6\left(b^3d^3p\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/18*(2*(b^3*d^3*p + b^3*d^3*q)*r*x^3 + 3*(2*a*b^2*d^3*p - (b^3*c*d^2 - 3*
a*b^2*d^3)*q)*r*x^2 + 6*(a^2*b*d^3*p + (b^3*c^2*d - 3*a*b^2*c*d^2 + 3*a^2*b
*d^3)*q)*r*x - 6*(b^3*d^3*p*r*x^3 + 3*a*b^2*d^3*p*r*x^2 + 3*a^2*b*d^3*p*r*x
 + a^3*d^3*p*r)*log(b*x + a) - 6*(b^3*d^3*q*r*x^3 + 3*a*b^2*d^3*q*r*x^2 + 3
*a^2*b*d^3*q*r*x + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2)*q*r)*log(d*x +
 c) - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x)*log(e) - 6*(b^3*d^3
*r*x^3 + 3*a*b^2*d^3*r*x^2 + 3*a^2*b*d^3*r*x)*log(f))/(b*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30847, size = 566, normalized size = 3.96

$$-\frac{1}{9}\left(b^2pr + b^2qr - 3\,b^2r\log\left(f\right) - 3\,b^2\right)x^3 - \frac{\left(2\,abdpr - b^2cqr + 3\,abdqr - 6\,abdr\log\left(f\right) - 6\,abd\right)x^2}{6\,d} + \frac{1}{3}\left(b^2prx^3 + 3\,abprx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] -1/9*(b^2*p*r + b^2*q*r - 3*b^2*r*log(f) - 3*b^2)*x^3 - 1/6*(2*a*b*d*p*r -
b^2*c*q*r + 3*a*b*d*q*r - 6*a*b*d*r*log(f) - 6*a*b*d)*x^2/d + 1/3*(b^2*p*r*
x^3 + 3*a*b*p*r*x^2 + 3*a^2*p*r*x)*log(b*x + a) + 1/3*(b^2*q*r*x^3 + 3*a*b*
q*r*x^2 + 3*a^2*q*r*x)*log(d*x + c) - 1/3*(a^2*d^2*p*r + b^2*c^2*q*r - 3*a*
b*c*d*q*r + 3*a^2*d^2*q*r - 3*a^2*d^2*r*log(f) - 3*a^2*d^2)*x/d^2 + 1/6*(a^
3*d^3*p*r + b^3*c^3*q*r - 3*a*b^2*c^2*d*q*r + 3*a^2*b*c*d^2*q*r)*log(abs(b*
d*x^2 + b*c*x + a*d*x + a*c))/(b*d^3) + 1/6*(a^3*b*c*d^3*p*r - a^4*d^4*p*r
- b^4*c^4*q*r + 4*a*b^3*c^3*d*q*r - 6*a^2*b^2*c^2*d^2*q*r + 3*a^3*b*c*d^3*q
*r)*log(abs((2*b*d*x + b*c + a*d - abs(b*c - a*d))/(2*b*d*x + b*c + a*d + a
bs(b*c - a*d))))/(b*d^3*abs(b*c - a*d))
```

3.10 $\int (a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

Optimal. Leaf size=116

$$-\frac{qr(bc-ad)^2\log(c+dx)}{2bd^2}+\frac{(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}+\frac{qrx(bc-ad)}{2d}-\frac{qr(a+bx)^2}{4b}-\frac{1}{2}aprx-\frac{1}{4}bprx^2$$

```
[Out] -(a*p*r*x)/2 + ((b*c - a*d)*q*r*x)/(2*d) - (b*p*r*x^2)/4 - (q*r*(a + b*x)^2
)/(4*b) - ((b*c - a*d)^2*q*r*Log[c + d*x])/(2*b*d^2) + ((a + b*x)^2*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b)
```

Rubi [A] time = 0.0432106, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2495, 43}

$$-\frac{qr(bc-ad)^2\log(c+dx)}{2bd^2}+\frac{(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}+\frac{qrx(bc-ad)}{2d}-\frac{qr(a+bx)^2}{4b}-\frac{1}{2}aprx-\frac{1}{4}bprx^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -(a*p*r*x)/2 + ((b*c - a*d)*q*r*x)/(2*d) - (b*p*r*x^2)/4 - (q*r*(a + b*x)^2
)/(4*b) - ((b*c - a*d)^2*q*r*Log[c + d*x])/(2*b*d^2) + ((a + b*x)^2*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b)
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx &= \frac{(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}-\frac{1}{2}(pr)\int (a+bx)\,dx-\frac{(dqr)\int}{}\\ &= -\frac{1}{2}aprx-\frac{1}{4}bprx^2+\frac{(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b}-\frac{(dqr)\int}{}\\ &= -\frac{1}{2}aprx+\frac{(bc-ad)qrx}{2d}-\frac{1}{4}bprx^2-\frac{qr(a+bx)^2}{4b}-\frac{(bc-ad)^2qr\log(c+}{2bd^2}\end{aligned}$$

Mathematica [A] time = 0.19288, size = 105, normalized size = 0.91

$$\frac{a^2pr\log(a+bx)}{2b}-\frac{dx\left(r(2ad(p+2q)-2bcq+bdx(p+q))-2d(2a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)+2cqr(bc-2ad)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (a^2*p*r*Log[a + b*x])/(2*b) - (2*c*(b*c - 2*a*d)*q*r*Log[c + d*x] + d*x*(r
*(-2*b*c*q + 2*a*d*(p + 2*q) + b*d*(p + q)*x) - 2*d*(2*a + b*x)*Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]))/(4*d^2)
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int (bx+a)\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Maxima [A] time = 1.35446, size = 159, normalized size = 1.37

$$\frac{1}{2}\left(bx^2+2ax\right)\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)+\frac{\left(\frac{2a^2fp\log(bx+a)}{b}-\frac{bdf(p+q)x^2+2\left(adf(p+2q)-bcfq\right)x}{d}-\frac{2\left(bc^2fq-2acdfq\right)\log(dx+c)}{d^2}\right)r}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/4*(2*a^2*f*p*l
og(b*x + a)/b - (b*d*f*(p + q)*x^2 + 2*(a*d*f*(p + 2*q) - b*c*f*q)*x)/d - 2
*(b*c^2*f*q - 2*a*c*d*f*q)*log(d*x + c)/d^2)*r/f
```

Fricas [A] time = 0.820055, size = 435, normalized size = 3.75

$$-\frac{\left(b^2d^2p+b^2d^2q\right)rx^2+2\left(abd^2p-\left(b^2cd-2abd^2\right)q\right)rx-2\left(b^2d^2prx^2+2abd^2prx+a^2d^2pr\right)\log(bx+a)-2\left(b^2d^2qrx^2+\right.}{4bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/4*((b^2*d^2*p + b^2*d^2*q)*r*x^2 + 2*(a*b*d^2*p - (b^2*c*d - 2*a*b*d^2)*
q)*r*x - 2*(b^2*d^2*p*r*x^2 + 2*a*b*d^2*p*r*x + a^2*d^2*p*r)*log(b*x + a) -
 2*(b^2*d^2*q*r*x^2 + 2*a*b*d^2*q*r*x - (b^2*c^2 - 2*a*b*c*d)*q*r)*log(d*x
+ c) - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x)*log(e) - 2*(b^2*d^2*r*x^2 + 2*a*b*d^2*
```

```
r*x)*log(f))/(b*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35044, size = 381, normalized size = 3.28

$$-\frac{1}{4}\left(bpr+bqr-2\,br\log\left(f\right)-2\,b\right)x^2+\frac{1}{2}\left(bprx^2+2\,aprx\right)\log\left(bx+a\right)+\frac{1}{2}\left(bqrx^2+2\,aqrx\right)\log\left(dx+c\right)-\frac{\left(adpr-bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] -1/4*(b*p*r + b*q*r - 2*b*r*log(f) - 2*b)*x^2 + 1/2*(b*p*r*x^2 + 2*a*p*r*x)
*log(b*x + a) + 1/2*(b*q*r*x^2 + 2*a*q*r*x)*log(d*x + c) - 1/2*(a*d*p*r - b
*c*q*r + 2*a*d*q*r - 2*a*d*r*log(f) - 2*a*d)*x/d + 1/4*(a^2*d^2*p*r - b^2*c
^2*q*r + 2*a*b*c*d*q*r)*log(abs(b*d*x^2 + b*c*x + a*d*x + a*c))/(b*d^2) + 1
/4*(a^2*b*c*d^2*p*r - a^3*d^3*p*r + b^3*c^3*q*r - 3*a*b^2*c^2*d*q*r + 2*a^2
*b*c*d^2*q*r)*log(abs((2*b*d*x + b*c + a*d - abs(b*c - a*d))/(2*b*d*x + b*c
 + a*d + abs(b*c - a*d))))/(b*d^2*abs(b*c - a*d))
```

3.11 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx}\,dx$

Optimal. Leaf size=107

$$-\frac{qr\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b}+\frac{\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}-\frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b}-\frac{pr\log^2(a+bx)}{2b}$$

```
[Out] -(p*r*Log[a + b*x]^2)/(2*b) - (q*r*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*
d)])/b + (Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/b - (q*r*PolyL
og[2, -((d*(a + b*x))/(b*c - a*d))])/b
```

Rubi [A] time = 0.0829128, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2494, 2390, 2301, 2394, 2393, 2391}

$$-\frac{qr\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b}+\frac{\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}-\frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b}-\frac{pr\log^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x),x]
```

```
[Out] -(p*r*Log[a + b*x]^2)/(2*b) - (q*r*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*
d)])/b + (Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/b - (q*r*PolyL
og[2, -((d*(a + b*x))/(b*c - a*d))])/b
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx}\,dx = \frac{\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} - (pr)\int \frac{\log(a+bx)}{a+bx}\,dx - \frac{(dqr)\int \frac{\log}{}}{b}$$
$$= -\frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b} - \frac{(pr}{}$$
$$= -\frac{pr\log^2(a+bx)}{2b} - \frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{\log(a+bx)\log\left(e\left(f(a+bx)^p\right.\right.}{b}$$
$$= -\frac{pr\log^2(a+bx)}{2b} - \frac{qr\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{b} + \frac{\log(a+bx)\log\left(e\left(f(a+bx)^p\right.\right.}{b}$$

Mathematica [A] time = 0.108976, size = 93, normalized size = 0.87

$$-\frac{2qr\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)+\log(a+bx)\left(-2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+2qr\log\left(\frac{b(c+dx)}{bc-ad}\right)+pr\log(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x),x]
```

```
[Out] -(Log[a + b*x]*(p*r*Log[a + b*x] + 2*q*r*Log[(b*(c + d*x))/(b*c - a*d)] - 2
*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + 2*q*r*PolyLog[2, (d*(a + b*x))/(-(
b*c) + a*d)])/(2*b)
```

Maple [F] time = 0.436, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x)
```

Maxima [A] time = 3.13703, size = 221, normalized size = 2.07

$$-\frac{\left(\frac{2\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)fq}{b}-\frac{fp\log(bx+a)^2+2\,fq\log(bx+a)\log(dx+c)}{b}\right)r}{2\,f}-\frac{\left(fp\log\left(bx+a\right)+fq\log\left(dx+c\right)\right)r\log\left(bx\right.}{bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*f*q/b - (f*p*log(b*x + a)^2 + 2*f*q*log(b*x + a)*log(d*x
 + c))/b)*r/f - (f*p*log(b*x + a) + f*q*log(d*x + c))*r*log(b*x + a)/(b*f)
+ log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(b*x + a)/b
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*x + a), x)
```

3.12 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}\,dx$

Optimal. Leaf size=95

$$-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)}+\frac{dqr\log(a+bx)}{b(bc-ad)}-\frac{dqr\log(c+dx)}{b(bc-ad)}-\frac{pr}{b(a+bx)}$$

```
[Out] -((p*r)/(b*(a + b*x))) + (d*q*r*Log[a + b*x])/(b*(b*c - a*d)) - (d*q*r*Log[
c + d*x])/(b*(b*c - a*d)) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(b*(a + b*
x))
```

Rubi [A] time = 0.0374773, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2495, 32, 36, 31}

$$-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)}+\frac{dqr\log(a+bx)}{b(bc-ad)}-\frac{dqr\log(c+dx)}{b(bc-ad)}-\frac{pr}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^2,x]
```

```
[Out] -((p*r)/(b*(a + b*x))) + (d*q*r*Log[a + b*x])/(b*(b*c - a*d)) - (d*q*r*Log[
c + d*x])/(b*(b*c - a*d)) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(b*(a + b*
x))
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}\,dx = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + (pr)\int \frac{1}{(a+bx)^2}\,dx + \frac{(dqr)\int \frac{1}{(a+bx)(c+dx)}\,dx}{b}$$
$$= -\frac{pr}{b(a+bx)} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + \frac{(dqr)\int \frac{1}{a+bx}\,dx}{bc-ad} - \frac{(d^2qr)\int \frac{1}{c+dx}\,dx}{b(bc-ad)}$$
$$= -\frac{pr}{b(a+bx)} + \frac{dqr\log(a+bx)}{b(bc-ad)} - \frac{dqr\log(c+dx)}{b(bc-ad)} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)}$$

Mathematica [A] time = 0.0576862, size = 89, normalized size = 0.94

$$\frac{r\left(\frac{dq\log(a+bx)}{bc-ad} - \frac{dq\log(c+dx)}{bc-ad} - \frac{p}{a+bx}\right)}{b} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^2,x]
```

```
[Out] (r*(-(p/(a + b*x)) + (d*q*Log[a + b*x])/(b*c - a*d) - (d*q*Log[c + d*x])/(b
*c - a*d)))/b - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(b*(a + b*x))
```

Maple [F] time = 0.421, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)}{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x)
```

Maxima [A] time = 1.26499, size = 134, normalized size = 1.41

$$\frac{\left(dfq\left(\frac{\log(bx+a)}{bc-ad} - \frac{\log(dx+c)}{bc-ad}\right) - \frac{bfp}{b^2x+ab}\right)r}{bf} - \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (d*f*q*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)) - b*f*p/(b^2*x
 + a*b))*r/(b*f) - log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((b*x + a)*b)
```

Fricas [A] time = 0.902385, size = 267, normalized size = 2.81

$$-\frac{(bc-ad)pr + (bc-ad)r\log\left(f\right) - \left(bdqrx + \left(adq - (bc-ad)p\right)r\right)\log(bx+a) + \left(bdqrx + bcqr\right)\log(dx+c) + (bc-ad)\,\mathrm{lo}}{ab^2c - a^2bd + \left(b^3c - ab^2d\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -((b*c - a*d)*p*r + (b*c - a*d)*r*log(f) - (b*d*q*r*x + (a*d*q - (b*c - a*d
)*p)*r)*log(b*x + a) + (b*d*q*r*x + b*c*q*r)*log(d*x + c) + (b*c - a*d)*log
(e))/(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19727, size = 151, normalized size = 1.59

$$\frac{dqr\log(bx+a)}{b^2c-abd}-\frac{dqr\log(dx+c)}{b^2c-abd}-\frac{pr\log(bx+a)}{b^2x+ab}-\frac{qr\log(dx+c)}{b^2x+ab}-\frac{pr+r\log(f)+1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] d*q*r*log(b*x + a)/(b^2*c - a*b*d) - d*q*r*log(d*x + c)/(b^2*c - a*b*d) - p
*r*log(b*x + a)/(b^2*x + a*b) - q*r*log(d*x + c)/(b^2*x + a*b) - (p*r + r*l
og(f) + 1)/(b^2*x + a*b)
```

3.13 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}\,dx$

Optimal. Leaf size=135

$$-\frac{d^2qr\log(a+bx)}{2b(bc-ad)^2}+\frac{d^2qr\log(c+dx)}{2b(bc-ad)^2}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}-\frac{dqr}{2b(a+bx)(bc-ad)}-\frac{pr}{4b(a+bx)^2}$$

```
[Out] -(p*r)/(4*b*(a + b*x)^2) - (d*q*r)/(2*b*(b*c - a*d)*(a + b*x)) - (d^2*q*r*L
og[a + b*x])/(2*b*(b*c - a*d)^2) + (d^2*q*r*Log[c + d*x])/(2*b*(b*c - a*d)^
2) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(2*b*(a + b*x)^2)
```

Rubi [A] time = 0.0561178, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2495, 32, 44}

$$-\frac{d^2qr\log(a+bx)}{2b(bc-ad)^2}+\frac{d^2qr\log(c+dx)}{2b(bc-ad)^2}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}-\frac{dqr}{2b(a+bx)(bc-ad)}-\frac{pr}{4b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^3,x]
```


```
[Out] -(p*r)/(4*b*(a + b*x)^2) - (d*q*r)/(2*b*(b*c - a*d)*(a + b*x)) - (d^2*q*r*L
og[a + b*x])/(2*b*(b*c - a*d)^2) + (d^2*q*r*Log[c + d*x])/(2*b*(b*c - a*d)^
2) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(2*b*(a + b*x)^2)
```


Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}dx = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} + \frac{1}{2}(pr)\int \frac{1}{(a+bx)^3}dx + \frac{(dqr)\int \frac{1}{(a+bx)^2(c+dx)}d}{2b}$$

$$= -\frac{pr}{4b(a+bx)^2} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} + \frac{(dqr)\int\left(\frac{b}{(bc-ad)(a+bx)^2} - \frac{b}{(bc-ad)^2}\right.}{2b}$$

$$= -\frac{pr}{4b(a+bx)^2} - \frac{dqr}{2b(bc-ad)(a+bx)} - \frac{d^2qr\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2qr\log(c+dx)}{2b(bc-ad)^2} - \frac{\text{lo}}{}$$

Mathematica [A] time = 0.272963, size = 116, normalized size = 0.86

$$\frac{r\left(-\frac{d^2q\log(a+bx)}{(bc-ad)^2}+\frac{d^2q\log(c+dx)}{(bc-ad)^2}-\frac{p-\frac{2dq(a+bx)}{ad-bc}}{2(a+bx)^2}\right)-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^3,x]
```

```
[Out] (r*(-(p - (2*d*q*(a + b*x))/(-(b*c) + a*d))/(2*(a + b*x)^2) - (d^2*q*Log[a
+ b*x])/(b*c - a*d)^2 + (d^2*q*Log[c + d*x])/(b*c - a*d)^2) - Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]/(a + b*x)^2)/(2*b)
```

Maple [F] time = 0.424, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)}{(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x)
```

Maxima [A] time = 1.23822, size = 223, normalized size = 1.65

$$-\frac{\left(2\,dfq\left(\frac{d\log(bx+a)}{b^2c^2-2\,abcd+a^2d^2}-\frac{d\log(dx+c)}{b^2c^2-2\,abcd+a^2d^2}+\frac{1}{abc-a^2d+\left(b^2c-abd\right)x}\right)+\frac{bfp}{b^3x^2+2\,ab^2x+a^2b}\right)r}{4\,bf}-\frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)}{2\,(bx+a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*d*f*q*(d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - d*log(d*x +
 c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
) + b*f*p/(b^3*x^2 + 2*a*b^2*x + a^2*b))*r/(b*f) - 1/2*log(((b*x + a)^p*(d*
x + c)^q*f)^r*e)/((b*x + a)^2*b)
```

Fricas [B] time = 0.87075, size = 689, normalized size = 5.1

$$-\frac{2\left(b^{2}cd-abd^{2}\right)qrx+2\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)r\log\left(f\right)+\left(\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)p+2\left(abcd-a^{2}d^{2}\right)q\right)r+2\left(b^{2}d^{2}qrx^{2}+\right.}{4\left(a^{2}b^{3}c^{2}-2\,a^{3}b^{2}cd+a^{4}bd^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(b^2*c*d - a*b*d^2)*q*r*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*r*log
(f) + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*p + 2*(a*b*c*d - a^2*d^2)*q)*r + 2*(
b^2*d^2*q*r*x^2 + 2*a*b*d^2*q*r*x + (a^2*d^2*q + (b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*p)*r)*log(b*x + a) - 2*(b^2*d^2*q*r*x^2 + 2*a*b*d^2*q*r*x - (b^2*c^2
- 2*a*b*c*d)*q*r)*log(d*x + c) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(e))/
(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3
*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19189, size = 332, normalized size = 2.46

$$-\frac{d^{2}qr\log\left(bx+a\right)}{2\left(b^{3}c^{2}-2\,ab^{2}cd+a^{2}bd^{2}\right)}+\frac{d^{2}qr\log\left(dx+c\right)}{2\left(b^{3}c^{2}-2\,ab^{2}cd+a^{2}bd^{2}\right)}-\frac{pr\log\left(bx+a\right)}{2\left(b^{3}x^{2}+2\,ab^{2}x+a^{2}b\right)}-\frac{qr\log\left(dx+c\right)}{2\left(b^{3}x^{2}+2\,ab^{2}x+a^{2}b\right)}-\frac{2\,bdqrx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*d^2*q*r*log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + 1/2*d^2*q*r
*log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 1/2*p*r*log(b*x + a)/(b
^3*x^2 + 2*a*b^2*x + a^2*b) - 1/2*q*r*log(d*x + c)/(b^3*x^2 + 2*a*b^2*x + a
^2*b) - 1/4*(2*b*d*q*r*x + b*c*p*r - a*d*p*r + 2*a*d*q*r + 2*b*c*r*log(f) -
 2*a*d*r*log(f) + 2*b*c - 2*a*d)/(b^4*c*x^2 - a*b^3*d*x^2 + 2*a*b^3*c*x - 2
*a^2*b^2*d*x + a^2*b^2*c - a^3*b*d)
```

3.14 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4} dx$

Optimal. Leaf size=164

$$\frac{d^2qr}{3b(a+bx)(bc-ad)^2}+\frac{d^3qr\log(a+bx)}{3b(bc-ad)^3}-\frac{d^3qr\log(c+dx)}{3b(bc-ad)^3}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3}-\frac{dqr}{6b(a+bx)^2(bc-ad)}-$$

```
[Out] -(p*r)/(9*b*(a + b*x)^3) - (d*q*r)/(6*b*(b*c - a*d)*(a + b*x)^2) + (d^2*q*r
)/(3*b*(b*c - a*d)^2*(a + b*x)) + (d^3*q*r*Log[a + b*x])/(3*b*(b*c - a*d)^3
) - (d^3*q*r*Log[c + d*x])/(3*b*(b*c - a*d)^3) - Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]/(3*b*(a + b*x)^3)
```

Rubi [A] time = 0.0693974, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2495, 32, 44}

$$\frac{d^2qr}{3b(a+bx)(bc-ad)^2}+\frac{d^3qr\log(a+bx)}{3b(bc-ad)^3}-\frac{d^3qr\log(c+dx)}{3b(bc-ad)^3}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3}-\frac{dqr}{6b(a+bx)^2(bc-ad)}-$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^4,x]
```

```
[Out] -(p*r)/(9*b*(a + b*x)^3) - (d*q*r)/(6*b*(b*c - a*d)*(a + b*x)^2) + (d^2*q*r
)/(3*b*(b*c - a*d)^2*(a + b*x)) + (d^3*q*r*Log[a + b*x])/(3*b*(b*c - a*d)^3
) - (d^3*q*r*Log[c + d*x])/(3*b*(b*c - a*d)^3) - Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]/(3*b*(a + b*x)^3)
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4}dx = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3} + \frac{1}{3}(pr)\int \frac{1}{(a+bx)^4}dx + \frac{(dqr)\int \frac{1}{(a+bx)^3(c+dx)}dx}{3b}$$

$$= -\frac{pr}{9b(a+bx)^3} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3} + \frac{(dqr)\int \left(\frac{b}{(bc-ad)(a+bx)^3} - \frac{bd}{(bc-ad)^2(a+\ldots}\right.}{\ldots}$$

$$= -\frac{pr}{9b(a+bx)^3} - \frac{dqr}{6b(bc-ad)(a+bx)^2} + \frac{d^2qr}{3b(bc-ad)^2(a+bx)} + \frac{d^3qr\log(a+bx)}{3b(bc-ad)^3} - \ldots$$

Mathematica [A] time = 0.391948, size = 141, normalized size = 0.86

$$\frac{r\left(\frac{\frac{6d^2q(a+bx)^2}{(bc-ad)^2}+\frac{3dq(a+bx)}{ad-bc}-2p}{6(a+bx)^3}+\frac{d^3q\log(a+bx)}{(bc-ad)^3}-\frac{d^3q\log(c+dx)}{(bc-ad)^3}\right)-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^4,x]
```

```
[Out] (r*((-2*p + (3*d*q*(a + b*x))/(-(b*c) + a*d) + (6*d^2*q*(a + b*x)^2)/(b*c -
 a*d)^2)/(6*(a + b*x)^3) + (d^3*q*Log[a + b*x])/(b*c - a*d)^3 - (d^3*q*Log[
c + d*x])/(b*c - a*d)^3) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^3
)/(3*b)
```

Maple [F] time = 0.425, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)}{(bx+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x)
```

Maxima [A] time = 1.27755, size = 390, normalized size = 2.38

$$\frac{\left(3\left(\frac{2\,d^2\log(bx+a)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}-\frac{2\,d^2\log(dx+c)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}+\frac{2\,bdx-bc+3\,ad}{a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x}\right)dfq - \ldots}{18\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/18*(3*(2*d^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3) - 2*d^2*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3) + (2*b*d*x - b*c + 3*a*d)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^
2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d
```

```
^2)*x))*d*f*q - 2*b*f*p/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b))*r/(b
*f) - 1/3*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((b*x + a)^3*b)
```

Fricas [B] time = 0.854387, size = 1177, normalized size = 7.18

$$\frac{6\left(b^3cd^2 - ab^2d^3\right)qrx^2 - 3\left(b^3c^2d - 6\,ab^2cd^2 + 5\,a^2bd^3\right)qrx - 6\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)r\log\left(f\right) - \left(2\left(b^3c^3 - \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b^3*c*d^2 - a*b^2*d^3)*q*r*x^2 - 3*(b^3*c^2*d - 6*a*b^2*c*d^2 + 5*
a^2*b*d^3)*q*r*x - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*r*
log(f) - (2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*p + 3*(a*b^
2*c^2*d - 4*a^2*b*c*d^2 + 3*a^3*d^3)*q)*r + 6*(b^3*d^3*q*r*x^3 + 3*a*b^2*d^
3*q*r*x^2 + 3*a^2*b*d^3*q*r*x + (a^3*d^3*q - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*p)*r)*log(b*x + a) - 6*(b^3*d^3*q*r*x^3 + 3*a*b^2*d^3
*q*r*x^2 + 3*a^2*b*d^3*q*r*x + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2)*q*
r)*log(d*x + c) - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log
(e))/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^
3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a
^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*
b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21093, size = 633, normalized size = 3.86

$$\frac{d^3qr\log\left(bx+a\right)}{3\left(b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3\right)} - \frac{d^3qr\log\left(dx+c\right)}{3\left(b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3\right)} - \frac{pr\log\left(bx+a\right)}{3\left(b^4x^3 + 3\,ab^3x^2 + 3\,a^2b^2x + a^3b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/3*d^3*q*r*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b
*d^3) - 1/3*d^3*q*r*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2
 - a^3*b*d^3) - 1/3*p*r*log(b*x + a)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x +
 a^3*b) - 1/3*q*r*log(d*x + c)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b
) + 1/18*(6*b^2*d^2*q*r*x^2 - 3*b^2*c*d*q*r*x + 15*a*b*d^2*q*r*x - 2*b^2*c^
2*p*r + 4*a*b*c*d*p*r - 2*a^2*d^2*p*r - 3*a*b*c*d*q*r + 9*a^2*d^2*q*r - 6*b
^2*c^2*r*log(f) + 12*a*b*c*d*r*log(f) - 6*a^2*d^2*r*log(f) - 6*b^2*c^2 + 12
```

```
*a*b*c*d - 6*a^2*d^2)/(b^6*c^2*x^3 - 2*a*b^5*c*d*x^3 + a^2*b^4*d^2*x^3 + 3*
a*b^5*c^2*x^2 - 6*a^2*b^4*c*d*x^2 + 3*a^3*b^3*d^2*x^2 + 3*a^2*b^4*c^2*x - 6
*a^3*b^3*c*d*x + 3*a^4*b^2*d^2*x + a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)
```

3.15 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5} dx$

Optimal. Leaf size=193

$$-\frac{d^3qr}{4b(a+bx)(bc-ad)^3}+\frac{d^2qr}{8b(a+bx)^2(bc-ad)^2}-\frac{d^4qr\log(a+bx)}{4b(bc-ad)^4}+\frac{d^4qr\log(c+dx)}{4b(bc-ad)^4}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4}$$

```
[Out] -(p*r)/(16*b*(a + b*x)^4) - (d*q*r)/(12*b*(b*c - a*d)*(a + b*x)^3) + (d^2*q
*r)/(8*b*(b*c - a*d)^2*(a + b*x)^2) - (d^3*q*r)/(4*b*(b*c - a*d)^3*(a + b*x
)) - (d^4*q*r*Log[a + b*x])/(4*b*(b*c - a*d)^4) + (d^4*q*r*Log[c + d*x])/(4
*b*(b*c - a*d)^4) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(4*b*(a + b*x)^4)
```

Rubi [A] time = 0.0854361, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2495, 32, 44}

$$-\frac{d^3qr}{4b(a+bx)(bc-ad)^3}+\frac{d^2qr}{8b(a+bx)^2(bc-ad)^2}-\frac{d^4qr\log(a+bx)}{4b(bc-ad)^4}+\frac{d^4qr\log(c+dx)}{4b(bc-ad)^4}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^5,x]
```

```
[Out] -(p*r)/(16*b*(a + b*x)^4) - (d*q*r)/(12*b*(b*c - a*d)*(a + b*x)^3) + (d^2*q
*r)/(8*b*(b*c - a*d)^2*(a + b*x)^2) - (d^3*q*r)/(4*b*(b*c - a*d)^3*(a + b*x
)) - (d^4*q*r*Log[a + b*x])/(4*b*(b*c - a*d)^4) + (d^4*q*r*Log[c + d*x])/(4
*b*(b*c - a*d)^4) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(4*b*(a + b*x)^4)
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5}\,dx = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4} + \frac{1}{4}(pr)\int \frac{1}{(a+bx)^5}\,dx + \frac{(dqr)\int \frac{1}{(a+bx)^4(c+dx)}\,dx}{4b}$$

$$= -\frac{pr}{16b(a+bx)^4} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4} + \frac{(dqr)\int\left(\frac{b}{(bc-ad)(a+bx)^4} - \frac{bd}{(bc-ad)^2(a\cdots}\right.}{}$$

$$= -\frac{pr}{16b(a+bx)^4} - \frac{dqr}{12b(bc-ad)(a+bx)^3} + \frac{d^2qr}{8b(bc-ad)^2(a+bx)^2} - \frac{d^3qr}{4b(bc-ad)^3(a\cdots}$$

Mathematica [A] time = 0.406553, size = 164, normalized size = 0.85

$$\frac{r\left(\frac{-\frac{12d^3q(a+bx)^3}{(bc-ad)^3}+\frac{6d^2q(a+bx)^2}{(bc-ad)^2}+\frac{4dq(a+bx)}{ad-bc}-3p}{12(a+bx)^4}-\frac{d^4q\log(a+bx)}{(bc-ad)^4}+\frac{d^4q\log(c+dx)}{(bc-ad)^4}\right)-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^5,x]
```

```
[Out] (r*((-3*p + (4*d*q*(a + b*x))/(-(b*c) + a*d) + (6*d^2*q*(a + b*x)^2)/(b*c -
 a*d)^2 - (12*d^3*q*(a + b*x)^3)/(b*c - a*d)^3)/(12*(a + b*x)^4) - (d^4*q*L
og[a + b*x])/(b*c - a*d)^4 + (d^4*q*Log[c + d*x])/(b*c - a*d)^4) - Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]/(a + b*x)^4)/(4*b)
```

Maple [F] time = 0.424, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)}{(bx+a)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x)
```

Maxima [B] time = 1.27945, size = 620, normalized size = 3.21

$$-\frac{\left(2\left(\frac{6\,d^3\log(bx+a)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4} - \frac{6\,d^3\log(dx+c)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4} + \frac{6\,b^2d^2x\cdots}{a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2\cdots\right.}\right.\right.}{48\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(6*d^3*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4) - 6*d^3*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6
*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + (6*b^2*d^2*x^2 + 2*b^2*c^2 -
7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/(a^3*b^3*c^3 - 3*a^4*b^
```

```
2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*
d^2 - a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 -
 a^4*b^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^
5*b*d^3)*x))*d*f*q + 3*b*f*p/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3
*b^2*x + a^4*b))*r/(b*f) - 1/4*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((b*x +
 a)^4*b)
```

Fricas [B] time = 0.848858, size = 1758, normalized size = 9.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/48*(12*(b^4*c*d^3 - a*b^3*d^4)*q*r*x^3 - 6*(b^4*c^2*d^2 - 8*a*b^3*c*d^3
+ 7*a^2*b^2*d^4)*q*r*x^2 + 4*(b^4*c^3*d - 6*a*b^3*c^2*d^2 + 18*a^2*b^2*c*d^
3 - 13*a^3*b*d^4)*q*r*x + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4)*r*log(f) + (3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^
2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*p + 2*(2*a*b^3*c^3*d - 9*a^2*b^2*c^2*d
^2 + 18*a^3*b*c*d^3 - 11*a^4*d^4)*q)*r + 12*(b^4*d^4*q*r*x^4 + 4*a*b^3*d^4*
q*r*x^3 + 6*a^2*b^2*d^4*q*r*x^2 + 4*a^3*b*d^4*q*r*x + (a^4*d^4*q + (b^4*c^4
 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*p)*r)*log(b
*x + a) - 12*(b^4*d^4*q*r*x^4 + 4*a*b^3*d^4*q*r*x^3 + 6*a^2*b^2*d^4*q*r*x^2
 + 4*a^3*b*d^4*q*r*x - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3)*q*r)*log(d*x + c) + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d
^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(e))/(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^
6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*
a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2
*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^
2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3
*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^
3*c*d^3 + a^7*b^2*d^4)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34956, size = 1010, normalized size = 5.23

$$-\frac{d^4qr\log(bx+a)}{4\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)}+\frac{d^4qr\log(dx+c)}{4\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)}-\frac{}{4\left(b^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/4*d^4*q*r*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4) + 1/4*d^4*q*r*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^
3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*p*r*log(b*x +
a)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) - 1/4*q*r*
log(d*x + c)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
- 1/48*(12*b^3*d^3*q*r*x^3 - 6*b^3*c*d^2*q*r*x^2 + 42*a*b^2*d^3*q*r*x^2 + 4
*b^3*c^2*d*q*r*x - 20*a*b^2*c*d^2*q*r*x + 52*a^2*b*d^3*q*r*x + 3*b^3*c^3*p*
r - 9*a*b^2*c^2*d*p*r + 9*a^2*b*c*d^2*p*r - 3*a^3*d^3*p*r + 4*a*b^2*c^2*d*q
*r - 14*a^2*b*c*d^2*q*r + 22*a^3*d^3*q*r + 12*b^3*c^3*r*log(f) - 36*a*b^2*c
^2*d*r*log(f) + 36*a^2*b*c*d^2*r*log(f) - 12*a^3*d^3*r*log(f) + 12*b^3*c^3
- 36*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 12*a^3*d^3)/(b^8*c^3*x^4 - 3*a*b^7*c^2*
d*x^4 + 3*a^2*b^6*c*d^2*x^4 - a^3*b^5*d^3*x^4 + 4*a*b^7*c^3*x^3 - 12*a^2*b^
6*c^2*d*x^3 + 12*a^3*b^5*c*d^2*x^3 - 4*a^4*b^4*d^3*x^3 + 6*a^2*b^6*c^3*x^2
- 18*a^3*b^5*c^2*d*x^2 + 18*a^4*b^4*c*d^2*x^2 - 6*a^5*b^3*d^3*x^2 + 4*a^3*b
^5*c^3*x - 12*a^4*b^4*c^2*d*x + 12*a^5*b^3*c*d^2*x - 4*a^6*b^2*d^3*x + a^4*
b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)
```

3.16 $\int (a+bx)^4 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$

Optimal. Leaf size=920

$$-\frac{q^2r^2\log^2(c+dx)(bc-ad)^5}{5bd^5}-\frac{137q^2r^2\log(c+dx)(bc-ad)^5}{150bd^5}-\frac{2pqr^2\log(c+dx)(bc-ad)^5}{25bd^5}-\frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log}{5bd^5}$$

```
[Out] -(a*(b*c - a*d)^3*p*q*r^2*x)/(5*d^3) + (2*(b*c - a*d)^4*p*q*r^2*x)/(25*d^4)
 + (77*(b*c - a*d)^4*q^2*r^2*x)/(150*d^4) + (2*(b*c - a*d)^4*q*(p + q)*r^2*
x)/(5*d^4) - (b*(b*c - a*d)^3*p*q*r^2*x^2)/(10*d^3) - ((b*c - a*d)^3*p*q*r^
2*(a + b*x)^2)/(25*b*d^3) - (77*(b*c - a*d)^3*q^2*r^2*(a + b*x)^2)/(300*b*d
^3) + (16*(b*c - a*d)^2*p*q*r^2*(a + b*x)^3)/(225*b*d^2) + (47*(b*c - a*d)^
2*q^2*r^2*(a + b*x)^3)/(450*b*d^2) - (9*(b*c - a*d)*p*q*r^2*(a + b*x)^4)/(2
00*b*d) - (9*(b*c - a*d)*q^2*r^2*(a + b*x)^4)/(200*b*d) + (2*p^2*r^2*(a + b
*x)^5)/(125*b) + (4*p*q*r^2*(a + b*x)^5)/(125*b) + (2*q^2*r^2*(a + b*x)^5)/
(125*b) - (2*(b*c - a*d)^5*p*q*r^2*Log[c + d*x])/(25*b*d^5) - (137*(b*c - a
*d)^5*q^2*r^2*Log[c + d*x])/(150*b*d^5) - (2*(b*c - a*d)^5*p*q*r^2*Log[-((d
*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(5*b*d^5) - ((b*c - a*d)^5*q^2*r^2*
Log[c + d*x]^2)/(5*b*d^5) - (2*(b*c - a*d)^4*q*r*(a + b*x)*Log[e*(f*(a + b*
x)^p*(c + d*x)^q)^r])/(5*b*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)^2*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/(5*b*d^3) - (2*(b*c - a*d)^2*q*r*(a + b*x)^3*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(15*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)
^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(10*b*d) - (2*p*r*(a + b*x)^5*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(25*b) - (2*q*r*(a + b*x)^5*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(25*b) + (2*(b*c - a*d)^5*q*r*Log[c + d*x]*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/(5*b*d^5) + ((a + b*x)^5*Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]^2)/(5*b) - (2*(b*c - a*d)^5*p*q*r^2*PolyLog[2, (b*(c + d*
x))/(b*c - a*d)])/(5*b*d^5)
```

Rubi [A] time = 0.844897, antiderivative size = 920, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {2498, 2495, 32, 43, 2514, 2487, 31, 8, 2494, 2394, 2393, 2391, 2390, 2301}

$$-\frac{q^2r^2\log^2(c+dx)(bc-ad)^5}{5bd^5}-\frac{137q^2r^2\log(c+dx)(bc-ad)^5}{150bd^5}-\frac{2pqr^2\log(c+dx)(bc-ad)^5}{25bd^5}-\frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log}{5bd^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```


```
[Out] -(a*(b*c - a*d)^3*p*q*r^2*x)/(5*d^3) + (2*(b*c - a*d)^4*p*q*r^2*x)/(25*d^4)
 + (77*(b*c - a*d)^4*q^2*r^2*x)/(150*d^4) + (2*(b*c - a*d)^4*q*(p + q)*r^2*
x)/(5*d^4) - (b*(b*c - a*d)^3*p*q*r^2*x^2)/(10*d^3) - ((b*c - a*d)^3*p*q*r^
2*(a + b*x)^2)/(25*b*d^3) - (77*(b*c - a*d)^3*q^2*r^2*(a + b*x)^2)/(300*b*d
^3) + (16*(b*c - a*d)^2*p*q*r^2*(a + b*x)^3)/(225*b*d^2) + (47*(b*c - a*d)^
2*q^2*r^2*(a + b*x)^3)/(450*b*d^2) - (9*(b*c - a*d)*p*q*r^2*(a + b*x)^4)/(2
00*b*d) - (9*(b*c - a*d)*q^2*r^2*(a + b*x)^4)/(200*b*d) + (2*p^2*r^2*(a + b
*x)^5)/(125*b) + (4*p*q*r^2*(a + b*x)^5)/(125*b) + (2*q^2*r^2*(a + b*x)^5)/
(125*b) - (2*(b*c - a*d)^5*p*q*r^2*Log[c + d*x])/(25*b*d^5) - (137*(b*c - a
*d)^5*q^2*r^2*Log[c + d*x])/(150*b*d^5) - (2*(b*c - a*d)^5*p*q*r^2*Log[-((d
*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(5*b*d^5) - ((b*c - a*d)^5*q^2*r^2*
Log[c + d*x]^2)/(5*b*d^5) - (2*(b*c - a*d)^4*q*r*(a + b*x)*Log[e*(f*(a + b*
x)^p*(c + d*x)^q)^r])/(5*b*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)^2*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/(5*b*d^3) - (2*(b*c - a*d)^2*q*r*(a + b*x)^3*Lo
```

```
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(15*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)
^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(10*b*d) - (2*p*r*(a + b*x)^5*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(25*b) - (2*q*r*(a + b*x)^5*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(25*b) + (2*(b*c - a*d)^5*q*r*Log[c + d*x]*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/(5*b*d^5) + ((a + b*x)^5*Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]^2)/(5*b) - (2*(b*c - a*d)^5*p*q*r^2*PolyLog[2, (b*(c + d*
x))/(b*c - a*d)])/(5*b*d^5)
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 2487

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + (Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
```

```
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int (a+bx)^4 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \frac{(a+bx)^5 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5b} - \frac{1}{5}(2pr)\int (a+bx)^4 \log\left(e\left(f(\right.\right.$$

$$= -\frac{2pr(a+bx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{25b} + \frac{(a+bx)^5 \log^2\left(e\left(f(a+b\right.\right.}{5b}$$

$$= \frac{2p^2r^2(a+bx)^5}{125b} - \frac{2pr(a+bx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{25b} + \frac{(a+bx)^5}{}$$

$$= \frac{2(bc-ad)^4pqr^2x}{25d^4} - \frac{(bc-ad)^3pqr^2(a+bx)^2}{25bd^3} + \frac{2(bc-ad)^2pqr^2(a+bx)^3}{75bd^2}$$

$$= -\frac{a(bc-ad)^3pqr^2x}{5d^3} + \frac{2(bc-ad)^4pqr^2x}{25d^4} + \frac{2(bc-ad)^4q(p+q)r^2x}{5d^4} - \frac{b(bc}{}$$

$$= -\frac{a(bc-ad)^3pqr^2x}{5d^3} + \frac{2(bc-ad)^4pqr^2x}{25d^4} + \frac{77(bc-ad)^4q^2r^2x}{150d^4} + \frac{2(bc-ad}{}$$

$$= -\frac{a(bc-ad)^3pqr^2x}{5d^3} + \frac{2(bc-ad)^4pqr^2x}{25d^4} + \frac{77(bc-ad)^4q^2r^2x}{150d^4} + \frac{2(bc-ad}{}$$

<u>Mathematica</u> [B] time = 2.68567, size = 2508, normalized size = 2.73

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*a^5*p*q*r^2)/b + (2*a*b^3*c^4*p*q*r^2)/(5*d^4) - (2*a^2*b^2*c^3*p*q*r^2)
/d^3 + (4*a^3*b*c^2*p*q*r^2)/d^2 - (4*a^4*c*p*q*r^2)/d + (2*a^4*p^2*r^2*x)/
25 + (197*a^4*p*q*r^2*x)/150 + (12*b^4*c^4*p*q*r^2*x)/(25*d^4) - (11*a*b^3*
c^3*p*q*r^2*x)/(5*d^3) + (59*a^2*b^2*c^2*p*q*r^2*x)/(15*d^2) - (101*a^3*b*c
*p*q*r^2*x)/(30*d) + 2*a^4*q^2*r^2*x + (137*b^4*c^4*q^2*r^2*x)/(150*d^4) -
(25*a*b^3*c^3*q^2*r^2*x)/(6*d^3) + (22*a^2*b^2*c^2*q^2*r^2*x)/(3*d^2) - (6*
a^3*b*c*q^2*r^2*x)/d + (4*a^3*b*p^2*r^2*x^2)/25 + (283*a^3*b*p*q*r^2*x^2)/3
00 - (7*b^4*c^3*p*q*r^2*x^2)/(50*d^3) + (19*a*b^3*c^2*p*q*r^2*x^2)/(30*d^2)
 - (67*a^2*b^2*c*p*q*r^2*x^2)/(60*d) + a^3*b*q^2*r^2*x^2 - (77*b^4*c^3*q^2*
r^2*x^2)/(300*d^3) + (13*a*b^3*c^2*q^2*r^2*x^2)/(12*d^2) - (5*a^2*b^2*c*q^2
*r^2*x^2)/(3*d) + (4*a^2*b^2*p^2*r^2*x^3)/25 + (257*a^2*b^2*p*q*r^2*x^3)/45
0 + (16*b^4*c^2*p*q*r^2*x^3)/(225*d^2) - (29*a*b^3*c*p*q*r^2*x^3)/(90*d) +
(4*a^2*b^2*q^2*r^2*x^3)/9 + (47*b^4*c^2*q^2*r^2*x^3)/(450*d^2) - (7*a*b^3*c
*q^2*r^2*x^3)/(18*d) + (2*a*b^3*p^2*r^2*x^4)/25 + (41*a*b^3*p*q*r^2*x^4)/20
0 - (9*b^4*c*p*q*r^2*x^4)/(200*d) + (a*b^3*q^2*r^2*x^4)/8 - (9*b^4*c*q^2*r^
2*x^4)/(200*d) + (2*b^4*p^2*r^2*x^5)/125 + (4*b^4*p*q*r^2*x^5)/125 + (2*b^4
*q^2*r^2*x^5)/125 - (a^5*p^2*r^2*Log[a + b*x]^2)/(5*b) + (2*a^5*p*q*r^2*Log
[c + d*x])/b - (2*b^4*c^5*p*q*r^2*Log[c + d*x])/(25*d^5) + (2*a*b^3*c^4*p*q
*r^2*Log[c + d*x])/(5*d^4) - (4*a^2*b^2*c^3*p*q*r^2*Log[c + d*x])/(5*d^3) +
 (4*a^3*b*c^2*p*q*r^2*Log[c + d*x])/(5*d^2) - (2*a^4*c*p*q*r^2*Log[c + d*x]
)/(5*d) - (137*b^4*c^5*q^2*r^2*Log[c + d*x])/(150*d^5) + (25*a*b^3*c^4*q^2*
r^2*Log[c + d*x])/(6*d^4) - (22*a^2*b^2*c^3*q^2*r^2*Log[c + d*x])/(3*d^3) +
 (6*a^3*b*c^2*q^2*r^2*Log[c + d*x])/d^2 - (2*a^4*c*q^2*r^2*Log[c + d*x])/d
- (b^4*c^5*q^2*r^2*Log[c + d*x]^2)/(5*d^5) + (a*b^3*c^4*q^2*r^2*Log[c + d*x
]^2)/d^4 - (2*a^2*b^2*c^3*q^2*r^2*Log[c + d*x]^2)/d^3 + (2*a^3*b*c^2*q^2*r^
2*Log[c + d*x]^2)/d^2 - (a^4*c*q^2*r^2*Log[c + d*x]^2)/d - (2*a^5*p*r*Log[e
```

```
*(f*(a + b*x)^p*(c + d*x)^q)^r])/b - (2*a^4*p*r*x*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r])/5 - 2*a^4*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - (2*b^4*
c^4*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*d^4) + (2*a*b^3*c^3*q*r*
x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - (4*a^2*b^2*c^2*q*r*x*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (4*a^3*b*c*q*r*x*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r])/d - (4*a^3*b*p*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/
5 - 2*a^3*b*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (b^4*c^3*q*r*x^2
*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*d^3) - (a*b^3*c^2*q*r*x^2*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (2*a^2*b^2*c*q*r*x^2*Log[e*(f*(a + b*x
)^p*(c + d*x)^q)^r])/d - (4*a^2*b^2*p*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r])/5 - (4*a^2*b^2*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/3 - (2*
b^4*c^2*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(15*d^2) + (2*a*b^3*c
*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*d) - (2*a*b^3*p*r*x^4*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/5 - (a*b^3*q*r*x^4*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r])/2 + (b^4*c*q*r*x^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(
10*d) - (2*b^4*p*r*x^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/25 - (2*b^4*q*
r*x^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/25 + (2*b^4*c^5*q*r*Log[c + d*x
]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*d^5) - (2*a*b^3*c^4*q*r*Log[c +
d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^4 + (4*a^2*b^2*c^3*q*r*Log[c +
 d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - (4*a^3*b*c^2*q*r*Log[c +
d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (2*a^4*c*q*r*Log[c + d*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d + a^4*x*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]^2 + 2*a^3*b*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 2*a^2*b^
2*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + a*b^3*x^4*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r]^2 + (b^4*x^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/5 +
 (p*r*Log[a + b*x]*(a*d*(a^4*d^4*(288*p - 137*q) - 60*b^4*c^4*q + 270*a*b^3
*c^3*d*q - 470*a^2*b^2*c^2*d^2*q + 385*a^3*b*c*d^3*q)*r - 60*b*c*(b^4*c^4 -
 5*a*b^3*c^3*d + 10*a^2*b^2*c^2*d^2 - 10*a^3*b*c*d^3 + 5*a^4*d^4)*q*r*Log[c
 + d*x] + 60*(b*c - a*d)^5*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + 60*a^5*d^5*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(150*b*d^5) + (2*(b*c - a*d)^5*p*q*r
^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*d^5)
```

Maple [F] time = 0.414, size = 0, normalized size = 0.

$$\int (bx+a)^4 \left(\ln\left(e\left(f (bx+a)^p (dx+c)^q\right)^r\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((b*x+a)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Maxima [A] time = 1.57889, size = 1918, normalized size = 2.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)*log((
(b*x + a)^p*(d*x + c)^q*f)^r*e)^2 + 1/150*(60*a^5*f*p*log(b*x + a)/b - (12*
b^4*d^4*f*(p + q)*x^5 + 15*(a*b^3*d^4*f*(4*p + 5*q) - b^4*c*d^3*f*q)*x^4 +
```

```
20*(2*a^2*b^2*d^4*f*(3*p + 5*q) + b^4*c^2*d^2*f*q - 5*a*b^3*c*d^3*f*q)*x^3
+ 30*(2*a^3*b*d^4*f*(2*p + 5*q) - b^4*c^3*d*f*q + 5*a*b^3*c^2*d^2*f*q - 10*
a^2*b^2*c*d^3*f*q)*x^2 + 60*(a^4*d^4*f*(p + 5*q) + b^4*c^4*f*q - 5*a*b^3*c^
3*d*f*q + 10*a^2*b^2*c^2*d^2*f*q - 10*a^3*b*c*d^3*f*q)*x)/d^4 + 60*(b^4*c^5
*f*q - 5*a*b^3*c^4*d*f*q + 10*a^2*b^2*c^3*d^2*f*q - 10*a^3*b*c^2*d^3*f*q +
5*a^4*c*d^4*f*q)*log(d*x + c)/d^5)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f
 - 1/9000*r^2*(60*((12*p*q + 137*q^2)*b^4*c^5*f^2 - 5*(12*p*q + 125*q^2)*a*
b^3*c^4*d*f^2 + 20*(6*p*q + 55*q^2)*a^2*b^2*c^3*d^2*f^2 - 60*(2*p*q + 15*q^
2)*a^3*b*c^2*d^3*f^2 + 60*(p*q + 5*q^2)*a^4*c*d^4*f^2)*log(d*x + c)/d^5 - 3
600*(b^5*c^5*f^2*p*q - 5*a*b^4*c^4*d*f^2*p*q + 10*a^2*b^3*c^3*d^2*f^2*p*q -
 10*a^3*b^2*c^2*d^3*f^2*p*q + 5*a^4*b*c*d^4*f^2*p*q - a^5*d^5*f^2*p*q)*(log
(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c -
a*d)))/(b*d^5) - (144*(p^2 + 2*p*q + q^2)*b^5*d^5*f^2*x^5 - 1800*a^5*d^5*f^
2*p^2*log(b*x + a)^2 - 45*(9*(p*q + q^2)*b^5*c*d^4*f^2 - (16*p^2 + 41*p*q +
 25*q^2)*a*b^4*d^5*f^2)*x^4 + 20*((32*p*q + 47*q^2)*b^5*c^2*d^3*f^2 - 5*(29
*p*q + 35*q^2)*a*b^4*c*d^4*f^2 + (72*p^2 + 257*p*q + 200*q^2)*a^2*b^3*d^5*f
^2)*x^3 - 30*(7*(6*p*q + 11*q^2)*b^5*c^3*d^2*f^2 - 5*(38*p*q + 65*q^2)*a*b^
4*c^2*d^3*f^2 + 5*(67*p*q + 100*q^2)*a^2*b^3*c*d^4*f^2 - (48*p^2 + 283*p*q
+ 300*q^2)*a^3*b^2*d^5*f^2)*x^2 - 3600*(b^5*c^5*f^2*p*q - 5*a*b^4*c^4*d*f^2
*p*q + 10*a^2*b^3*c^3*d^2*f^2*p*q - 10*a^3*b^2*c^2*d^3*f^2*p*q + 5*a^4*b*c*
d^4*f^2*p*q)*log(b*x + a)*log(d*x + c) - 1800*(b^5*c^5*f^2*q^2 - 5*a*b^4*c^
4*d*f^2*q^2 + 10*a^2*b^3*c^3*d^2*f^2*q^2 - 10*a^3*b^2*c^2*d^3*f^2*q^2 + 5*a
^4*b*c*d^4*f^2*q^2)*log(d*x + c)^2 + 60*((72*p*q + 137*q^2)*b^5*c^4*d*f^2 -
 5*(66*p*q + 125*q^2)*a*b^4*c^3*d^2*f^2 + 10*(59*p*q + 110*q^2)*a^2*b^3*c^2
*d^3*f^2 - 5*(101*p*q + 180*q^2)*a^3*b^2*c*d^4*f^2 + (12*p^2 + 197*p*q + 30
0*q^2)*a^4*b*d^5*f^2)*x - 60*(60*a*b^4*c^4*d*f^2*p*q - 270*a^2*b^3*c^3*d^2*
f^2*p*q + 470*a^3*b^2*c^2*d^3*f^2*p*q - 385*a^4*b*c*d^4*f^2*p*q + (12*p^2 +
 137*p*q)*a^5*d^5*f^2)*log(b*x + a))/(b*d^5))/f^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^4x^4 + 4\,ab^3x^3 + 6\,a^2b^2x^2 + 4\,a^3bx + a^4\right)\log\left(\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
```

```
[Out] integral((b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(((b*
x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^4 \log \left(\left((bx+a)^p (dx+c)^q f \right)^r e \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^4*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.17 $\int (a+bx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

Optimal. Leaf size=805

$$\frac{q^2r^2\log^2(c+dx)(bc-ad)^4}{4bd^4}+\frac{25q^2r^2\log(c+dx)(bc-ad)^4}{24bd^4}+\frac{pqr^2\log(c+dx)(bc-ad)^4}{8bd^4}+\frac{pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx}{2bd^4}$$

```
[Out] (a*(b*c - a*d)^2*p*q*r^2*x)/(4*d^2) - ((b*c - a*d)^3*p*q*r^2*x)/(8*d^3) - (
13*(b*c - a*d)^3*q^2*r^2*x)/(24*d^3) - ((b*c - a*d)^3*q*(p + q)*r^2*x)/(2*d
^3) + (b*(b*c - a*d)^2*p*q*r^2*x^2)/(8*d^2) + ((b*c - a*d)^2*p*q*r^2*(a + b
*x)^2)/(16*b*d^2) + (13*(b*c - a*d)^2*q^2*r^2*(a + b*x)^2)/(48*b*d^2) - (7*
(b*c - a*d)*p*q*r^2*(a + b*x)^3)/(72*b*d) - (7*(b*c - a*d)*q^2*r^2*(a + b*x
)^3)/(72*b*d) + (p^2*r^2*(a + b*x)^4)/(32*b) + (p*q*r^2*(a + b*x)^4)/(16*b)
 + (q^2*r^2*(a + b*x)^4)/(32*b) + ((b*c - a*d)^4*p*q*r^2*Log[c + d*x])/(8*b
*d^4) + (25*(b*c - a*d)^4*q^2*r^2*Log[c + d*x])/(24*b*d^4) + ((b*c - a*d)^4
*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*b*d^4) + ((b*c
- a*d)^4*q^2*r^2*Log[c + d*x]^2)/(4*b*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)*L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*d^3) - ((b*c - a*d)^2*q*r*(a + b*
x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*b*d^2) + ((b*c - a*d)*q*r*(a
+ b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(6*b*d) - (p*r*(a + b*x)^4*L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(8*b) - (q*r*(a + b*x)^4*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(8*b) - ((b*c - a*d)^4*q*r*Log[c + d*x]*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/(2*b*d^4) + ((a + b*x)^4*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r]^2)/(4*b) + ((b*c - a*d)^4*p*q*r^2*PolyLog[2, (b*(c + d*x))/(
b*c - a*d)])/(2*b*d^4)
```

Rubi [A] time = 0.66471, antiderivative size = 805, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {2498, 2495, 32, 43, 2514, 2487, 31, 8, 2494, 2394, 2393, 2391, 2390, 2301}

$$\frac{q^2r^2\log^2(c+dx)(bc-ad)^4}{4bd^4}+\frac{25q^2r^2\log(c+dx)(bc-ad)^4}{24bd^4}+\frac{pqr^2\log(c+dx)(bc-ad)^4}{8bd^4}+\frac{pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx}{2bd^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```


```
[Out] (a*(b*c - a*d)^2*p*q*r^2*x)/(4*d^2) - ((b*c - a*d)^3*p*q*r^2*x)/(8*d^3) - (
13*(b*c - a*d)^3*q^2*r^2*x)/(24*d^3) - ((b*c - a*d)^3*q*(p + q)*r^2*x)/(2*d
^3) + (b*(b*c - a*d)^2*p*q*r^2*x^2)/(8*d^2) + ((b*c - a*d)^2*p*q*r^2*(a + b
*x)^2)/(16*b*d^2) + (13*(b*c - a*d)^2*q^2*r^2*(a + b*x)^2)/(48*b*d^2) - (7*
(b*c - a*d)*p*q*r^2*(a + b*x)^3)/(72*b*d) - (7*(b*c - a*d)*q^2*r^2*(a + b*x
)^3)/(72*b*d) + (p^2*r^2*(a + b*x)^4)/(32*b) + (p*q*r^2*(a + b*x)^4)/(16*b)
 + (q^2*r^2*(a + b*x)^4)/(32*b) + ((b*c - a*d)^4*p*q*r^2*Log[c + d*x])/(8*b
*d^4) + (25*(b*c - a*d)^4*q^2*r^2*Log[c + d*x])/(24*b*d^4) + ((b*c - a*d)^4
*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*b*d^4) + ((b*c
- a*d)^4*q^2*r^2*Log[c + d*x]^2)/(4*b*d^4) + ((b*c - a*d)^3*q*r*(a + b*x)*L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*d^3) - ((b*c - a*d)^2*q*r*(a + b*
x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*b*d^2) + ((b*c - a*d)*q*r*(a
+ b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(6*b*d) - (p*r*(a + b*x)^4*L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(8*b) - (q*r*(a + b*x)^4*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(8*b) - ((b*c - a*d)^4*q*r*Log[c + d*x]*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/(2*b*d^4) + ((a + b*x)^4*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r]^2)/(4*b) + ((b*c - a*d)^4*p*q*r^2*PolyLog[2, (b*(c + d*x))/(
```

```
b*c - a*d)])/(2*b*d^4)
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 2487

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + (Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int (a+bx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \frac{(a+bx)^4 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b} - \frac{1}{2}(pr)\int (a+bx)^3 \log\left(e\left(\right.\right.$$

$$= -\frac{pr(a+bx)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b} + \frac{(a+bx)^4 \log^2\left(e\left(f(a+\right.\right.}{4b}$$

$$= \frac{p^2r^2(a+bx)^4}{32b} - \frac{pr(a+bx)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b} + \frac{(a+bx)}{}$$

$$= -\frac{(bc-ad)^3pqr^2x}{8d^3} + \frac{(bc-ad)^2pqr^2(a+bx)^2}{16bd^2} - \frac{(bc-ad)pqr^2(a+bx)^3}{24bd}$$

$$= \frac{a(bc-ad)^2pqr^2x}{4d^2} - \frac{(bc-ad)^3pqr^2x}{8d^3} - \frac{(bc-ad)^3q(p+q)r^2x}{2d^3} + \frac{b(bc-}{}$$

$$= \frac{a(bc-ad)^2pqr^2x}{4d^2} - \frac{(bc-ad)^3pqr^2x}{8d^3} - \frac{13(bc-ad)^3q^2r^2x}{24d^3} - \frac{(bc-ad)^3}{2}$$

$$= \frac{a(bc-ad)^2pqr^2x}{4d^2} - \frac{(bc-ad)^3pqr^2x}{8d^3} - \frac{13(bc-ad)^3q^2r^2x}{24d^3} - \frac{(bc-ad)^3}{2}$$

<u>**Mathematica**</u> **[B]** time = 1.85919, size = 1853, normalized size = 2.3

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*a^4*p*q*r^2)/b - (a*b^2*c^3*p*q*r^2)/(2*d^3) + (2*a^2*b*c^2*p*q*r^2)/d^2
 - (3*a^3*c*p*q*r^2)/d + (a^3*p^2*r^2*x)/8 + (37*a^3*p*q*r^2*x)/24 - (5*b^3
*c^3*p*q*r^2*x)/(8*d^3) + (9*a*b^2*c^2*p*q*r^2*x)/(4*d^2) - (35*a^2*b*c*p*q
*r^2*x)/(12*d) + 2*a^3*q^2*r^2*x - (25*b^3*c^3*q^2*r^2*x)/(24*d^3) + (11*a*
b^2*c^2*q^2*r^2*x)/(3*d^2) - (9*a^2*b*c*q^2*r^2*x)/(2*d) + (3*a^2*b*p^2*r^2
*x^2)/16 + (41*a^2*b*p*q*r^2*x^2)/48 + (3*b^3*c^2*p*q*r^2*x^2)/(16*d^2) - (
2*a*b^2*c*p*q*r^2*x^2)/(3*d) + (3*a^2*b*q^2*r^2*x^2)/4 + (13*b^3*c^2*q^2*r^
2*x^2)/(48*d^2) - (5*a*b^2*c*q^2*r^2*x^2)/(6*d) + (a*b^2*p^2*r^2*x^3)/8 + (
25*a*b^2*p*q*r^2*x^3)/72 - (7*b^3*c*p*q*r^2*x^3)/(72*d) + (2*a*b^2*q^2*r^2*
x^3)/9 - (7*b^3*c*q^2*r^2*x^3)/(72*d) + (b^3*p^2*r^2*x^4)/32 + (b^3*p*q*r^2
*x^4)/16 + (b^3*q^2*r^2*x^4)/32 - (a^4*p^2*r^2*Log[a + b*x]^2)/(4*b) + (2*a
^4*p*q*r^2*Log[c + d*x])/b + (b^3*c^4*p*q*r^2*Log[c + d*x])/(8*d^4) - (a*b^
2*c^3*p*q*r^2*Log[c + d*x])/(2*d^3) + (3*a^2*b*c^2*p*q*r^2*Log[c + d*x])/(4
*d^2) - (a^3*c*p*q*r^2*Log[c + d*x])/(2*d) + (25*b^3*c^4*q^2*r^2*Log[c + d*
x])/(24*d^4) - (11*a*b^2*c^3*q^2*r^2*Log[c + d*x])/(3*d^3) + (9*a^2*b*c^2*q
^2*r^2*Log[c + d*x])/(2*d^2) - (2*a^3*c*q^2*r^2*Log[c + d*x])/d + (b^3*c^4*
q^2*r^2*Log[c + d*x]^2)/(4*d^4) - (a*b^2*c^3*q^2*r^2*Log[c + d*x]^2)/d^3 +
(3*a^2*b*c^2*q^2*r^2*Log[c + d*x]^2)/(2*d^2) - (a^3*c*q^2*r^2*Log[c + d*x]^
2)/d - (2*a^4*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/b - (a^3*p*r*x*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/2 - 2*a^3*q*r*x*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r] + (b^3*c^3*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*d^3)
- (2*a*b^2*c^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (3*a^2*b*c
*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d - (3*a^2*b*p*r*x^2*Log[e*(f*
(a + b*x)^p*(c + d*x)^q)^r])/4 - (3*a^2*b*q*r*x^2*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r])/2 - (b^3*c^2*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*
d^2) + (a*b^2*c*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d - (a*b^2*p*
```

```
r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/2 - (2*a*b^2*q*r*x^3*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/3 + (b^3*c*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r])/(6*d) - (b^3*p*r*x^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/8 - (b
^3*q*r*x^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/8 - (b^3*c^4*q*r*Log[c + d
*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*d^4) + (2*a*b^2*c^3*q*r*Log[c
+ d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - (3*a^2*b*c^2*q*r*Log[c +
 d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (2*a^3*c*q*r*Log[c + d*x]
*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d + a^3*x*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^2 + (3*a^2*b*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/2 + a*b
^2*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + (b^3*x^4*Log[e*(f*(a + b*x)
^p*(c + d*x)^q)^r]^2)/4 + (p*r*Log[a + b*x]*(a*d*(5*a^3*d^3*(9*p - 5*q) + 1
2*b^3*c^3*q - 42*a*b^2*c^2*d*q + 52*a^2*b*c*d^2*q)*r + 12*b*c*(b^3*c^3 - 4*
a*b^2*c^2*d + 6*a^2*b*c*d^2 - 4*a^3*d^3)*q*r*Log[c + d*x] - 12*(b*c - a*d)^
4*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + 12*a^4*d^4*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]))/(24*b*d^4) - ((b*c - a*d)^4*p*q*r^2*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)])/(2*b*d^4)
```

Maple [F] time = 0.402, size = 0, normalized size = 0.

$$\int (bx + a)^3 \left(\ln\left(e\left(f\,(bx + a)^p\,(dx + c)^q\right)^r\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((b*x+a)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Maxima [A] time = 1.47114, size = 1446, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
```

```
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)^2 + 1/24*(12*a^4*f*p*log(b*x + a)/b - (3*b^3*d^3*f*(p + q)*x^
4 + 4*(a*b^2*d^3*f*(3*p + 4*q) - b^3*c*d^2*f*q)*x^3 + 6*(3*a^2*b*d^3*f*(p +
 2*q) + b^3*c^2*d*f*q - 4*a*b^2*c*d^2*f*q)*x^2 + 12*(a^3*d^3*f*(p + 4*q) -
b^3*c^3*f*q + 4*a*b^2*c^2*d*f*q - 6*a^2*b*c*d^2*f*q)*x)/d^3 - 12*(b^3*c^4*f
*q - 4*a*b^2*c^3*d*f*q + 6*a^2*b*c^2*d^2*f*q - 4*a^3*c*d^3*f*q)*log(d*x + c
)/d^4)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f + 1/288*r^2*(12*((3*p*q + 2
5*q^2)*b^3*c^4*f^2 - 4*(3*p*q + 22*q^2)*a*b^2*c^3*d*f^2 + 18*(p*q + 6*q^2)*
a^2*b*c^2*d^2*f^2 - 12*(p*q + 4*q^2)*a^3*c*d^3*f^2)*log(d*x + c)/d^4 - 144*
(b^4*c^4*f^2*p*q - 4*a*b^3*c^3*d*f^2*p*q + 6*a^2*b^2*c^2*d^2*f^2*p*q - 4*a^
3*b*c*d^3*f^2*p*q + a^4*d^4*f^2*p*q)*(log(b*x + a)*log((b*d*x + a*d)/(b*c -
 a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*d^4) + (9*(p^2 + 2*p*q +
 q^2)*b^4*d^4*f^2*x^4 - 72*a^4*d^4*f^2*p^2*log(b*x + a)^2 - 4*(7*(p*q + q^2
)*b^4*c*d^3*f^2 - (9*p^2 + 25*p*q + 16*q^2)*a*b^3*d^4*f^2)*x^3 + 6*((9*p*q
+ 13*q^2)*b^4*c^2*d^2*f^2 - 8*(4*p*q + 5*q^2)*a*b^3*c*d^3*f^2 + (9*p^2 + 41
*p*q + 36*q^2)*a^2*b^2*d^4*f^2)*x^2 + 144*(b^4*c^4*f^2*p*q - 4*a*b^3*c^3*d*
f^2*p*q + 6*a^2*b^2*c^2*d^2*f^2*p*q - 4*a^3*b*c*d^3*f^2*p*q)*log(b*x + a)*l
og(d*x + c) + 72*(b^4*c^4*f^2*q^2 - 4*a*b^3*c^3*d*f^2*q^2 + 6*a^2*b^2*c^2*d
```

```
^2*f^2*q^2 - 4*a^3*b*c*d^3*f^2*q^2)*log(d*x + c)^2 - 12*(5*(3*p*q + 5*q^2)*
b^4*c^3*d*f^2 - 2*(27*p*q + 44*q^2)*a*b^3*c^2*d^2*f^2 + 2*(35*p*q + 54*q^2)
*a^2*b^2*c*d^3*f^2 - (3*p^2 + 37*p*q + 48*q^2)*a^3*b*d^4*f^2)*x + 12*(12*a*
b^3*c^3*d*f^2*p*q - 42*a^2*b^2*c^2*d^2*f^2*p*q + 52*a^3*b*c*d^3*f^2*p*q - (
3*p^2 + 25*p*q)*a^4*d^4*f^2)*log(b*x + a))/(b*d^4))/f^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3\right)\log\left(\left((bx + a)^p(dx + c)^q f\right)^r e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
```

```
[Out] integral((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*log(((b*x + a)^p*(d*x +
c)^q*f)^r*e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^3 \log\left(\left((bx + a)^p(dx + c)^q f\right)^r e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^3*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.18 $\int (a+bx)^2 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$

Optimal. Leaf size=686

$$-\frac{2pqr^2(bc-ad)^3\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3bd^3}+\frac{2qr(bc-ad)^3\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3bd^3}-\frac{2qr(a+bx)(bc-ad)^2}{}$$

```
[Out] -(a*(b*c - a*d)*p*q*r^2*x)/(3*d) + (2*(b*c - a*d)^2*p*q*r^2*x)/(9*d^2) + (5
*(b*c - a*d)^2*q^2*r^2*x)/(9*d^2) + (2*(b*c - a*d)^2*q*(p + q)*r^2*x)/(3*d^
2) - (b*(b*c - a*d)*p*q*r^2*x^2)/(6*d) - ((b*c - a*d)*p*q*r^2*(a + b*x)^2)/
(9*b*d) - (5*(b*c - a*d)*q^2*r^2*(a + b*x)^2)/(18*b*d) + (2*p^2*r^2*(a + b*
x)^3)/(27*b) + (4*p*q*r^2*(a + b*x)^3)/(27*b) + (2*q^2*r^2*(a + b*x)^3)/(27
*b) - (2*(b*c - a*d)^3*p*q*r^2*Log[c + d*x])/(9*b*d^3) - (11*(b*c - a*d)^3*
q^2*r^2*Log[c + d*x])/(9*b*d^3) - (2*(b*c - a*d)^3*p*q*r^2*Log[-((d*(a + b*
x))/(b*c - a*d))]*Log[c + d*x])/(3*b*d^3) - ((b*c - a*d)^3*q^2*r^2*Log[c +
d*x]^2)/(3*b*d^3) - (2*(b*c - a*d)^2*q*r*(a + b*x)*Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r])/(3*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)^2*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r])/(3*b*d) - (2*p*r*(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r])/(9*b) - (2*q*r*(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/
(9*b) + (2*(b*c - a*d)^3*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)
^r])/(3*b*d^3) + ((a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(3*b)
 - (2*(b*c - a*d)^3*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*b*d^3
)
```

Rubi [A] time = 0.532864, antiderivative size = 686, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {2498, 2495, 32, 43, 2514, 2487, 31, 8, 2494, 2394, 2393, 2391, 2390, 2301}

$$-\frac{2pqr^2(bc-ad)^3\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3bd^3}+\frac{2qr(bc-ad)^3\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3bd^3}-\frac{2qr(a+bx)(bc-ad)^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```


```
[Out] -(a*(b*c - a*d)*p*q*r^2*x)/(3*d) + (2*(b*c - a*d)^2*p*q*r^2*x)/(9*d^2) + (5
*(b*c - a*d)^2*q^2*r^2*x)/(9*d^2) + (2*(b*c - a*d)^2*q*(p + q)*r^2*x)/(3*d^
2) - (b*(b*c - a*d)*p*q*r^2*x^2)/(6*d) - ((b*c - a*d)*p*q*r^2*(a + b*x)^2)/
(9*b*d) - (5*(b*c - a*d)*q^2*r^2*(a + b*x)^2)/(18*b*d) + (2*p^2*r^2*(a + b*
x)^3)/(27*b) + (4*p*q*r^2*(a + b*x)^3)/(27*b) + (2*q^2*r^2*(a + b*x)^3)/(27
*b) - (2*(b*c - a*d)^3*p*q*r^2*Log[c + d*x])/(9*b*d^3) - (11*(b*c - a*d)^3*
q^2*r^2*Log[c + d*x])/(9*b*d^3) - (2*(b*c - a*d)^3*p*q*r^2*Log[-((d*(a + b*
x))/(b*c - a*d))]*Log[c + d*x])/(3*b*d^3) - ((b*c - a*d)^3*q^2*r^2*Log[c +
d*x]^2)/(3*b*d^3) - (2*(b*c - a*d)^2*q*r*(a + b*x)*Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r])/(3*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)^2*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r])/(3*b*d) - (2*p*r*(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r])/(9*b) - (2*q*r*(a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/
(9*b) + (2*(b*c - a*d)^3*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)
^r])/(3*b*d^3) + ((a + b*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(3*b)
 - (2*(b*c - a*d)^3*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*b*d^3
)
```


Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 2487

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + (Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
```

```
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int (a+bx)^2 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \frac{(a+bx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b} - \frac{1}{3}(2pr)\int (a+bx)^2 \log\left(e\right.$$

$$= -\frac{2pr(a+bx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b} + \frac{(a+bx)^3 \log^2\left(e\left(f(a\right.\right.}{3b}$$

$$= \frac{2p^2r^2(a+bx)^3}{27b} - \frac{2pr(a+bx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b} + \frac{(a+b}{}$$

$$= \frac{2(bc-ad)^2pqr^2x}{9d^2} - \frac{(bc-ad)pqr^2(a+bx)^2}{9bd} + \frac{2p^2r^2(a+bx)^3}{27b} + \frac{2pqr^2}{}$$

$$= -\frac{a(bc-ad)pqr^2x}{3d} + \frac{2(bc-ad)^2pqr^2x}{9d^2} + \frac{2(bc-ad)^2q(p+q)r^2x}{3d^2} - \frac{b(b}{}$$

$$= -\frac{a(bc-ad)pqr^2x}{3d} + \frac{2(bc-ad)^2pqr^2x}{9d^2} + \frac{5(bc-ad)^2q^2r^2x}{9d^2} + \frac{2(bc-a}{}$$

$$= -\frac{a(bc-ad)pqr^2x}{3d} + \frac{2(bc-ad)^2pqr^2x}{9d^2} + \frac{5(bc-ad)^2q^2r^2x}{9d^2} + \frac{2(bc-a}{}$$

<u>Mathematica</u> [A] time = 1.12677, size = 1211, normalized size = 1.77

$$\frac{1}{54}\left(\frac{108pqr^2a^3}{b} - \frac{18p^2r^2\log^2(a+bx)a^3}{b} + \frac{108pqr^2\log(c+dx)a^3}{b} - \frac{108pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)a^3}{b} - \frac{108cpqr^2}{d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] ((108*a^3*p*q*r^2)/b + (36*a*b*c^2*p*q*r^2)/d^2 - (108*a^2*c*p*q*r^2)/d + 1
2*a^2*p^2*r^2*x + 102*a^2*p*q*r^2*x + (48*b^2*c^2*p*q*r^2*x)/d^2 - (126*a*b
*c*p*q*r^2*x)/d + 108*a^2*q^2*r^2*x + (66*b^2*c^2*q^2*r^2*x)/d^2 - (162*a*b
*c*q^2*r^2*x)/d + 12*a*b*p^2*r^2*x^2 + 39*a*b*p*q*r^2*x^2 - (15*b^2*c*p*q*r
^2*x^2)/d + 27*a*b*q^2*r^2*x^2 - (15*b^2*c*q^2*r^2*x^2)/d + 4*b^2*p^2*r^2*x
^3 + 8*b^2*p*q*r^2*x^3 + 4*b^2*q^2*r^2*x^3 - (18*a^3*p^2*r^2*Log[a + b*x]^2
)/b + (108*a^3*p*q*r^2*Log[c + d*x])/b - (12*b^2*c^3*p*q*r^2*Log[c + d*x])/
d^3 + (36*a*b*c^2*p*q*r^2*Log[c + d*x])/d^2 - (36*a^2*c*p*q*r^2*Log[c + d*x
])/d - (66*b^2*c^3*q^2*r^2*Log[c + d*x])/d^3 + (162*a*b*c^2*q^2*r^2*Log[c +
 d*x])/d^2 - (108*a^2*c*q^2*r^2*Log[c + d*x])/d - (18*b^2*c^3*q^2*r^2*Log[c
 + d*x]^2)/d^3 + (54*a*b*c^2*q^2*r^2*Log[c + d*x]^2)/d^2 - (54*a^2*c*q^2*r^
2*Log[c + d*x]^2)/d - (108*a^3*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/b
- 36*a^2*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 108*a^2*q*r*x*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r] - (36*b^2*c^2*q*r*x*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r])/d^2 + (108*a*b*c*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d
 - 36*a*b*p*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 54*a*b*q*r*x^2*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (18*b^2*c*q*r*x^2*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r])/d - 12*b^2*p*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] -
12*b^2*q*r*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (36*b^2*c^3*q*r*Log[c
 + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^3 - (108*a*b*c^2*q*r*Log[c
+ d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d^2 + (108*a^2*c*q*r*Log[c + d
*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/d + 54*a^2*x*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^2 + 54*a*b*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 18
```

```
*b^2*x^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + (6*p*r*Log[a + b*x]*(a*d*
(a^2*d^2*(16*p - 11*q) - 6*b^2*c^2*q + 15*a*b*c*d*q)*r - 6*b*c*(b^2*c^2 - 3
*a*b*c*d + 3*a^2*d^2)*q*r*Log[c + d*x] + 6*(b*c - a*d)^3*q*r*Log[(b*(c + d*
x))/(b*c - a*d)] + 6*a^3*d^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(b*d^3)
 + (36*(b*c - a*d)^3*p*q*r^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*d
^3))/54
```

Maple [F] time = 0.404, size = 0, normalized size = 0.

$$\int (bx+a)^2 \left(\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((b*x+a)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Maxima [A] time = 1.42788, size = 1038, normalized size = 1.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2
+ 1/9*(6*a^3*f*p*log(b*x + a)/b - (2*b^2*d^2*f*(p + q)*x^3 + 3*(a*b*d^2*f*(
2*p + 3*q) - b^2*c*d*f*q)*x^2 + 6*(a^2*d^2*f*(p + 3*q) + b^2*c^2*f*q - 3*a*
b*c*d*f*q)*x)/d^2 + 6*(b^2*c^3*f*q - 3*a*b*c^2*d*f*q + 3*a^2*c*d^2*f*q)*log
(d*x + c)/d^3)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f - 1/54*r^2*(6*((2*p
*q + 11*q^2)*b^2*c^3*f^2 - 3*(2*p*q + 9*q^2)*a*b*c^2*d*f^2 + 6*(p*q + 3*q^2
)*a^2*c*d^2*f^2)*log(d*x + c)/d^3 - 36*(b^3*c^3*f^2*p*q - 3*a*b^2*c^2*d*f^2
*p*q + 3*a^2*b*c*d^2*f^2*p*q - a^3*d^3*f^2*p*q)*(log(b*x + a)*log((b*d*x +
a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*d^3) - (4*(p^
2 + 2*p*q + q^2)*b^3*d^3*f^2*x^3 - 18*a^3*d^3*f^2*p^2*log(b*x + a)^2 - 3*(5
*(p*q + q^2)*b^3*c*d^2*f^2 - (4*p^2 + 13*p*q + 9*q^2)*a*b^2*d^3*f^2)*x^2 -
36*(b^3*c^3*f^2*p*q - 3*a*b^2*c^2*d*f^2*p*q + 3*a^2*b*c*d^2*f^2*p*q)*log(b*
x + a)*log(d*x + c) - 18*(b^3*c^3*f^2*q^2 - 3*a*b^2*c^2*d*f^2*q^2 + 3*a^2*b
*c*d^2*f^2*q^2)*log(d*x + c)^2 + 6*((8*p*q + 11*q^2)*b^3*c^2*d*f^2 - 3*(7*p
*q + 9*q^2)*a*b^2*c*d^2*f^2 + (2*p^2 + 17*p*q + 18*q^2)*a^2*b*d^3*f^2)*x -
6*(6*a*b^2*c^2*d*f^2*p*q - 15*a^2*b*c*d^2*f^2*p*q + (2*p^2 + 11*p*q)*a^3*d^
3*f^2)*log(b*x + a))/(b*d^3))/f^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^2 + 2\,abx + a^2\right)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)^2 \log\left(\left((bx+a)^p (dx+c)^q f\right)^r e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.19 $\int (a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx$

Optimal. Leaf size=540

$$\frac{pqr^2(bc-ad)^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd^2}-\frac{qr(bc-ad)^2\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{bd^2}+\frac{pqr^2(bc-ad)^2\log(c+dx)}{2bd^2}$$

```
[Out] (a*p^2*r^2*x)/2 + (a*p*q*r^2*x)/2 - ((b*c - a*d)*p*q*r^2*x)/(2*d) - ((b*c -
 a*d)*q^2*r^2*x)/(2*d) - ((b*c - a*d)*q*(p + q)*r^2*x)/d + (b*p^2*r^2*x^2)/
4 + (b*p*q*r^2*x^2)/4 + (p*q*r^2*(a + b*x)^2)/(4*b) + (q^2*r^2*(a + b*x)^2)
/(4*b) + ((b*c - a*d)^2*p*q*r^2*Log[c + d*x])/(2*b*d^2) + (3*(b*c - a*d)^2*
q^2*r^2*Log[c + d*x])/(2*b*d^2) + ((b*c - a*d)^2*p*q*r^2*Log[-((d*(a + b*x)
)/(b*c - a*d))]*Log[c + d*x])/(b*d^2) + ((b*c - a*d)^2*q^2*r^2*Log[c + d*x]
^2)/(2*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r])/(b*d) - (p*r*(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b)
- (q*r*(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b) - ((b*c - a*
d)^2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*d^2) + ((a +
 b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(2*b) + ((b*c - a*d)^2*p*q*
r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*d^2)
```

Rubi [A] time = 0.390924, antiderivative size = 540, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {2498, 2495, 43, 2514, 2487, 31, 8, 2494, 2394, 2393, 2391, 2390, 2301}

$$\frac{pqr^2(bc-ad)^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd^2}-\frac{qr(bc-ad)^2\log(c+dx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{bd^2}+\frac{pqr^2(bc-ad)^2\log(c+dx)}{2bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```


```
[Out] (a*p^2*r^2*x)/2 + (a*p*q*r^2*x)/2 - ((b*c - a*d)*p*q*r^2*x)/(2*d) - ((b*c -
 a*d)*q^2*r^2*x)/(2*d) - ((b*c - a*d)*q*(p + q)*r^2*x)/d + (b*p^2*r^2*x^2)/
4 + (b*p*q*r^2*x^2)/4 + (p*q*r^2*(a + b*x)^2)/(4*b) + (q^2*r^2*(a + b*x)^2)
/(4*b) + ((b*c - a*d)^2*p*q*r^2*Log[c + d*x])/(2*b*d^2) + (3*(b*c - a*d)^2*
q^2*r^2*Log[c + d*x])/(2*b*d^2) + ((b*c - a*d)^2*p*q*r^2*Log[-((d*(a + b*x)
)/(b*c - a*d))]*Log[c + d*x])/(b*d^2) + ((b*c - a*d)^2*q^2*r^2*Log[c + d*x]
^2)/(2*b*d^2) + ((b*c - a*d)*q*r*(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r])/(b*d) - (p*r*(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b)
- (q*r*(a + b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b) - ((b*c - a*
d)^2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*d^2) + ((a +
 b*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(2*b) + ((b*c - a*d)^2*p*q*
r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*d^2)
```


Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[s, 0] && NeQ[m, -1]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 2487

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + (Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int (a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx = \frac{(a+bx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b} - (pr)\int (a+bx)\log\left(e\left(f(a+b\right.\right.$$

$$= -\frac{pr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b} + \frac{(a+bx)^2\log^2\left(e\left(f(a+bx)\right.\right.}{2b}$$

$$= \frac{1}{2}ap^2r^2x + \frac{1}{4}bp^2r^2x^2 - \frac{pr(a+bx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b} + \frac{(a+}{}$$

$$= \frac{1}{2}ap^2r^2x - \frac{(bc-ad)pqr^2x}{2d} + \frac{1}{4}bp^2r^2x^2 + \frac{pqr^2(a+bx)^2}{4b} + \frac{(bc-ad)^2pqr^2}{2bd^2}$$

$$= \frac{1}{2}ap^2r^2x + \frac{1}{2}apqr^2x - \frac{(bc-ad)pqr^2x}{2d} - \frac{(bc-ad)q(p+q)r^2x}{d} + \frac{1}{4}bp^2r^2x^2$$

$$= \frac{1}{2}ap^2r^2x + \frac{1}{2}apqr^2x - \frac{(bc-ad)pqr^2x}{2d} - \frac{(bc-ad)q^2r^2x}{2d} - \frac{(bc-ad)q(p+}{d}$$

$$= \frac{1}{2}ap^2r^2x + \frac{1}{2}apqr^2x - \frac{(bc-ad)pqr^2x}{2d} - \frac{(bc-ad)q^2r^2x}{2d} - \frac{(bc-ad)q(p+}{d}$$

Mathematica [A] time = 0.570108, size = 781, normalized size = 1.45

$$\frac{-4pqr^2(bc-ad)^2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right) - 8a^2d^2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - 2a^2d^2p^2r^2\log^2(a+bx) + 8a^2d^2pqr^2\log(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (-4*a*b*c*d*p*q*r^2 + 8*a^2*d^2*p*q*r^2 + 2*a*b*d^2*p^2*r^2*x - 6*b^2*c*d*p
*q*r^2*x + 10*a*b*d^2*p*q*r^2*x - 6*b^2*c*d*q^2*r^2*x + 8*a*b*d^2*q^2*r^2*x
 + b^2*d^2*p^2*r^2*x^2 + 2*b^2*d^2*p*q*r^2*x^2 + b^2*d^2*q^2*r^2*x^2 - 2*a^
2*d^2*p^2*r^2*Log[a + b*x]^2 + 2*b^2*c^2*p*q*r^2*Log[c + d*x] - 4*a*b*c*d*p
*q*r^2*Log[c + d*x] + 8*a^2*d^2*p*q*r^2*Log[c + d*x] + 6*b^2*c^2*q^2*r^2*Lo
g[c + d*x] - 8*a*b*c*d*q^2*r^2*Log[c + d*x] + 2*b^2*c^2*q^2*r^2*Log[c + d*x
]^2 - 4*a*b*c*d*q^2*r^2*Log[c + d*x]^2 - 8*a^2*d^2*p*r*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r] - 4*a*b*d^2*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 4
*b^2*c*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 8*a*b*d^2*q*r*x*Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b^2*d^2*p*r*x^2*Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r] - 2*b^2*d^2*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*
b^2*c^2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 8*a*b*c*d*q
*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 4*a*b*d^2*x*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^2 + 2*b^2*d^2*x^2*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r]^2 + 2*p*r*Log[a + b*x]*(2*b*c*(b*c - 2*a*d)*q*r*Log[c + d*x] - 2*(
b*c - a*d)^2*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(3*a*d*(p - q)*r + 2*
b*c*q*r + 2*a*d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])) - 4*(b*c - a*d)^2*p*
q*r^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b*d^2)
```

Maple [F] time = 0.166, size = 0, normalized size = 0.

$$\int \left(bx+a\right)\left(\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((b*x+a)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Maxima [A] time = 1.5037, size = 680, normalized size = 1.26

$$\frac{1}{2}\left(bx^{2}+2\,ax\right)\log\left(\left(\left(bx+a\right)^{p}\left(dx+c\right)^{q}f\right)^{r}e\right)^{2}+\frac{\left(\frac{2\,a^{2}fp\log\left(bx+a\right)}{b}-\frac{bdf\left(p+q\right)x^{2}+2\left(adf\left(p+2\,q\right)-bcfq\right)x}{d}-\frac{2\left(bc^{2}fq-2\,acdfq\right)\log\left(dx+c\right)}{d^{2}}\right.}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2 + 1/2*(2*a^2*f*p
*log(b*x + a)/b - (b*d*f*(p + q)*x^2 + 2*(a*d*f*(p + 2*q) - b*c*f*q)*x)/d -
 2*(b*c^2*f*q - 2*a*c*d*f*q)*log(d*x + c)/d^2)*r*log(((b*x + a)^p*(d*x + c)
^q*f)^r*e)/f + 1/4*r^2*(2*((p*q + 3*q^2)*b*c^2*f^2 - 2*(p*q + 2*q^2)*a*c*d*
f^2)*log(d*x + c)/d^2 - 4*(b^2*c^2*f^2*p*q - 2*a*b*c*d*f^2*p*q + a^2*d^2*f^
2*p*q)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a
*d)/(b*c - a*d)))/(b*d^2) - (2*a^2*d^2*f^2*p^2*log(b*x + a)^2 - (p^2 + 2*p*
q + q^2)*b^2*d^2*f^2*x^2 - 4*(b^2*c^2*f^2*p*q - 2*a*b*c*d*f^2*p*q)*log(b*x
+ a)*log(d*x + c) - 2*(b^2*c^2*f^2*q^2 - 2*a*b*c*d*f^2*q^2)*log(d*x + c)^2
+ 2*(3*(p*q + q^2)*b^2*c*d*f^2 - (p^2 + 5*p*q + 4*q^2)*a*b*d^2*f^2)*x - 2*(
2*a*b*c*d*f^2*p*q - (p^2 + 3*p*q)*a^2*d^2*f^2)*log(b*x + a))/(b*d^2))/f^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx+a)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.20 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx}\,dx$

Optimal. Leaf size=431

$$-\frac{1}{4}\left(-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)\right)\left(8\left(\frac{qr\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b}+\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{}\right.\right.$$

```
[Out] Log[(a + b*x)^(p*r)]^3/(3*b*p*r) - (q*Log[(a + b*x)^(p*r)]^2*Log[(b*(c + d*
x))/(b*c - a*d)])/(b*p) + (Log[(a + b*x)^(p*r)]^2*Log[(c + d*x)^(q*r)])/(b*
p*r) + (Log[-((d*(a + b*x))/(b*c - a*d))]*Log[(c + d*x)^(q*r)]^2)/b - (2*q*
r*Log[(a + b*x)^(p*r)]*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/b + (2*q*r
*Log[(c + d*x)^(q*r)]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b - ((Log[(a +
 b*x)^(p*r)] + Log[(c + d*x)^(q*r)] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])
*((Log[(a + b*x)^(p*r)] - Log[(c + d*x)^(q*r)] + Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r])^2/(b*p*r) + 8*((Log[-((d*(a + b*x))/(b*c - a*d))]*Log[(c + d*x)
^(q*r)])/b + (q*r*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b)))/4 + (2*p*q*r^
2*PolyLog[3, -((d*(a + b*x))/(b*c - a*d))])/b - (2*q^2*r^2*PolyLog[3, (b*(c
 + d*x))/(b*c - a*d)])/b
```

Rubi [A] time = 0.491518, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.484, Rules used = {2496, 6742, 2390, 2302, 30, 2433, 2375, 2317, 2374, 6589, 2396, 2394, 2393, 2391, 6686}

$$-\frac{1}{4}\left(-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)+\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)\right)\left(8\left(\frac{qr\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b}+\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x),x]
```


```
[Out] Log[(a + b*x)^(p*r)]^3/(3*b*p*r) - (q*Log[(a + b*x)^(p*r)]^2*Log[(b*(c + d*
x))/(b*c - a*d)])/(b*p) + (Log[(a + b*x)^(p*r)]^2*Log[(c + d*x)^(q*r)])/(b*
p*r) + (Log[-((d*(a + b*x))/(b*c - a*d))]*Log[(c + d*x)^(q*r)]^2)/b - (2*q*
r*Log[(a + b*x)^(p*r)]*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/b + (2*q*r
*Log[(c + d*x)^(q*r)]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b - ((Log[(a +
 b*x)^(p*r)] + Log[(c + d*x)^(q*r)] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])
*((Log[(a + b*x)^(p*r)] - Log[(c + d*x)^(q*r)] + Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r])^2/(b*p*r) + 8*((Log[-((d*(a + b*x))/(b*c - a*d))]*Log[(c + d*x)
^(q*r)])/b + (q*r*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b)))/4 + (2*p*q*r^
2*PolyLog[3, -((d*(a + b*x))/(b*c - a*d))])/b - (2*q^2*r^2*PolyLog[3, (b*(c
 + d*x))/(b*c - a*d)])/b
```


Rule 2496

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^2/((g_.) + (h_.)*(x_)), x_Symbol] :> Int[(Log[(a + b*x)^(p*r)] + Lo
g[(c + d*x)^(q*r)])^2/(g + h*x), x] + Simp[(Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r] - Log[(a + b*x)^(p*r)] - Log[(c + d*x)^(q*r)])*(2*Int[Log[(c + d*x)^(
q*r)]/(g + h*x), x] + Int[(Log[(a + b*x)^(p*r)] - Log[(c + d*x)^(q*r)] + Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(g + h*x), x]), x] /; FreeQ[{a, b, c, d
, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0] && EqQ[b*g - a*h, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{a+bx}\,dx = \int \frac{(\log((a+bx)^{pr})+\log((c+dx)^{qr}))^2}{a+bx}\,dx - \left(\log((a+bx)^{pr})+\log((c+dx)^{qr}) - \right.$$

$$= -\left(\log((a+bx)^{pr})+\log((c+dx)^{qr})-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)\left(\frac{\left(\log((a+\right.}{}\right.$$

$$= 2\int \frac{\log((a+bx)^{pr})\log((c+dx)^{qr})}{a+bx}\,dx - \left(\log((a+bx)^{pr})+\log((c+dx)^{qr})-\log\right.$$

$$= \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log^2((c+dx)^{qr})}{b} - \left(\log((a+bx)^{pr})+\log((c+dx)^{qr})-\log\left(e\left(f(a\right.\right.\right.$$

$$= \frac{\log^2((a+bx)^{pr})\log((c+dx)^{qr})}{bpr} + \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log^2((c+dx)^{qr})}{b} - \left(\log((a+bx\right.$$

$$= \frac{\log^3((a+bx)^{pr})}{3bpr} - \frac{q\log^2((a+bx)^{pr})\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bp} + \frac{\log^2((a+bx)^{pr})\log((c+dx}{bpr}$$

$$= \frac{\log^3((a+bx)^{pr})}{3bpr} - \frac{q\log^2((a+bx)^{pr})\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bp} + \frac{\log^2((a+bx)^{pr})\log((c+dx}{bpr}$$

$$= \frac{\log^3((a+bx)^{pr})}{3bpr} - \frac{q\log^2((a+bx)^{pr})\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bp} + \frac{\log^2((a+bx)^{pr})\log((c+dx}{bpr}$$

Mathematica [A] time = 0.17125, size = 460, normalized size = 1.07

$$\frac{6qr\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)\left(\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)-pr\log(a+bx)\right)+6pqr^2\text{PolyLog}\left(3,\frac{d(a+bx)}{ad-bc}\right)-6pqr^2\log(a+bx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x),x]
```

```
[Out] (p^2*r^2*Log[a + b*x]^3 + 6*p*q*r^2*Log[a + b*x]^2*Log[c + d*x] - 6*p*q*r^2
*Log[a + b*x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 3*q^2*r^2*Lo
g[a + b*x]*Log[c + d*x]^2 - 3*q^2*r^2*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log
[c + d*x]^2 - 3*p*q*r^2*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)] - 3*p
*r*Log[a + b*x]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 6*q*r*Log[a + b*x]
*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 6*q*r*Log[(d*(a + b*x)
)/(-(b*c) + a*d)]*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 3*Log
[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 6*p*q*r^2*Log[a + b*x]*P
```

```
olyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 6*q*r*(-(p*r*Log[a + b*x]) + Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 6
*p*q*r^2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*q^2*r^2*PolyLog[3, (b
*(c + d*x))/(b*c - a*d)])/(3*b)
```

Maple [F] time = 0.416, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)\right)^{2}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log(bx+a)\log\left(\left((dx+c)^{q}\right)^{r}\right)^{2}}{b}+\int \frac{\left(\log(e)^{2}+2\,\log(e)\log\left(f^{r}\right)+\log\left(f^{r}\right)^{2}\right)bdx+\left(\log(e)^{2}+2\,\log(e)\log\left(f^{r}\right)+\mathrm{lo}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] log(b*x + a)*log(((d*x + c)^q)^r)^2/b + integrate(((log(e)^2 + 2*log(e)*log
(f^r) + log(f^r)^2)*b*d*x + (log(e)^2 + 2*log(e)*log(f^r) + log(f^r)^2)*b*c
 + (b*d*x + b*c)*log(((b*x + a)^p)^r)^2 + 2*(b*d*x*(log(e) + log(f^r)) + b*
c*(log(e) + log(f^r)))*log(((b*x + a)^p)^r) + 2*(b*d*x*(log(e) + log(f^r))
+ b*c*(log(e) + log(f^r)) - (b*d*q*r*x + a*d*q*r)*log(b*x + a) + (b*d*x + b
*c)*log(((b*x + a)^p)^r))*log(((d*x + c)^q)^r))/(b^2*d*x^2 + a*b*c + (b^2*c
 + a*b*d)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)^{2}}{bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a), x)
```

3.21 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}\,dx$

Optimal. Leaf size=465

$$\frac{2dpqr^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)}-\frac{2dq^2r^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)}-\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)}+\frac{2dqr\log(a+bx)\log(}{b(b}$$

```
[Out] (-2*p^2*r^2)/(b*(a + b*x)) + (2*d*p*q*r^2*Log[a + b*x])/(b*(b*c - a*d)) - (
d*p*q*r^2*Log[a + b*x]^2)/(b*(b*c - a*d)) - (2*d*p*q*r^2*Log[c + d*x])/(b*(
b*c - a*d)) + (2*d*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/
(b*(b*c - a*d)) + (d*q^2*r^2*Log[c + d*x]^2)/(b*(b*c - a*d)) - (2*d*q^2*r^2
*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)) - (2*p*r*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(a + b*x)) + (2*d*q*r*Log[a + b*x]*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)) - (2*d*q*r*Log[c + d*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)) - Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]^2/(b*(a + b*x)) - (2*d*q^2*r^2*PolyLog[2, -((d*(a + b*x))
/(b*c - a*d))])/(b*(b*c - a*d)) + (2*d*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)])/(b*(b*c - a*d))
```

Rubi [A] time = 0.387124, antiderivative size = 465, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {2498, 2495, 32, 36, 31, 2514, 2494, 2390, 2301, 2394, 2393, 2391}

$$\frac{2dpqr^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)}-\frac{2dq^2r^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)}-\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)}+\frac{2dqr\log(a+bx)\log(}{b(b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^2,x]
```


```
[Out] (-2*p^2*r^2)/(b*(a + b*x)) + (2*d*p*q*r^2*Log[a + b*x])/(b*(b*c - a*d)) - (
d*p*q*r^2*Log[a + b*x]^2)/(b*(b*c - a*d)) - (2*d*p*q*r^2*Log[c + d*x])/(b*(
b*c - a*d)) + (2*d*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/
(b*(b*c - a*d)) + (d*q^2*r^2*Log[c + d*x]^2)/(b*(b*c - a*d)) - (2*d*q^2*r^2
*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)) - (2*p*r*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(a + b*x)) + (2*d*q*r*Log[a + b*x]*Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)) - (2*d*q*r*Log[c + d*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)) - Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]^2/(b*(a + b*x)) - (2*d*q^2*r^2*PolyLog[2, -((d*(a + b*x))
/(b*c - a*d))])/(b*(b*c - a*d)) + (2*d*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)])/(b*(b*c - a*d))
```


Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}\,dx = -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + (2pr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^2}\,dx$$

$$= -\frac{2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} + \frac{(2dqr)}{}$$

$$= -\frac{2p^2r^2}{b(a+bx)} - \frac{2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(a+bx)} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right.\right.}{b(a+bx)}$$

$$= -\frac{2p^2r^2}{b(a+bx)} + \frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{2dpqr^2\log(c+dx)}{b(bc-ad)} - \frac{2pr\log\left(e\left(f(a+bx\right.\right.}{b(a+bx}$$

$$= -\frac{2p^2r^2}{b(a+bx)} + \frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{2dpqr^2\log(c+dx)}{b(bc-ad)} + \frac{2dpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right.}{b(bc-a}$$

$$= -\frac{2p^2r^2}{b(a+bx)} + \frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{dpqr^2\log^2(a+bx)}{b(bc-ad)} - \frac{2dpqr^2\log(c+dx)}{b(bc-ad)} -$$

$$= -\frac{2p^2r^2}{b(a+bx)} + \frac{2dpqr^2\log(a+bx)}{b(bc-ad)} - \frac{dpqr^2\log^2(a+bx)}{b(bc-ad)} - \frac{2dpqr^2\log(c+dx)}{b(bc-ad)} -$$

Mathematica [A] time = 0.836383, size = 411, normalized size = 0.88

$$\frac{-2dqr^2(p+q)(a+bx)\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right) - bc\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) + ad\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - 2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^2,x]
```

```
[Out] (-2*b*c*p^2*r^2 + 2*a*d*p^2*r^2 - d*p*q*r^2*(a + b*x)*Log[a + b*x]^2 - 2*a*
d*p*q*r^2*Log[c + d*x] - 2*b*d*p*q*r^2*x*Log[c + d*x] + a*d*q^2*r^2*Log[c +
 d*x]^2 + b*d*q^2*r^2*x*Log[c + d*x]^2 - 2*b*c*p*r*Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r] + 2*a*d*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*a*d*q*r*L
og[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b*d*q*r*x*Log[c + d*x]
*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - b*c*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^2 + a*d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 2*d*q*r*(a + b*x)*Lo
g[a + b*x]*(p*r + p*r*Log[c + d*x] - (p + q)*r*Log[(b*(c + d*x))/(b*c - a*d
)] + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) - 2*d*q*(p + q)*r^2*(a + b*x)*Po
lyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(b*c - a*d)*(a + b*x))
```

Maple [F] time = 0.41, size = 0, normalized size = 0.

$$\int \frac{\left(\ln \left(e \left(f \,(b x+a)^{p} \,(d x+c)^{q}\right)^{r}\right)\right)^{2}}{(b x+a)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x)
```

Maxima [A] time = 1.34225, size = 529, normalized size = 1.14

$$\frac{2\left(\frac{dfq\log(bx+a)}{bc-ad}-\frac{dfq\log(dx+c)}{bc-ad}-\frac{fp}{bx+a}\right)r\log\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)}{bf}-\frac{\left(\frac{2\,df^2pq\log(dx+c)}{bc-ad}+\frac{2\left(pq+q^2\right)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\right.\right.}{bc-ad}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x, algorithm="maxima
")
```

```
[Out] 2*(d*f*q*log(b*x + a)/(b*c - a*d) - d*f*q*log(d*x + c)/(b*c - a*d) - f*p/(b
*x + a))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) - (2*d*f^2*p*q*log(d*
x + c)/(b*c - a*d) + 2*(p*q + q^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a
*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*d*f^2/(b*c - a*d) + (2*b*c*f^
2*p^2 - 2*a*d*f^2*p^2 + (b*d*f^2*p*q*x + a*d*f^2*p*q)*log(b*x + a)^2 - 2*(b
*d*f^2*p*q*x + a*d*f^2*p*q)*log(b*x + a)*log(d*x + c) - (b*d*f^2*q^2*x + a*
d*f^2*q^2)*log(d*x + c)^2 - 2*(b*d*f^2*p*q*x + a*d*f^2*p*q)*log(b*x + a))/(
a*b*c - a^2*d + (b^2*c - a*b*d)*x))*r^2/(b*f^2) - log(((b*x + a)^p*(d*x + c
)^q*f)^r*e)^2/((b*x + a)*b)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^qf\right)^r e\right)^2}{b^2x^2+2\,abx+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^2*x^2 + 2*a*b*x + a^2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^2, x)
```

3.22 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}\,dx$

Optimal. Leaf size=632

$$-\frac{d^2pqr^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)^2}+\frac{d^2q^2r^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)^2}-\frac{d^2qr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)^2}+\frac{d^2qr\log}{}$$

```
[Out] -(p^2*r^2)/(4*b*(a + b*x)^2) - (3*d*p*q*r^2)/(2*b*(b*c - a*d)*(a + b*x)) -
(d^2*p*q*r^2*Log[a + b*x])/(2*b*(b*c - a*d)^2) + (d^2*q^2*r^2*Log[a + b*x])
/(b*(b*c - a*d)^2) + (d^2*p*q*r^2*Log[a + b*x]^2)/(2*b*(b*c - a*d)^2) + (d^
2*p*q*r^2*Log[c + d*x])/(2*b*(b*c - a*d)^2) - (d^2*q^2*r^2*Log[c + d*x])/(b
*(b*c - a*d)^2) - (d^2*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*
x])/(b*(b*c - a*d)^2) - (d^2*q^2*r^2*Log[c + d*x]^2)/(2*b*(b*c - a*d)^2) +
(d^2*q^2*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^2)
 - (p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(a + b*x)^2) - (d*q*r*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)*(a + b*x)) - (d^2*q*r*Lo
g[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)^2) + (d^2*q
*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)^2) - L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(2*b*(a + b*x)^2) + (d^2*q^2*r^2*Poly
Log[2, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)^2) - (d^2*p*q*r^2*Poly
Log[2, (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^2)
```

Rubi [A] time = 0.48971, antiderivative size = 632, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {2498, 2495, 32, 44, 2514, 36, 31, 2494, 2390, 2301, 2394, 2393, 2391}

$$-\frac{d^2pqr^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b(bc-ad)^2}+\frac{d^2q^2r^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{b(bc-ad)^2}-\frac{d^2qr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b(bc-ad)^2}+\frac{d^2qr\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^3,x]
```


```
[Out] -(p^2*r^2)/(4*b*(a + b*x)^2) - (3*d*p*q*r^2)/(2*b*(b*c - a*d)*(a + b*x)) -
(d^2*p*q*r^2*Log[a + b*x])/(2*b*(b*c - a*d)^2) + (d^2*q^2*r^2*Log[a + b*x])
/(b*(b*c - a*d)^2) + (d^2*p*q*r^2*Log[a + b*x]^2)/(2*b*(b*c - a*d)^2) + (d^
2*p*q*r^2*Log[c + d*x])/(2*b*(b*c - a*d)^2) - (d^2*q^2*r^2*Log[c + d*x])/(b
*(b*c - a*d)^2) - (d^2*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*
x])/(b*(b*c - a*d)^2) - (d^2*q^2*r^2*Log[c + d*x]^2)/(2*b*(b*c - a*d)^2) +
(d^2*q^2*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^2)
 - (p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(a + b*x)^2) - (d*q*r*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)*(a + b*x)) - (d^2*q*r*Lo
g[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)^2) + (d^2*q
*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(b*(b*c - a*d)^2) - L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(2*b*(a + b*x)^2) + (d^2*q^2*r^2*Poly
Log[2, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)^2) - (d^2*p*q*r^2*Poly
Log[2, (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)^2)
```


Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
```

```
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}\,dx = -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} + (pr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^3}\,dx + \ldots$$

$$= -\frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} + \frac{(dqr)\int \ldots}{\ldots}$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2b(a+bx)^2}\ldots$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2pqr^2\log(c+dx)}{2b(bc-ad)^2} - \ldots$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{3dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2pqr^2\log(c+dx)}{2b(bc-ad)^2} - \ldots$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{3dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2q^2r^2\log(a+bx)}{b(bc-ad)^2} + \ldots$$

$$= -\frac{p^2r^2}{4b(a+bx)^2} - \frac{3dpqr^2}{2b(bc-ad)(a+bx)} - \frac{d^2pqr^2\log(a+bx)}{2b(bc-ad)^2} + \frac{d^2q^2r^2\log(a+bx)}{b(bc-ad)^2} + \ldots$$

Mathematica [A] time = 1.60547, size = 872, normalized size = 1.38

$$-\frac{c^2p^2r^2b^2 + 2d^2q^2r^2x^2\log^2(c+dx)b^2 + 2c^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)b^2 + 6cdpqr^2xb^2 + 4d^2q^2r^2x^2\log(c+dx)b^2 - \ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^3,x]
```

```
[Out] -(b^2*c^2*p^2*r^2 - 2*a*b*c*d*p^2*r^2 + a^2*d^2*p^2*r^2 + 6*a*b*c*d*p*q*r^2
 - 6*a^2*d^2*p*q*r^2 + 6*b^2*c*d*p*q*r^2*x - 6*a*b*d^2*p*q*r^2*x - 2*d^2*p*
q*r^2*(a + b*x)^2*Log[a + b*x]^2 - 2*a^2*d^2*p*q*r^2*Log[c + d*x] + 4*a^2*d
^2*q^2*r^2*Log[c + d*x] - 4*a*b*d^2*p*q*r^2*x*Log[c + d*x] + 8*a*b*d^2*q^2*
r^2*x*Log[c + d*x] - 2*b^2*d^2*p*q*r^2*x^2*Log[c + d*x] + 4*b^2*d^2*q^2*r^2
*x^2*Log[c + d*x] + 2*a^2*d^2*q^2*r^2*Log[c + d*x]^2 + 4*a*b*d^2*q^2*r^2*x*
Log[c + d*x]^2 + 2*b^2*d^2*q^2*r^2*x^2*Log[c + d*x]^2 - 2*d^2*q*r*(a + b*x)
^2*Log[a + b*x]*(-(p*r) + 2*q*r - 2*p*r*Log[c + d*x] + 2*(p + q)*r*Log[(b*(
c + d*x))/(b*c - a*d)] - 2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + 2*b^2*c^
2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a*b*c*d*p*r*Log[e*(f*(a + b*
x)^p*(c + d*x)^q)^r] + 2*a^2*d^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] +
 4*a*b*c*d*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a^2*d^2*q*r*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r] + 4*b^2*c*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r] - 4*a*b*d^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*a^2*d^
2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 8*a*b*d^2*q*r*x*L
og[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 4*b^2*d^2*q*r*x^2*Log[c
+ d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b^2*c^2*Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]^2 - 4*a*b*c*d*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + 2*
a^2*d^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 4*d^2*q*(p + q)*r^2*(a + b
*x)^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b*(b*c - a*d)^2*(a + b*x
)^2)
```

Maple [F] time = 0.412, size = 0, normalized size = 0.

$$\int \frac{\left(\ln \left(e \left(f \left(bx+a\right)^{p} \left(dx+c\right)^{q}\right)^{r}\right)\right)^{2}}{\left(bx+a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x)
```

Maxima [A] time = 1.48079, size = 1019, normalized size = 1.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(2*d^2*f*q*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 2*d^2*f*q*lo
g(d*x + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + (2*b*d*f*q*x - a*d*f*(p - 2*q)
 + b*c*f*p)/(a^2*b*c - a^3*d + (b^3*c - a*b^2*d)*x^2 + 2*(a*b^2*c - a^2*b*d
)*x))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) + 1/4*(4*(p*q + q^2)*(lo
g(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c -
 a*d)))*d^2*f^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 2*(p*q - 2*q^2)*d^2*f^2*l
og(d*x + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - (b^2*c^2*f^2*p^2 - 2*(p^2 - 3
*p*q)*a*b*c*d*f^2 + (p^2 - 6*p*q)*a^2*d^2*f^2 - 2*(b^2*d^2*f^2*p*q*x^2 + 2*
a*b*d^2*f^2*p*q*x + a^2*d^2*f^2*p*q)*log(b*x + a)^2 + 4*(b^2*d^2*f^2*p*q*x^
```

```
2 + 2*a*b*d^2*f^2*p*q*x + a^2*d^2*f^2*p*q)*log(b*x + a)*log(d*x + c) + 2*(b
^2*d^2*f^2*q^2*x^2 + 2*a*b*d^2*f^2*q^2*x + a^2*d^2*f^2*q^2)*log(d*x + c)^2
+ 6*(b^2*c*d*f^2*p*q - a*b*d^2*f^2*p*q)*x + 2*((p*q - 2*q^2)*b^2*d^2*f^2*x^
2 + 2*(p*q - 2*q^2)*a*b*d^2*f^2*x + (p*q - 2*q^2)*a^2*d^2*f^2)*log(b*x + a)
)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d
^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x))*r^2/(b*f^2) - 1/2*l
og(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((b*x + a)^2*b)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^3*x^3 + 3*a*b^2*x^2 + 3*
a^2*b*x + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^3, x)
```

3.23 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4}\,dx$

Optimal. Leaf size=764

$$\frac{2d^3pqr^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3b(bc-ad)^3}-\frac{2d^3q^2r^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{3b(bc-ad)^3}+\frac{2d^3qr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(bc-ad)^3}-\frac{2d}{\ }$$

```
[Out] (-2*p^2*r^2)/(27*b*(a + b*x)^3) - (5*d*p*q*r^2)/(18*b*(b*c - a*d)*(a + b*x)
^2) + (8*d^2*p*q*r^2)/(9*b*(b*c - a*d)^2*(a + b*x)) - (d^2*q^2*r^2)/(3*b*(b
*c - a*d)^2*(a + b*x)) + (2*d^3*p*q*r^2*Log[a + b*x])/(9*b*(b*c - a*d)^3) -
 (d^3*q^2*r^2*Log[a + b*x])/(b*(b*c - a*d)^3) - (d^3*p*q*r^2*Log[a + b*x]^2
)/(3*b*(b*c - a*d)^3) - (2*d^3*p*q*r^2*Log[c + d*x])/(9*b*(b*c - a*d)^3) +
(d^3*q^2*r^2*Log[c + d*x])/(b*(b*c - a*d)^3) + (2*d^3*p*q*r^2*Log[-((d*(a +
 b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b*(b*c - a*d)^3) + (d^3*q^2*r^2*Log[c
 + d*x]^2)/(3*b*(b*c - a*d)^3) - (2*d^3*q^2*r^2*Log[a + b*x]*Log[(b*(c + d*
x))/(b*c - a*d)])/(3*b*(b*c - a*d)^3) - (2*p*r*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r])/(9*b*(a + b*x)^3) - (d*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/
(3*b*(b*c - a*d)*(a + b*x)^2) + (2*d^2*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r])/(3*b*(b*c - a*d)^2*(a + b*x)) + (2*d^3*q*r*Log[a + b*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(3*b*(b*c - a*d)^3) - (2*d^3*q*r*Log[c + d*x]*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*b*(b*c - a*d)^3) - Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]^2/(3*b*(a + b*x)^3) - (2*d^3*q^2*r^2*PolyLog[2, -((d*(a +
 b*x))/(b*c - a*d))])/(3*b*(b*c - a*d)^3) + (2*d^3*p*q*r^2*PolyLog[2, (b*(c
 + d*x))/(b*c - a*d)])/(3*b*(b*c - a*d)^3)
```

Rubi [A] time = 0.606462, antiderivative size = 764, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {2498, 2495, 32, 44, 2514, 36, 31, 2494, 2390, 2301, 2394, 2393, 2391}

$$\frac{2d^3pqr^2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{3b(bc-ad)^3}-\frac{2d^3q^2r^2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)}{3b(bc-ad)^3}+\frac{2d^3qr\log(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(bc-ad)^3}-\frac{2d}{\ }$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^4,x]
```


```
[Out] (-2*p^2*r^2)/(27*b*(a + b*x)^3) - (5*d*p*q*r^2)/(18*b*(b*c - a*d)*(a + b*x)
^2) + (8*d^2*p*q*r^2)/(9*b*(b*c - a*d)^2*(a + b*x)) - (d^2*q^2*r^2)/(3*b*(b
*c - a*d)^2*(a + b*x)) + (2*d^3*p*q*r^2*Log[a + b*x])/(9*b*(b*c - a*d)^3) -
 (d^3*q^2*r^2*Log[a + b*x])/(b*(b*c - a*d)^3) - (d^3*p*q*r^2*Log[a + b*x]^2
)/(3*b*(b*c - a*d)^3) - (2*d^3*p*q*r^2*Log[c + d*x])/(9*b*(b*c - a*d)^3) +
(d^3*q^2*r^2*Log[c + d*x])/(b*(b*c - a*d)^3) + (2*d^3*p*q*r^2*Log[-((d*(a +
 b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b*(b*c - a*d)^3) + (d^3*q^2*r^2*Log[c
 + d*x]^2)/(3*b*(b*c - a*d)^3) - (2*d^3*q^2*r^2*Log[a + b*x]*Log[(b*(c + d*
x))/(b*c - a*d)])/(3*b*(b*c - a*d)^3) - (2*p*r*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r])/(9*b*(a + b*x)^3) - (d*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/
(3*b*(b*c - a*d)*(a + b*x)^2) + (2*d^2*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r])/(3*b*(b*c - a*d)^2*(a + b*x)) + (2*d^3*q*r*Log[a + b*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(3*b*(b*c - a*d)^3) - (2*d^3*q*r*Log[c + d*x]*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*b*(b*c - a*d)^3) - Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]^2/(3*b*(a + b*x)^3) - (2*d^3*q^2*r^2*PolyLog[2, -((d*(a +
 b*x))/(b*c - a*d))])/(3*b*(b*c - a*d)^3) + (2*d^3*p*q*r^2*PolyLog[2, (b*(c
 + d*x))/(b*c - a*d)])/(3*b*(b*c - a*d)^3)
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4}\,dx = -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3} + \frac{1}{3}(2pr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^4}\,dx$$

$$= -\frac{2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b(a+bx)^3} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3b(a+bx)^3} + \frac{(2dqr)}{}$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{2pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{9b(a+bx)^3} - \frac{\log^2\left(e\left(f(a+bx)^p(c+\right.\right.}{3b(a+bx)^3}$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{dpqr^2}{9b(bc-ad)(a+bx)^2} + \frac{2d^2pqr^2}{9b(bc-ad)^2(a+bx)} + \frac{2d^3pqr^2\log(}{9b(bc-a}$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{5dpqr^2}{18b(bc-ad)(a+bx)^2} + \frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)} + \frac{2d^3pqr^2\log}{9b(bc-}$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{5dpqr^2}{18b(bc-ad)(a+bx)^2} + \frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)} - \frac{d^2q^2}{3b(bc-ad)^2}$$

$$= -\frac{2p^2r^2}{27b(a+bx)^3} - \frac{5dpqr^2}{18b(bc-ad)(a+bx)^2} + \frac{8d^2pqr^2}{9b(bc-ad)^2(a+bx)} - \frac{d^2q^2}{3b(bc-ad)^2}$$

Mathematica [A] time = 2.73125, size = 1407, normalized size = 1.84

$$-\frac{4c^3p^2r^2b^3 + 18cd^2q^2r^2x^2b^3 - 48cd^2pqr^2x^2b^3 - 18d^3q^2r^2x^3\log^2(c+dx)b^3 + 18c^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)b^3 + 15c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^4,x]
```

```
[Out] -(4*b^3*c^3*p^2*r^2 - 12*a*b^2*c^2*d*p^2*r^2 + 12*a^2*b*c*d^2*p^2*r^2 - 4*a
^3*d^3*p^2*r^2 + 15*a*b^2*c^2*d*p*q*r^2 - 78*a^2*b*c*d^2*p*q*r^2 + 63*a^3*d
^3*p*q*r^2 + 18*a^2*b*c*d^2*q^2*r^2 - 18*a^3*d^3*q^2*r^2 + 15*b^3*c^2*d*p*q
*r^2*x - 126*a*b^2*c*d^2*p*q*r^2*x + 111*a^2*b*d^3*p*q*r^2*x + 36*a*b^2*c*d
^2*q^2*r^2*x - 36*a^2*b*d^3*q^2*r^2*x - 48*b^3*c*d^2*p*q*r^2*x^2 + 48*a*b^2
*d^3*p*q*r^2*x^2 + 18*b^3*c*d^2*q^2*r^2*x^2 - 18*a*b^2*d^3*q^2*r^2*x^2 + 18
*d^3*p*q*r^2*(a + b*x)^3*Log[a + b*x]^2 + 12*a^3*d^3*p*q*r^2*Log[c + d*x] -
 54*a^3*d^3*q^2*r^2*Log[c + d*x] + 36*a^2*b*d^3*p*q*r^2*x*Log[c + d*x] - 16
2*a^2*b*d^3*q^2*r^2*x*Log[c + d*x] + 36*a*b^2*d^3*p*q*r^2*x^2*Log[c + d*x]
- 162*a*b^2*d^3*q^2*r^2*x^2*Log[c + d*x] + 12*b^3*d^3*p*q*r^2*x^3*Log[c + d
*x] - 54*b^3*d^3*q^2*r^2*x^3*Log[c + d*x] - 18*a^3*d^3*q^2*r^2*Log[c + d*x]
^2 - 54*a^2*b*d^3*q^2*r^2*x*Log[c + d*x]^2 - 54*a*b^2*d^3*q^2*r^2*x^2*Log[c
 + d*x]^2 - 18*b^3*d^3*q^2*r^2*x^3*Log[c + d*x]^2 + 12*b^3*c^3*p*r*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r] - 36*a*b^2*c^2*d*p*r*Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r] + 36*a^2*b*c*d^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 12*
a^3*d^3*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*a*b^2*c^2*d*q*r*Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r] - 72*a^2*b*c*d^2*q*r*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r] + 54*a^3*d^3*q*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*b
^3*c^2*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 108*a*b^2*c*d^2*q*r*x
*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 90*a^2*b*d^3*q*r*x*Log[e*(f*(a + b*
x)^p*(c + d*x)^q)^r] - 36*b^3*c*d^2*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r] + 36*a*b^2*d^3*q*r*x^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 36*a^3*
d^3*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 108*a^2*b*d^3*q
*r*x*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 108*a*b^2*d^3*q*r*
x^2*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 36*b^3*d^3*q*r*x^3*
Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*b^3*c^3*Log[e*(f*(a
+ b*x)^p*(c + d*x)^q)^r]^2 - 54*a*b^2*c^2*d*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^2 + 54*a^2*b*c*d^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 18*a^3*d^
3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 6*d^3*q*r*(a + b*x)^3*Log[a + b*
x]*(2*p*r - 9*q*r + 6*p*r*Log[c + d*x] - 6*(p + q)*r*Log[(b*(c + d*x))/(b*c
 - a*d)] + 6*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + 36*d^3*q*(p + q)*r^2*(
a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(54*b*(b*c - a*d)^3*(a
 + b*x)^3)
```

Maple [F] time = 0.414, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)\right)^2}{(bx+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x)
```

Maxima [A] time = 1.76099, size = 1690, normalized size = 2.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x, algorithm="maxima
")
```

```
[Out] 1/9*(6*d^3*f*q*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3) - 6*d^3*f*q*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3) + (6*b^2*d^2*f*q*x^2 + a*b*c*d*f*(4*p - 3*q) - a^2*d^2*f*(2*p - 9*q)
 - 2*b^2*c^2*f*p - 3*(b^2*c*d*f*q - 5*a*b*d^2*f*q)*x)/(a^3*b^2*c^2 - 2*a^4*
b*c*d + a^5*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^3 + 3*(a*b^4*c^2
- 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2 + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b
*d^2)*x))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) - 1/54*(36*(p*q + q^
2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/
(b*c - a*d)))*d^3*f^2/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) +
 6*(2*p*q - 9*q^2)*d^3*f^2*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3) + (4*b^3*c^3*f^2*p^2 - 3*(4*p^2 - 5*p*q)*a*b^2*c^2*d*f^2 +
 6*(2*p^2 - 13*p*q + 3*q^2)*a^2*b*c*d^2*f^2 - (4*p^2 - 63*p*q + 18*q^2)*a^3
*d^3*f^2 - 6*((8*p*q - 3*q^2)*b^3*c*d^2*f^2 - (8*p*q - 3*q^2)*a*b^2*d^3*f^2
)*x^2 + 18*(b^3*d^3*f^2*p*q*x^3 + 3*a*b^2*d^3*f^2*p*q*x^2 + 3*a^2*b*d^3*f^2
*p*q*x + a^3*d^3*f^2*p*q)*log(b*x + a)^2 - 36*(b^3*d^3*f^2*p*q*x^3 + 3*a*b^
2*d^3*f^2*p*q*x^2 + 3*a^2*b*d^3*f^2*p*q*x + a^3*d^3*f^2*p*q)*log(b*x + a)*l
og(d*x + c) - 18*(b^3*d^3*f^2*q^2*x^3 + 3*a*b^2*d^3*f^2*q^2*x^2 + 3*a^2*b*d
^3*f^2*q^2*x + a^3*d^3*f^2*q^2)*log(d*x + c)^2 + 3*(5*b^3*c^2*d*f^2*p*q - 6
*(7*p*q - 2*q^2)*a*b^2*c*d^2*f^2 + (37*p*q - 12*q^2)*a^2*b*d^3*f^2)*x - 6*(
(2*p*q - 9*q^2)*b^3*d^3*f^2*x^3 + 3*(2*p*q - 9*q^2)*a*b^2*d^3*f^2*x^2 + 3*(
2*p*q - 9*q^2)*a^2*b*d^3*f^2*x + (2*p*q - 9*q^2)*a^3*d^3*f^2)*log(b*x + a))
/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*
b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c
^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*
d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*x))*r^2/(b*f^2) - 1/3*log(((b*x + a)^p*(d*
x + c)^q*f)^r*e)^2/((b*x + a)^3*b)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^4*x^4 + 4*a*b^3*x^3 + 6*
a^2*b^2*x^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^4, x)
```

3.24 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5}\,dx$

Optimal. Leaf size=884

$$\frac{pqr^2\log^2(a+bx)d^4}{4b(bc-ad)^4} - \frac{q^2r^2\log^2(c+dx)d^4}{4b(bc-ad)^4} + \frac{11q^2r^2\log(a+bx)d^4}{12b(bc-ad)^4} - \frac{pqr^2\log(a+bx)d^4}{8b(bc-ad)^4} - \frac{11q^2r^2\log(c+dx)d^4}{12b(bc-ad)^4} + \frac{pqr^2}{8}$$

```
[Out] -(p^2*r^2)/(32*b*(a + b*x)^4) - (7*d*p*q*r^2)/(72*b*(b*c - a*d)*(a + b*x)^3
) + (3*d^2*p*q*r^2)/(16*b*(b*c - a*d)^2*(a + b*x)^2) - (d^2*q^2*r^2)/(12*b*
(b*c - a*d)^2*(a + b*x)^2) - (5*d^3*p*q*r^2)/(8*b*(b*c - a*d)^3*(a + b*x))
+ (5*d^3*q^2*r^2)/(12*b*(b*c - a*d)^3*(a + b*x)) - (d^4*p*q*r^2*Log[a + b*x
])/(8*b*(b*c - a*d)^4) + (11*d^4*q^2*r^2*Log[a + b*x])/(12*b*(b*c - a*d)^4)
 + (d^4*p*q*r^2*Log[a + b*x]^2)/(4*b*(b*c - a*d)^4) + (d^4*p*q*r^2*Log[c +
d*x])/(8*b*(b*c - a*d)^4) - (11*d^4*q^2*r^2*Log[c + d*x])/(12*b*(b*c - a*d)
^4) - (d^4*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*b*(b*
c - a*d)^4) - (d^4*q^2*r^2*Log[c + d*x]^2)/(4*b*(b*c - a*d)^4) + (d^4*q^2*r
^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(2*b*(b*c - a*d)^4) - (p*r*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(8*b*(a + b*x)^4) - (d*q*r*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/(6*b*(b*c - a*d)*(a + b*x)^3) + (d^2*q*r*Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*b*(b*c - a*d)^2*(a + b*x)^2) - (d^3*q*r*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(b*c - a*d)^3*(a + b*x)) - (d^4*
q*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(b*c - a*d)^4)
+ (d^4*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(b*c - a
*d)^4) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(4*b*(a + b*x)^4) + (d^4*q^
2*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(2*b*(b*c - a*d)^4) - (d^4*
p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(2*b*(b*c - a*d)^4)
```

Rubi [A] time = 0.737891, antiderivative size = 884, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.419, Rules used = {2498, 2495, 32, 44, 2514, 36, 31, 2494, 2390, 2301, 2394, 2393, 2391}

$$\frac{pqr^2\log^2(a+bx)d^4}{4b(bc-ad)^4} - \frac{q^2r^2\log^2(c+dx)d^4}{4b(bc-ad)^4} + \frac{11q^2r^2\log(a+bx)d^4}{12b(bc-ad)^4} - \frac{pqr^2\log(a+bx)d^4}{8b(bc-ad)^4} - \frac{11q^2r^2\log(c+dx)d^4}{12b(bc-ad)^4} + \frac{pqr^2}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^5,x]
```

```
[Out] -(p^2*r^2)/(32*b*(a + b*x)^4) - (7*d*p*q*r^2)/(72*b*(b*c - a*d)*(a + b*x)^3
) + (3*d^2*p*q*r^2)/(16*b*(b*c - a*d)^2*(a + b*x)^2) - (d^2*q^2*r^2)/(12*b*
(b*c - a*d)^2*(a + b*x)^2) - (5*d^3*p*q*r^2)/(8*b*(b*c - a*d)^3*(a + b*x))
+ (5*d^3*q^2*r^2)/(12*b*(b*c - a*d)^3*(a + b*x)) - (d^4*p*q*r^2*Log[a + b*x
])/(8*b*(b*c - a*d)^4) + (11*d^4*q^2*r^2*Log[a + b*x])/(12*b*(b*c - a*d)^4)
 + (d^4*p*q*r^2*Log[a + b*x]^2)/(4*b*(b*c - a*d)^4) + (d^4*p*q*r^2*Log[c +
d*x])/(8*b*(b*c - a*d)^4) - (11*d^4*q^2*r^2*Log[c + d*x])/(12*b*(b*c - a*d)
^4) - (d^4*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*b*(b*
c - a*d)^4) - (d^4*q^2*r^2*Log[c + d*x]^2)/(4*b*(b*c - a*d)^4) + (d^4*q^2*r
^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(2*b*(b*c - a*d)^4) - (p*r*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(8*b*(a + b*x)^4) - (d*q*r*Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r])/(6*b*(b*c - a*d)*(a + b*x)^3) + (d^2*q*r*Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*b*(b*c - a*d)^2*(a + b*x)^2) - (d^3*q*r*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(b*c - a*d)^3*(a + b*x)) - (d^4*
q*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(b*c - a*d)^4)
+ (d^4*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*b*(b*c - a
```

```
*d)^4) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(4*b*(a + b*x)^4) + (d^4*q^
2*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(2*b*(b*c - a*d)^4) - (d^4*
p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(2*b*(b*c - a*d)^4)
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFx_), x_Symbol] :> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
```

```
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5}\,dx = -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4} + \frac{1}{2}(pr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)^5}\,dx + \cdots$$

$$= -\frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b(a+bx)^4} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4b(a+bx)^4} + \frac{(dqr)\int\left(\cdots\right.}{\cdots}$$

$$= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{pr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{8b(a+bx)^4} - \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)\cdots\right.}{4b(a+bx)^4}$$

$$= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{dpqr^2}{24b(bc-ad)(a+bx)^3} + \frac{d^2pqr^2}{16b(bc-ad)^2(a+bx)^2} - \frac{d^3pqr^2\cdots}{8b(bc-ad)^3(\cdots}$$

$$= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{7dpqr^2}{72b(bc-ad)(a+bx)^3} + \frac{3d^2pqr^2}{16b(bc-ad)^2(a+bx)^2} - \frac{5d^3pqr\cdots}{8b(bc-ad)^3(\cdots}$$

$$= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{7dpqr^2}{72b(bc-ad)(a+bx)^3} + \frac{3d^2pqr^2}{16b(bc-ad)^2(a+bx)^2} - \frac{d^2q^2r\cdots}{12b(bc-ad)^2\cdots}$$

$$= -\frac{p^2r^2}{32b(a+bx)^4} - \frac{7dpqr^2}{72b(bc-ad)(a+bx)^3} + \frac{3d^2pqr^2}{16b(bc-ad)^2(a+bx)^2} - \frac{d^2q^2r\cdots}{12b(bc-ad)^2\cdots}$$

Mathematica [B] time = 7.02508, size = 14573, normalized size = 16.49

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(a + b*x)^5,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.418, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)\right)^2}{(bx+a)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x)
```

Maxima [B] time = 2.1752, size = 2452, normalized size = 2.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x, algorithm="maxima
")
```

```
[Out] -1/24*(12*d^4*f*q*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4) - 12*d^4*f*q*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^
3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + (12*b^3*d^3*f*q*x^3 -
a*b^2*c^2*d*f*(9*p - 4*q) + a^2*b*c*d^2*f*(9*p - 14*q) - a^3*d^3*f*(3*p - 2
2*q) + 3*b^3*c^3*f*p - 6*(b^3*c*d^2*f*q - 7*a*b^2*d^3*f*q)*x^2 + 4*(b^3*c^2
*d*f*q - 5*a*b^2*c*d^2*f*q + 13*a^2*b*d^3*f*q)*x)/(a^4*b^3*c^3 - 3*a^5*b^2*
c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^
2 - a^3*b^4*d^3)*x^4 + 4*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a
^4*b^3*d^3)*x^3 + 6*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*
b^2*d^3)*x^2 + 4*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d
^3)*x))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(b*f) + 1/288*(144*(p*q + q^
2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/
(b*c - a*d)))*d^4*f^2/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4) + 12*(3*p*q - 22*q^2)*d^4*f^2*log(d*x + c)/(b^4*c^4 - 4*
a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - (9*b^4*c^4*f^2
*p^2 - 4*(9*p^2 - 7*p*q)*a*b^3*c^3*d*f^2 + 6*(9*p^2 - 23*p*q + 4*q^2)*a^2*b
^2*c^2*d^2*f^2 - 12*(3*p^2 - 31*p*q + 14*q^2)*a^3*b*c*d^3*f^2 + (9*p^2 - 26
2*p*q + 144*q^2)*a^4*d^4*f^2 + 60*((3*p*q - 2*q^2)*b^4*c*d^3*f^2 - (3*p*q -
 2*q^2)*a*b^3*d^4*f^2)*x^3 - 6*((9*p*q - 4*q^2)*b^4*c^2*d^2*f^2 - 4*(27*p*q
 - 17*q^2)*a*b^3*c*d^3*f^2 + (99*p*q - 64*q^2)*a^2*b^2*d^4*f^2)*x^2 - 72*(b
^4*d^4*f^2*p*q*x^4 + 4*a*b^3*d^4*f^2*p*q*x^3 + 6*a^2*b^2*d^4*f^2*p*q*x^2 +
4*a^3*b*d^4*f^2*p*q*x + a^4*d^4*f^2*p*q)*log(b*x + a)^2 + 144*(b^4*d^4*f^2*
p*q*x^4 + 4*a*b^3*d^4*f^2*p*q*x^3 + 6*a^2*b^2*d^4*f^2*p*q*x^2 + 4*a^3*b*d^4
*f^2*p*q*x + a^4*d^4*f^2*p*q)*log(b*x + a)*log(d*x + c) + 72*(b^4*d^4*f^2*q
^2*x^4 + 4*a*b^3*d^4*f^2*q^2*x^3 + 6*a^2*b^2*d^4*f^2*q^2*x^2 + 4*a^3*b*d^4*
f^2*q^2*x + a^4*d^4*f^2*q^2)*log(d*x + c)^2 + 4*(7*b^4*c^3*d*f^2*p*q - 12*(
4*p*q - q^2)*a*b^3*c^2*d^2*f^2 + 6*(35*p*q - 19*q^2)*a^2*b^2*c*d^3*f^2 - (1
69*p*q - 102*q^2)*a^3*b*d^4*f^2)*x + 12*((3*p*q - 22*q^2)*b^4*d^4*f^2*x^4 +
 4*(3*p*q - 22*q^2)*a*b^3*d^4*f^2*x^3 + 6*(3*p*q - 22*q^2)*a^2*b^2*d^4*f^2*
x^2 + 4*(3*p*q - 22*q^2)*a^3*b*d^4*f^2*x + (3*p*q - 22*q^2)*a^4*d^4*f^2)*lo
g(b*x + a))/(a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*
d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*
d^3 + a^4*b^4*d^4)*x^4 + 4*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2
 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d +
6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4
*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x))*r^2/(
b*f^2) - 1/4*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((b*x + a)^4*b)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{b^5x^5+5\,ab^4x^4+10\,a^2b^3x^3+10\,a^3b^2x^2+5\,a^4bx+a^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b^5*x^5 + 5*a*b^4*x^4 + 10
*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{(bx+a)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(b*x + a)^5, x)
```

3.25 $\int (g+hx)^4 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$

Optimal. Leaf size=334

$$-\frac{prx(bg-ah)^4}{5b^4}-\frac{pr(g+hx)^2(bg-ah)^3}{10b^3h}-\frac{pr(g+hx)^3(bg-ah)^2}{15b^2h}-\frac{pr(bg-ah)^5\log(a+bx)}{5b^5h}+\frac{(g+hx)^5\log\left(e\left(f(a+\right.\right.}{5h}$$

```
[Out] -((b*g - a*h)^4*p*r*x)/(5*b^4) - ((d*g - c*h)^4*q*r*x)/(5*d^4) - ((b*g - a*
h)^3*p*r*(g + h*x)^2)/(10*b^3*h) - ((d*g - c*h)^3*q*r*(g + h*x)^2)/(10*d^3*
h) - ((b*g - a*h)^2*p*r*(g + h*x)^3)/(15*b^2*h) - ((d*g - c*h)^2*q*r*(g + h
*x)^3)/(15*d^2*h) - ((b*g - a*h)*p*r*(g + h*x)^4)/(20*b*h) - ((d*g - c*h)*q
*r*(g + h*x)^4)/(20*d*h) - (p*r*(g + h*x)^5)/(25*h) - (q*r*(g + h*x)^5)/(25
*h) - ((b*g - a*h)^5*p*r*Log[a + b*x])/(5*b^5*h) - ((d*g - c*h)^5*q*r*Log[c
 + d*x])/(5*d^5*h) + ((g + h*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*
h)
```

Rubi [A] time = 0.185563, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2495, 43}

$$-\frac{prx(bg-ah)^4}{5b^4}-\frac{pr(g+hx)^2(bg-ah)^3}{10b^3h}-\frac{pr(g+hx)^3(bg-ah)^2}{15b^2h}-\frac{pr(bg-ah)^5\log(a+bx)}{5b^5h}+\frac{(g+hx)^5\log\left(e\left(f(a+\right.\right.}{5h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```


```
[Out] -((b*g - a*h)^4*p*r*x)/(5*b^4) - ((d*g - c*h)^4*q*r*x)/(5*d^4) - ((b*g - a*
h)^3*p*r*(g + h*x)^2)/(10*b^3*h) - ((d*g - c*h)^3*q*r*(g + h*x)^2)/(10*d^3*
h) - ((b*g - a*h)^2*p*r*(g + h*x)^3)/(15*b^2*h) - ((d*g - c*h)^2*q*r*(g + h
*x)^3)/(15*d^2*h) - ((b*g - a*h)*p*r*(g + h*x)^4)/(20*b*h) - ((d*g - c*h)*q
*r*(g + h*x)^4)/(20*d*h) - (p*r*(g + h*x)^5)/(25*h) - (q*r*(g + h*x)^5)/(25
*h) - ((b*g - a*h)^5*p*r*Log[a + b*x])/(5*b^5*h) - ((d*g - c*h)^5*q*r*Log[c
 + d*x])/(5*d^5*h) + ((g + h*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5*
h)
```


Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (g+hx)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \frac{(g+hx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5h} - \frac{(bpr)\int \frac{(g+hx)^5}{a+bx} dx}{5h} - \frac{(dqr)\int \frac{(g+}{c+}}{5h}$$

$$= \frac{(g+hx)^5 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{5h} - \frac{(bpr)\int \left(\frac{h(bg-ah)^4}{b^5} + \frac{(bg-ah)^5}{b^5(a+bx)} + \cdots\right.}{}$$

$$= -\frac{(bg-ah)^4 prx}{5b^4} - \frac{(dg-ch)^4 qrx}{5d^4} - \frac{(bg-ah)^3 pr(g+hx)^2}{10b^3h} - \frac{(dg-ch)^3 qr(\cdots}{10d^3h}$$

Mathematica [A] time = 0.351812, size = 275, normalized size = 0.82

$$\frac{-\frac{pr\left(30b^2(g+hx)^2(bg-ah)^3+20b^3(g+hx)^3(bg-ah)^2+15b^4(g+hx)^4(bg-ah)+60bhx(bg-ah)^4+60(bg-ah)^5\log(a+bx)+12b^5(g+hx)^5\right)}{60b^5} + (g+hx)^5\log\left(e\left(f(a+\cdots\right.\right.}{5h}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (-(p*r*(60*b*h*(b*g - a*h)^4*x + 30*b^2*(b*g - a*h)^3*(g + h*x)^2 + 20*b^3*
(b*g - a*h)^2*(g + h*x)^3 + 15*b^4*(b*g - a*h)*(g + h*x)^4 + 12*b^5*(g + h*
x)^5 + 60*(b*g - a*h)^5*Log[a + b*x]))/(60*b^5) - (q*r*(60*d*h*(d*g - c*h)^
4*x + 30*d^2*(d*g - c*h)^3*(g + h*x)^2 + 20*d^3*(d*g - c*h)^2*(g + h*x)^3 +
 15*d^4*(d*g - c*h)*(g + h*x)^4 + 12*d^5*(g + h*x)^5 + 60*(d*g - c*h)^5*Log
[c + d*x]))/(60*d^5) + (g + h*x)^5*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(5
*h)
```

Maple [F] time = 0.393, size = 0, normalized size = 0.

$$\int \left(hx+g\right)^4 \ln\left(e\left(f\left(bx+a\right)^p\left(dx+c\right)^q\right)^r\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((h*x+g)^4*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Maxima [B] time = 1.20345, size = 842, normalized size = 2.52

$$\frac{1}{5}\left(h^4x^5 + 5gh^3x^4 + 10g^2h^2x^3 + 10g^3hx^2 + 5g^4x\right)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right) + \frac{r\left(\frac{60\left(5ab^4fg^4p - 10a^2b^3fg^3hp + 10a^3b^2fg^2h^2p - 5\cdots\right.}{b^5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/5*(h^4*x^5 + 5*g*h^3*x^4 + 10*g^2*h^2*x^3 + 10*g^3*h*x^2 + 5*g^4*x)*log((
(b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/300*r*(60*(5*a*b^4*f*g^4*p - 10*a^2*b^3
*f*g^3*h*p + 10*a^3*b^2*f*g^2*h^2*p - 5*a^4*b*f*g*h^3*p + a^5*f*h^4*p)*log(
b*x + a)/b^5 + 60*(5*c*d^4*f*g^4*q - 10*c^2*d^3*f*g^3*h*q + 10*c^3*d^2*f*g^
```

```
2*h^2*q - 5*c^4*d*f*g*h^3*q + c^5*f*h^4*q)*log(d*x + c)/d^5 - (12*b^4*d^4*f
*h^4*(p + q)*x^5 - 15*(a*b^3*d^4*f*h^4*p - (5*d^4*f*g*h^3*(p + q) - c*d^3*f
*h^4*q)*b^4)*x^4 - 20*(5*a*b^3*d^4*f*g*h^3*p - a^2*b^2*d^4*f*h^4*p - (10*d^
4*f*g^2*h^2*(p + q) - 5*c*d^3*f*g*h^3*q + c^2*d^2*f*h^4*q)*b^4)*x^3 - 30*(1
0*a*b^3*d^4*f*g^2*h^2*p - 5*a^2*b^2*d^4*f*g*h^3*p + a^3*b*d^4*f*h^4*p - (10
*d^4*f*g^3*h*(p + q) - 10*c*d^3*f*g^2*h^2*q + 5*c^2*d^2*f*g*h^3*q - c^3*d*f
*h^4*q)*b^4)*x^2 - 60*(10*a*b^3*d^4*f*g^3*h*p - 10*a^2*b^2*d^4*f*g^2*h^2*p
+ 5*a^3*b*d^4*f*g*h^3*p - a^4*d^4*f*h^4*p - (5*d^4*f*g^4*(p + q) - 10*c*d^3
*f*g^3*h*q + 10*c^2*d^2*f*g^2*h^2*q - 5*c^3*d*f*g*h^3*q + c^4*f*h^4*q)*b^4)
*x)/(b^4*d^4))/f
```

Fricas [B] time = 1.54248, size = 1932, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/300*(12*(b^5*d^5*h^4*p + b^5*d^5*h^4*q)*r*x^5 + 15*((5*b^5*d^5*g*h^3 - a
*b^4*d^5*h^4)*p + (5*b^5*d^5*g*h^3 - b^5*c*d^4*h^4)*q)*r*x^4 + 20*((10*b^5*
d^5*g^2*h^2 - 5*a*b^4*d^5*g*h^3 + a^2*b^3*d^5*h^4)*p + (10*b^5*d^5*g^2*h^2
- 5*b^5*c*d^4*g*h^3 + b^5*c^2*d^3*h^4)*q)*r*x^3 + 30*((10*b^5*d^5*g^3*h - 1
0*a*b^4*d^5*g^2*h^2 + 5*a^2*b^3*d^5*g*h^3 - a^3*b^2*d^5*h^4)*p + (10*b^5*d^
5*g^3*h - 10*b^5*c*d^4*g^2*h^2 + 5*b^5*c^2*d^3*g*h^3 - b^5*c^3*d^2*h^4)*q)*
r*x^2 + 60*((5*b^5*d^5*g^4 - 10*a*b^4*d^5*g^3*h + 10*a^2*b^3*d^5*g^2*h^2 -
5*a^3*b^2*d^5*g*h^3 + a^4*b*d^5*h^4)*p + (5*b^5*d^5*g^4 - 10*b^5*c*d^4*g^3*
h + 10*b^5*c^2*d^3*g^2*h^2 - 5*b^5*c^3*d^2*g*h^3 + b^5*c^4*d*h^4)*q)*r*x -
60*(b^5*d^5*h^4*p*r*x^5 + 5*b^5*d^5*g*h^3*p*r*x^4 + 10*b^5*d^5*g^2*h^2*p*r*
x^3 + 10*b^5*d^5*g^3*h*p*r*x^2 + 5*b^5*d^5*g^4*p*r*x + (5*a*b^4*d^5*g^4 - 1
0*a^2*b^3*d^5*g^3*h + 10*a^3*b^2*d^5*g^2*h^2 - 5*a^4*b*d^5*g*h^3 + a^5*d^5*
h^4)*p*r)*log(b*x + a) - 60*(b^5*d^5*h^4*q*r*x^5 + 5*b^5*d^5*g*h^3*q*r*x^4
+ 10*b^5*d^5*g^2*h^2*q*r*x^3 + 10*b^5*d^5*g^3*h*q*r*x^2 + 5*b^5*d^5*g^4*q*r
*x + (5*b^5*c*d^4*g^4 - 10*b^5*c^2*d^3*g^3*h + 10*b^5*c^3*d^2*g^2*h^2 - 5*b
^5*c^4*d*g*h^3 + b^5*c^5*h^4)*q*r)*log(d*x + c) - 60*(b^5*d^5*h^4*x^5 + 5*b
^5*d^5*g*h^3*x^4 + 10*b^5*d^5*g^2*h^2*x^3 + 10*b^5*d^5*g^3*h*x^2 + 5*b^5*d^
5*g^4*x)*log(e) - 60*(b^5*d^5*h^4*r*x^5 + 5*b^5*d^5*g*h^3*r*x^4 + 10*b^5*d^
5*g^2*h^2*r*x^3 + 10*b^5*d^5*g^3*h*r*x^2 + 5*b^5*d^5*g^4*r*x)*log(f))/(b^5*
d^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**4*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.52229, size = 1715, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] -1/25*(h^4*p*r + h^4*q*r - 5*h^4*r*log(f) - 5*h^4)*x^5 - 1/20*(5*b*d*g*h^3*
p*r - a*d*h^4*p*r + 5*b*d*g*h^3*q*r - b*c*h^4*q*r - 20*b*d*g*h^3*r*log(f) -
 20*b*d*g*h^3)*x^4/(b*d) + 1/5*(h^4*p*r*x^5 + 5*g*h^3*p*r*x^4 + 10*g^2*h^2*
p*r*x^3 + 10*g^3*h*p*r*x^2 + 5*g^4*p*r*x)*log(b*x + a) + 1/5*(h^4*q*r*x^5 +
 5*g*h^3*q*r*x^4 + 10*g^2*h^2*q*r*x^3 + 10*g^3*h*q*r*x^2 + 5*g^4*q*r*x)*log
(d*x + c) - 1/15*(10*b^2*d^2*g^2*h^2*p*r - 5*a*b*d^2*g*h^3*p*r + a^2*d^2*h^
4*p*r + 10*b^2*d^2*g^2*h^2*q*r - 5*b^2*c*d*g*h^3*q*r + b^2*c^2*h^4*q*r - 30
*b^2*d^2*g^2*h^2*r*log(f) - 30*b^2*d^2*g^2*h^2)*x^3/(b^2*d^2) - 1/10*(10*b^
3*d^3*g^3*h*p*r - 10*a*b^2*d^3*g^2*h^2*p*r + 5*a^2*b*d^3*g*h^3*p*r - a^3*d^
3*h^4*p*r + 10*b^3*d^3*g^3*h*q*r - 10*b^3*c*d^2*g^2*h^2*q*r + 5*b^3*c^2*d*g
*h^3*q*r - b^3*c^3*h^4*q*r - 20*b^3*d^3*g^3*h*r*log(f) - 20*b^3*d^3*g^3*h)*
x^2/(b^3*d^3) - 1/5*(5*b^4*d^4*g^4*p*r - 10*a*b^3*d^4*g^3*h*p*r + 10*a^2*b^
2*d^4*g^2*h^2*p*r - 5*a^3*b*d^4*g*h^3*p*r + a^4*d^4*h^4*p*r + 5*b^4*d^4*g^4
*q*r - 10*b^4*c*d^3*g^3*h*q*r + 10*b^4*c^2*d^2*g^2*h^2*q*r - 5*b^4*c^3*d*g*
h^3*q*r + b^4*c^4*h^4*q*r - 5*b^4*d^4*g^4*r*log(f) - 5*b^4*d^4*g^4)*x/(b^4*
d^4) + 1/10*(5*a*b^4*d^5*g^4*p*r - 10*a^2*b^3*d^5*g^3*h*p*r + 10*a^3*b^2*d^
5*g^2*h^2*p*r - 5*a^4*b*d^5*g*h^3*p*r + a^5*d^5*h^4*p*r + 5*b^5*c*d^4*g^4*q
*r - 10*b^5*c^2*d^3*g^3*h*q*r + 10*b^5*c^3*d^2*g^2*h^2*q*r - 5*b^5*c^4*d*g*
h^3*q*r + b^5*c^5*h^4*q*r)*log(abs(b*d*x^2 + b*c*x + a*d*x + a*c))/(b^5*d^5
) + 1/10*(5*a*b^5*c*d^5*g^4*p*r - 5*a^2*b^4*d^6*g^4*p*r - 10*a^2*b^4*c*d^5*
g^3*h*p*r + 10*a^3*b^3*d^6*g^3*h*p*r + 10*a^3*b^3*c*d^5*g^2*h^2*p*r - 10*a^
4*b^2*d^6*g^2*h^2*p*r - 5*a^4*b^2*c*d^5*g*h^3*p*r + 5*a^5*b*d^6*g*h^3*p*r +
 a^5*b*c*d^5*h^4*p*r - a^6*d^6*h^4*p*r - 5*b^6*c^2*d^4*g^4*q*r + 5*a*b^5*c*
d^5*g^4*q*r + 10*b^6*c^3*d^3*g^3*h*q*r - 10*a*b^5*c^2*d^4*g^3*h*q*r - 10*b^
6*c^4*d^2*g^2*h^2*q*r + 10*a*b^5*c^3*d^3*g^2*h^2*q*r + 5*b^6*c^5*d*g*h^3*q*
r - 5*a*b^5*c^4*d^2*g*h^3*q*r - b^6*c^6*h^4*q*r + a*b^5*c^5*d*h^4*q*r)*log(
abs((2*b*d*x + b*c + a*d - abs(-b*c + a*d))/(2*b*d*x + b*c + a*d + abs(-b*c
 + a*d))))/(b^5*d^5*abs(-b*c + a*d))
```

3.26 $\int (g+hx)^3 \log \left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$

Optimal. Leaf size=276

$$-\frac{prx(bg-ah)^3}{4b^3}-\frac{pr(g+hx)^2(bg-ah)^2}{8b^2h}-\frac{pr(bg-ah)^4\log(a+bx)}{4b^4h}+\frac{(g+hx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h}-\frac{pr(g}{}$$

```
[Out] -((b*g - a*h)^3*p*r*x)/(4*b^3) - ((d*g - c*h)^3*q*r*x)/(4*d^3) - ((b*g - a*
h)^2*p*r*(g + h*x)^2)/(8*b^2*h) - ((d*g - c*h)^2*q*r*(g + h*x)^2)/(8*d^2*h)
 - ((b*g - a*h)*p*r*(g + h*x)^3)/(12*b*h) - ((d*g - c*h)*q*r*(g + h*x)^3)/(
12*d*h) - (p*r*(g + h*x)^4)/(16*h) - (q*r*(g + h*x)^4)/(16*h) - ((b*g - a*h
)^4*p*r*Log[a + b*x])/(4*b^4*h) - ((d*g - c*h)^4*q*r*Log[c + d*x])/(4*d^4*h
) + ((g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*h)
```

Rubi [A] time = 0.127419, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2495, 43}

$$-\frac{prx(bg-ah)^3}{4b^3}-\frac{pr(g+hx)^2(bg-ah)^2}{8b^2h}-\frac{pr(bg-ah)^4\log(a+bx)}{4b^4h}+\frac{(g+hx)^4\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h}-\frac{pr(g}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -((b*g - a*h)^3*p*r*x)/(4*b^3) - ((d*g - c*h)^3*q*r*x)/(4*d^3) - ((b*g - a*
h)^2*p*r*(g + h*x)^2)/(8*b^2*h) - ((d*g - c*h)^2*q*r*(g + h*x)^2)/(8*d^2*h)
 - ((b*g - a*h)*p*r*(g + h*x)^3)/(12*b*h) - ((d*g - c*h)*q*r*(g + h*x)^3)/(
12*d*h) - (p*r*(g + h*x)^4)/(16*h) - (q*r*(g + h*x)^4)/(16*h) - ((b*g - a*h
)^4*p*r*Log[a + b*x])/(4*b^4*h) - ((d*g - c*h)^4*q*r*Log[c + d*x])/(4*d^4*h
) + ((g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*h)
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (g+hx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx = \frac{(g+hx)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h} - \frac{(bpr)\int \frac{(g+hx)^4}{a+bx}dx}{4h} - \frac{(dqr)\int \frac{(g+}{c+}}{4h}$$

$$= \frac{(g+hx)^4 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h} - \frac{(bpr)\int\left(\frac{h(bg-ah)^3}{b^4} + \frac{(bg-ah)^4}{b^4(a+bx)} + \frac{h}{}\right.}{}$$

$$= -\frac{(bg-ah)^3prx}{4b^3} - \frac{(dg-ch)^3qrx}{4d^3} - \frac{(bg-ah)^2pr(g+hx)^2}{8b^2h} - \frac{(dg-ch)^2qr(}{8d^2h}$$

Mathematica [A] time = 0.310036, size = 231, normalized size = 0.84

$$\frac{\frac{1}{12}r\left(-\frac{p\left(6b^2(g+hx)^2(bg-ah)^2+4b^3(g+hx)^3(bg-ah)+12bhx(bg-ah)^3+12(bg-ah)^4\log(a+bx)+3b^4(g+hx)^4\right)}{b^4} - \frac{q\left(6d^2(g+hx)^2(dg-ch)^2+4d^3(g+hx)^3(dg-ch)+12dhx}{d^4}\right.}{4h}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] ((r*(-((p*(12*b*h*(b*g - a*h)^3*x + 6*b^2*(b*g - a*h)^2*(g + h*x)^2 + 4*b^3
*(b*g - a*h)*(g + h*x)^3 + 3*b^4*(g + h*x)^4 + 12*(b*g - a*h)^4*Log[a + b*x
]))/b^4) - (q*(12*d*h*(d*g - c*h)^3*x + 6*d^2*(d*g - c*h)^2*(g + h*x)^2 + 4
*d^3*(d*g - c*h)*(g + h*x)^3 + 3*d^4*(g + h*x)^4 + 12*(d*g - c*h)^4*Log[c +
 d*x]))/d^4))/12 + (g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*h)
```

Maple [F] time = 0.397, size = 0, normalized size = 0.

$$\int \left(hx+g\right)^3 \ln\left(e\left(f\left(bx+a\right)^p\left(dx+c\right)^q\right)^r\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Maxima [A] time = 1.21627, size = 582, normalized size = 2.11

$$\frac{1}{4}\left(h^3x^4+4gh^2x^3+6g^2hx^2+4g^3x\right)\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right) + \frac{r\left(\frac{12\left(4ab^3fg^3p-6a^2b^2fg^2hp+4a^3bfgh^2p-a^4fh^3p\right)\log(bx+a)}{b^4} + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/4*(h^3*x^4 + 4*g*h^2*x^3 + 6*g^2*h*x^2 + 4*g^3*x)*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e) + 1/48*r*(12*(4*a*b^3*f*g^3*p - 6*a^2*b^2*f*g^2*h*p + 4*a^3*b
*f*g*h^2*p - a^4*f*h^3*p)*log(b*x + a)/b^4 + 12*(4*c*d^3*f*g^3*q - 6*c^2*d^
2*f*g^2*h*q + 4*c^3*d*f*g*h^2*q - c^4*f*h^3*q)*log(d*x + c)/d^4 - (3*b^3*d^
3*f*h^3*(p + q)*x^4 - 4*(a*b^2*d^3*f*h^3*p - (4*d^3*f*g*h^2*(p + q) - c*d^2
```

```
*f*h^3*q)*b^3)*x^3 - 6*(4*a*b^2*d^3*f*g*h^2*p - a^2*b*d^3*f*h^3*p - (6*d^3*
f*g^2*h*(p + q) - 4*c*d^2*f*g*h^2*q + c^2*d*f*h^3*q)*b^3)*x^2 - 12*(6*a*b^2
*d^3*f*g^2*h*p - 4*a^2*b*d^3*f*g*h^2*p + a^3*d^3*f*h^3*p - (4*d^3*f*g^3*(p
+ q) - 6*c*d^2*f*g^2*h*q + 4*c^2*d*f*g*h^2*q - c^3*f*h^3*q)*b^3)*x)/(b^3*d^
3))/f
```

Fricas [B] time = 1.24885, size = 1377, normalized size = 4.99

$$-\frac{3\left(b^4d^4h^3p + b^4d^4h^3q\right)rx^4 + 4\left(\left(4\,b^4d^4gh^2 - ab^3d^4h^3\right)p + \left(4\,b^4d^4gh^2 - b^4cd^3h^3\right)q\right)rx^3 + 6\left(\left(6\,b^4d^4g^2h - 4\,ab^3d^4gh^2 + \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/48*(3*(b^4*d^4*h^3*p + b^4*d^4*h^3*q)*r*x^4 + 4*((4*b^4*d^4*g*h^2 - a*b^
3*d^4*h^3)*p + (4*b^4*d^4*g*h^2 - b^4*c*d^3*h^3)*q)*r*x^3 + 6*((6*b^4*d^4*g
^2*h - 4*a*b^3*d^4*g*h^2 + a^2*b^2*d^4*h^3)*p + (6*b^4*d^4*g^2*h - 4*b^4*c*
d^3*g*h^2 + b^4*c^2*d^2*h^3)*q)*r*x^2 + 12*((4*b^4*d^4*g^3 - 6*a*b^3*d^4*g^
2*h + 4*a^2*b^2*d^4*g*h^2 - a^3*b*d^4*h^3)*p + (4*b^4*d^4*g^3 - 6*b^4*c*d^3
*g^2*h + 4*b^4*c^2*d^2*g*h^2 - b^4*c^3*d*h^3)*q)*r*x - 12*(b^4*d^4*h^3*p*r*
x^4 + 4*b^4*d^4*g*h^2*p*r*x^3 + 6*b^4*d^4*g^2*h*p*r*x^2 + 4*b^4*d^4*g^3*p*r
*x + (4*a*b^3*d^4*g^3 - 6*a^2*b^2*d^4*g^2*h + 4*a^3*b*d^4*g*h^2 - a^4*d^4*h
^3)*p*r)*log(b*x + a) - 12*(b^4*d^4*h^3*q*r*x^4 + 4*b^4*d^4*g*h^2*q*r*x^3 +
 6*b^4*d^4*g^2*h*q*r*x^2 + 4*b^4*d^4*g^3*q*r*x + (4*b^4*c*d^3*g^3 - 6*b^4*c
^2*d^2*g^2*h + 4*b^4*c^3*d*g*h^2 - b^4*c^4*h^3)*q*r)*log(d*x + c) - 12*(b^4
*d^4*h^3*x^4 + 4*b^4*d^4*g*h^2*x^3 + 6*b^4*d^4*g^2*h*x^2 + 4*b^4*d^4*g^3*x)
*log(e) - 12*(b^4*d^4*h^3*r*x^4 + 4*b^4*d^4*g*h^2*r*x^3 + 6*b^4*d^4*g^2*h*r
*x^2 + 4*b^4*d^4*g^3*r*x)*log(f))/(b^4*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32799, size = 1254, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] -1/16*(h^3*p*r + h^3*q*r - 4*h^3*r*log(f) - 4*h^3)*x^4 - 1/12*(4*b*d*g*h^2*
p*r - a*d*h^3*p*r + 4*b*d*g*h^2*q*r - b*c*h^3*q*r - 12*b*d*g*h^2*r*log(f) -
 12*b*d*g*h^2)*x^3/(b*d) + 1/4*(h^3*p*r*x^4 + 4*g*h^2*p*r*x^3 + 6*g^2*h*p*r
*x^2 + 4*g^3*p*r*x)*log(b*x + a) + 1/4*(h^3*q*r*x^4 + 4*g*h^2*q*r*x^3 + 6*g
```

```
^2*h*q*r*x^2 + 4*g^3*q*r*x)*log(d*x + c) - 1/8*(6*b^2*d^2*g^2*h*p*r - 4*a*b
*d^2*g*h^2*p*r + a^2*d^2*h^3*p*r + 6*b^2*d^2*g^2*h*q*r - 4*b^2*c*d*g*h^2*q*
r + b^2*c^2*h^3*q*r - 12*b^2*d^2*g^2*h*r*log(f) - 12*b^2*d^2*g^2*h)*x^2/(b^
2*d^2) - 1/4*(4*b^3*d^3*g^3*p*r - 6*a*b^2*d^3*g^2*h*p*r + 4*a^2*b*d^3*g*h^2
*p*r - a^3*d^3*h^3*p*r + 4*b^3*d^3*g^3*q*r - 6*b^3*c*d^2*g^2*h*q*r + 4*b^3*
c^2*d*g*h^2*q*r - b^3*c^3*h^3*q*r - 4*b^3*d^3*g^3*r*log(f) - 4*b^3*d^3*g^3)
*x/(b^3*d^3) + 1/8*(4*a*b^3*d^4*g^3*p*r - 6*a^2*b^2*d^4*g^2*h*p*r + 4*a^3*b
*d^4*g*h^2*p*r - a^4*d^4*h^3*p*r + 4*b^4*c*d^3*g^3*q*r - 6*b^4*c^2*d^2*g^2*
h*q*r + 4*b^4*c^3*d*g*h^2*q*r - b^4*c^4*h^3*q*r)*log(abs(b*d*x^2 + b*c*x +
a*d*x + a*c))/(b^4*d^4) + 1/8*(4*a*b^4*c*d^4*g^3*p*r - 4*a^2*b^3*d^5*g^3*p*
r - 6*a^2*b^3*c*d^4*g^2*h*p*r + 6*a^3*b^2*d^5*g^2*h*p*r + 4*a^3*b^2*c*d^4*g
*h^2*p*r - 4*a^4*b*d^5*g*h^2*p*r - a^4*b*c*d^4*h^3*p*r + a^5*d^5*h^3*p*r -
4*b^5*c^2*d^3*g^3*q*r + 4*a*b^4*c*d^4*g^3*q*r + 6*b^5*c^3*d^2*g^2*h*q*r - 6
*a*b^4*c^2*d^3*g^2*h*q*r - 4*b^5*c^4*d*g*h^2*q*r + 4*a*b^4*c^3*d^2*g*h^2*q*
r + b^5*c^5*h^3*q*r - a*b^4*c^4*d*h^3*q*r)*log(abs((2*b*d*x + b*c + a*d - a
bs(-b*c + a*d))/(2*b*d*x + b*c + a*d + abs(-b*c + a*d))))/(b^4*d^4*abs(-b*c
 + a*d))
```

3.27 $\int (g+hx)^2 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

Optimal. Leaf size=218

$$-\frac{prx(bg-ah)^2}{3b^2}-\frac{pr(bg-ah)^3\log(a+bx)}{3b^3h}+\frac{(g+hx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h}-\frac{pr(g+hx)^2(bg-ah)}{6bh}-\frac{qrx(dg}{3}$$

```
[Out] -((b*g - a*h)^2*p*r*x)/(3*b^2) - ((d*g - c*h)^2*q*r*x)/(3*d^2) - ((b*g - a*
h)*p*r*(g + h*x)^2)/(6*b*h) - ((d*g - c*h)*q*r*(g + h*x)^2)/(6*d*h) - (p*r*
(g + h*x)^3)/(9*h) - (q*r*(g + h*x)^3)/(9*h) - ((b*g - a*h)^3*p*r*Log[a + b
*x])/(3*b^3*h) - ((d*g - c*h)^3*q*r*Log[c + d*x])/(3*d^3*h) + ((g + h*x)^3*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*h)
```

Rubi [A] time = 0.0987066, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2495, 43}

$$-\frac{prx(bg-ah)^2}{3b^2}-\frac{pr(bg-ah)^3\log(a+bx)}{3b^3h}+\frac{(g+hx)^3\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h}-\frac{pr(g+hx)^2(bg-ah)}{6bh}-\frac{qrx(dg}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -((b*g - a*h)^2*p*r*x)/(3*b^2) - ((d*g - c*h)^2*q*r*x)/(3*d^2) - ((b*g - a*
h)*p*r*(g + h*x)^2)/(6*b*h) - ((d*g - c*h)*q*r*(g + h*x)^2)/(6*d*h) - (p*r*
(g + h*x)^3)/(9*h) - (q*r*(g + h*x)^3)/(9*h) - ((b*g - a*h)^3*p*r*Log[a + b
*x])/(3*b^3*h) - ((d*g - c*h)^3*q*r*Log[c + d*x])/(3*d^3*h) + ((g + h*x)^3*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*h)
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (g+hx)^2 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \frac{(g+hx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h} - \frac{(bpr)\int \frac{(g+hx)^3}{a+bx}\,dx}{3h} - \frac{(dqr)\int \frac{(g+\ldots}{c+\ldots}}{3h}$$

$$= \frac{(g+hx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h} - \frac{(bpr)\int\left(\frac{h(bg-ah)^2}{b^3} + \frac{(bg-ah)^3}{b^3(a+bx)} + \ldots\right.}{3h}$$

$$= -\frac{(bg-ah)^2prx}{3b^2} - \frac{(dg-ch)^2qrx}{3d^2} - \frac{(bg-ah)pr(g+hx)^2}{6bh} - \frac{(dg-ch)qr(g\ldots}{6dh}$$

Mathematica [A] time = 0.253959, size = 209, normalized size = 0.96

$$\frac{(g+hx)^3 \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - \frac{r\left(b\left(6a^2d^3h^3px - 3abd^3hp\left(g^2+6ghx+h^2x^2\right)+b^2d\left(6c^2h^3qx - 3cdhq\left(g^2+6ghx+h^2x^2\right)+d^2(p+q)\left(18g^2hx+5g^3+9\ldots\right.\right.\right.}{6b^3d^3}}{3h}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (-(r*(6*d^3*(b*g - a*h)^3*p*Log[a + b*x] + b*(6*a^2*d^3*h^3*p*x - 3*a*b*d^3
*h*p*(g^2 + 6*g*h*x + h^2*x^2) + b^2*d*(6*c^2*h^3*q*x - 3*c*d*h*q*(g^2 + 6*
g*h*x + h^2*x^2) + d^2*(p + q)*(5*g^3 + 18*g^2*h*x + 9*g*h^2*x^2 + 2*h^3*x^
3)) + 6*b^2*(d*g - c*h)^3*q*Log[c + d*x])))/(6*b^3*d^3) + (g + h*x)^3*Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*h)
```

Maple [F] time = 0.431, size = 0, normalized size = 0.

$$\int \left(hx+g\right)^2 \ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Maxima [A] time = 1.25796, size = 363, normalized size = 1.67

$$\frac{1}{3}\left(h^2x^3 + 3\,ghx^2 + 3\,g^2x\right)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right) + \frac{r\left(\frac{6\left(3\,ab^2fg^2p - 3\,a^2bfghp + a^3fh^2p\right)\log(bx+a)}{b^3} + \frac{6\left(3\,cd^2fg^2q - 3\,c^2dfghq + c^3fh^2\ldots\right.}{d^3}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/3*(h^2*x^3 + 3*g*h*x^2 + 3*g^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) +
1/18*r*(6*(3*a*b^2*f*g^2*p - 3*a^2*b*f*g*h*p + a^3*f*h^2*p)*log(b*x + a)/b^
3 + 6*(3*c*d^2*f*g^2*q - 3*c^2*d*f*g*h*q + c^3*f*h^2*q)*log(d*x + c)/d^3 -
(2*b^2*d^2*f*h^2*(p + q)*x^3 - 3*(a*b*d^2*f*h^2*p - (3*d^2*f*g*h*(p + q) -
c*d*f*h^2*q)*b^2)*x^2 - 6*(3*a*b*d^2*f*g*h*p - a^2*d^2*f*h^2*p - (3*d^2*f*g
```

```
^2*(p + q) - 3*c*d*f*g*h*q + c^2*f*h^2*q)*b^2)*x)/(b^2*d^2))/f
```

Fricas [B] time = 1.16108, size = 917, normalized size = 4.21

$$-\frac{2\left(b^3d^3h^2p + b^3d^3h^2q\right)rx^3 + 3\left(\left(3\,b^3d^3gh - ab^2d^3h^2\right)p + \left(3\,b^3d^3gh - b^3cd^2h^2\right)q\right)rx^2 + 6\left(\left(3\,b^3d^3g^2 - 3\,ab^2d^3gh + a^2b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/18*(2*(b^3*d^3*h^2*p + b^3*d^3*h^2*q)*r*x^3 + 3*((3*b^3*d^3*g*h - a*b^2*
d^3*h^2)*p + (3*b^3*d^3*g*h - b^3*c*d^2*h^2)*q)*r*x^2 + 6*((3*b^3*d^3*g^2 -
 3*a*b^2*d^3*g*h + a^2*b*d^3*h^2)*p + (3*b^3*d^3*g^2 - 3*b^3*c*d^2*g*h + b^
3*c^2*d*h^2)*q)*r*x - 6*(b^3*d^3*h^2*p*r*x^3 + 3*b^3*d^3*g*h*p*r*x^2 + 3*b^
3*d^3*g^2*p*r*x + (3*a*b^2*d^3*g^2 - 3*a^2*b*d^3*g*h + a^3*d^3*h^2)*p*r)*lo
g(b*x + a) - 6*(b^3*d^3*h^2*q*r*x^3 + 3*b^3*d^3*g*h*q*r*x^2 + 3*b^3*d^3*g^2
*q*r*x + (3*b^3*c*d^2*g^2 - 3*b^3*c^2*d*g*h + b^3*c^3*h^2)*q*r)*log(d*x + c
) - 6*(b^3*d^3*h^2*x^3 + 3*b^3*d^3*g*h*x^2 + 3*b^3*d^3*g^2*x)*log(e) - 6*(b
^3*d^3*h^2*r*x^3 + 3*b^3*d^3*g*h*r*x^2 + 3*b^3*d^3*g^2*r*x)*log(f))/(b^3*d^
3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30521, size = 837, normalized size = 3.84

$$-\frac{1}{9}\left(h^2pr + h^2qr - 3\,h^2r\log\left(f\right) - 3\,h^2\right)x^3 + \frac{1}{3}\left(h^2prx^3 + 3\,ghprx^2 + 3\,g^2prx\right)\log\left(bx + a\right) + \frac{1}{3}\left(h^2qrx^3 + 3\,ghqrx^2 + 3\,g\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] -1/9*(h^2*p*r + h^2*q*r - 3*h^2*r*log(f) - 3*h^2)*x^3 + 1/3*(h^2*p*r*x^3 +
3*g*h*p*r*x^2 + 3*g^2*p*r*x)*log(b*x + a) + 1/3*(h^2*q*r*x^3 + 3*g*h*q*r*x^
2 + 3*g^2*q*r*x)*log(d*x + c) - 1/6*(3*b*d*g*h*p*r - a*d*h^2*p*r + 3*b*d*g*
h*q*r - b*c*h^2*q*r - 6*b*d*g*h*r*log(f) - 6*b*d*g*h)*x^2/(b*d) - 1/3*(3*b^
2*d^2*g^2*p*r - 3*a*b*d^2*g*h*p*r + a^2*d^2*h^2*p*r + 3*b^2*d^2*g^2*q*r - 3
*b^2*c*d*g*h*q*r + b^2*c^2*h^2*q*r - 3*b^2*d^2*g^2*r*log(f) - 3*b^2*d^2*g^2
)*x/(b^2*d^2) + 1/6*(3*a*b^2*d^3*g^2*p*r - 3*a^2*b*d^3*g*h*p*r + a^3*d^3*h^
2*p*r + 3*b^3*c*d^2*g^2*q*r - 3*b^3*c^2*d*g*h*q*r + b^3*c^3*h^2*q*r)*log(ab
s(b*d*x^2 + b*c*x + a*d*x + a*c))/(b^3*d^3) + 1/6*(3*a*b^3*c*d^3*g^2*p*r -
3*a^2*b^2*d^4*g^2*p*r - 3*a^2*b^2*c*d^3*g*h*p*r + 3*a^3*b*d^4*g*h*p*r + a^3
```

```
*b*c*d^3*h^2*p*r - a^4*d^4*h^2*p*r - 3*b^4*c^2*d^2*g^2*q*r + 3*a*b^3*c*d^3*
g^2*q*r + 3*b^4*c^3*d*g*h*q*r - 3*a*b^3*c^2*d^2*g*h*q*r - b^4*c^4*h^2*q*r +
 a*b^3*c^3*d*h^2*q*r)*log(abs((2*b*d*x + b*c + a*d - abs(-b*c + a*d))/(2*b*
d*x + b*c + a*d + abs(-b*c + a*d))))/(b^3*d^3*abs(-b*c + a*d))
```

3.28 $\int (g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

Optimal. Leaf size=160

$$-\frac{pr(bg-ah)^2\log(a+bx)}{2b^2h}+\frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}-\frac{prx(bg-ah)}{2b}-\frac{qr(dg-ch)^2\log(c+dx)}{2d^2h}-\frac{qrx}{}$$

```
[Out] -((b*g - a*h)*p*r*x)/(2*b) - ((d*g - c*h)*q*r*x)/(2*d) - (p*r*(g + h*x)^2)/
(4*h) - (q*r*(g + h*x)^2)/(4*h) - ((b*g - a*h)^2*p*r*Log[a + b*x])/(2*b^2*h
) - ((d*g - c*h)^2*q*r*Log[c + d*x])/(2*d^2*h) + ((g + h*x)^2*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(2*h)
```

Rubi [A] time = 0.0687969, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2495, 43}

$$-\frac{pr(bg-ah)^2\log(a+bx)}{2b^2h}+\frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}-\frac{prx(bg-ah)}{2b}-\frac{qr(dg-ch)^2\log(c+dx)}{2d^2h}-\frac{qrx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -((b*g - a*h)*p*r*x)/(2*b) - ((d*g - c*h)*q*r*x)/(2*d) - (p*r*(g + h*x)^2)/
(4*h) - (q*r*(g + h*x)^2)/(4*h) - ((b*g - a*h)^2*p*r*Log[a + b*x])/(2*b^2*h
) - ((d*g - c*h)^2*q*r*Log[c + d*x])/(2*d^2*h) + ((g + h*x)^2*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/(2*h)
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx &= \frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}-\frac{(bpr)\int \frac{(g+hx)^2}{a+bx}\,dx}{2h}-\frac{(dqr)\int}{2}\\ &= \frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h}-\frac{(bpr)\int \left(\frac{h(bg-ah)}{b^2}+\frac{(bg-ah)^2}{b^2(a+bx)}+\right.}{2h}\\ &= -\frac{(bg-ah)prx}{2b}-\frac{(dg-ch)qrx}{2d}-\frac{pr(g+hx)^2}{4h}-\frac{qr(g+hx)^2}{4h}-\frac{(bg-ah)}{}\end{aligned}$$

Mathematica [A] time = 0.196375, size = 120, normalized size = 0.75

$$-\frac{b\left(dx\left(r(-2adhp-2bchq+bd(p+q)(4g+hx))-2bd(2g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)+2bcqr(ch-2dg)\log(c+\ldots}{4b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -(2*a*d^2*(-2*b*g + a*h)*p*r*Log[a + b*x] + b*(2*b*c*(-2*d*g + c*h)*q*r*Log
[c + d*x] + d*x*(r*(-2*a*d*h*p - 2*b*c*h*q + b*d*(p + q)*(4*g + h*x)) - 2*b
*d*(2*g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])))/(4*b^2*d^2)
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int (hx+g)\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

Maxima [A] time = 1.20219, size = 193, normalized size = 1.21

$$\frac{1}{2}\left(hx^2+2gx\right)\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)+\frac{r\left(\frac{2\left(2abfgp-a^2fhp\right)\log(bx+a)}{b^2}+\frac{2\left(2cdfgq-c^2fhq\right)\log(dx+c)}{d^2}-\frac{bdfh(p+q)x^2-2\left(adfhp-(2\ldots\right.}{bd}\right.}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] 1/2*(h*x^2 + 2*g*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) + 1/4*r*(2*(2*a*b*
f*g*p - a^2*f*h*p)*log(b*x + a)/b^2 + 2*(2*c*d*f*g*q - c^2*f*h*q)*log(d*x +
 c)/d^2 - (b*d*f*h*(p + q)*x^2 - 2*(a*d*f*h*p - (2*d*f*g*(p + q) - c*f*h*q)
*b)*x)/(b*d))/f
```

Fricas [A] time = 1.20831, size = 524, normalized size = 3.28

$$-\frac{\left(b^2d^2hp+b^2d^2hq\right)rx^2+2\left(\left(2b^2d^2g-abd^2h\right)p+\left(2b^2d^2g-b^2cdh\right)q\right)rx-2\left(b^2d^2hprx^2+2b^2d^2gprx+\left(2abd^2g-a^2d^2h\ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] -1/4*((b^2*d^2*h*p + b^2*d^2*h*q)*r*x^2 + 2*((2*b^2*d^2*g - a*b*d^2*h)*p +
(2*b^2*d^2*g - b^2*c*d*h)*q)*r*x - 2*(b^2*d^2*h*p*r*x^2 + 2*b^2*d^2*g*p*r*x
 + (2*a*b*d^2*g - a^2*d^2*h)*p*r)*log(b*x + a) - 2*(b^2*d^2*h*q*r*x^2 + 2*b
```

```
^2*d^2*g*q*r*x + (2*b^2*c*d*g - b^2*c^2*h)*q*r)*log(d*x + c) - 2*(b^2*d^2*h
*x^2 + 2*b^2*d^2*g*x)*log(e) - 2*(b^2*d^2*h*r*x^2 + 2*b^2*d^2*g*r*x)*log(f)
)/(b^2*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)

[Out] Timed out
```

Giac [B] time = 1.31039, size = 479, normalized size = 2.99

$$-\frac{1}{4}\left(hpr + hqr - 2\,hr\log\left(f\right) - 2\,h\right)x^2 + \frac{1}{2}\left(hprx^2 + 2\,gprx\right)\log\left(bx + a\right) + \frac{1}{2}\left(hqrx^2 + 2\,gqrx\right)\log\left(dx + c\right) - \frac{\left(2\,bdgpr - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")

[Out] -1/4*(h*p*r + h*q*r - 2*h*r*log(f) - 2*h)*x^2 + 1/2*(h*p*r*x^2 + 2*g*p*r*x)
*log(b*x + a) + 1/2*(h*q*r*x^2 + 2*g*q*r*x)*log(d*x + c) - 1/2*(2*b*d*g*p*r
 - a*d*h*p*r + 2*b*d*g*q*r - b*c*h*q*r - 2*b*d*g*r*log(f) - 2*b*d*g)*x/(b*d
) + 1/4*(2*a*b*d^2*g*p*r - a^2*d^2*h*p*r + 2*b^2*c*d*g*q*r - b^2*c^2*h*q*r)
*log(abs(b*d*x^2 + b*c*x + a*d*x + a*c))/(b^2*d^2) + 1/4*(2*a*b^2*c*d^2*g*p
*r - 2*a^2*b*d^3*g*p*r - a^2*b*c*d^2*h*p*r + a^3*d^3*h*p*r - 2*b^3*c^2*d*g*
q*r + 2*a*b^2*c*d^2*g*q*r + b^3*c^3*h*q*r - a*b^2*c^2*d*h*q*r)*log(abs((2*b
*d*x + b*c + a*d - abs(-b*c + a*d))/(2*b*d*x + b*c + a*d + abs(-b*c + a*d))
))/(b^2*d^2*abs(-b*c + a*d))
```

3.29 $\int \log \left(e \left(f (a+bx)^p (c+dx)^q\right)^r\right) \, dx$

Optimal. Leaf size=61

$$\frac{(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}+\frac{qr(bc-ad)\log(c+dx)}{bd}+rx(-(p+q))$$

```
[Out] -((p + q)*r*x) + ((b*c - a*d)*q*r*Log[c + d*x])/(b*d) + ((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/b
```

Rubi [A] time = 0.0148794, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2487, 31, 8}

$$\frac{(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}+\frac{qr(bc-ad)\log(c+dx)}{bd}+rx(-(p+q))$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] -((p + q)*r*x) + ((b*c - a*d)*q*r*Log[c + d*x])/(b*d) + ((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/b
```

Rule 2487

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + (Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx &= \frac{(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}+\frac{((bc-ad)qr)\int \frac{1}{c+dx}\,dx}{b}-((p+q)r)\int 1\,d\\ &= -(p+q)rx+\frac{(bc-ad)qr\log(c+dx)}{bd}+\frac{(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0628468, size = 57, normalized size = 0.93

$$x\left(\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)-r(p+q)\right)+\frac{apr\log(a+bx)}{b}+\frac{cqr\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r],x]
```

```
[Out] (a*p*r*Log[a + b*x])/b + (c*q*r*Log[c + d*x])/d + x*(-((p + q)*r) + Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])
```

Maple [A] time = 0.071, size = 61, normalized size = 1.

$$\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)x-rpx-rqx+\frac{rap\ln\left(bx+a\right)}{b}+\frac{rqc\ln\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x)
```

```
[Out] ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*x-r*p*x-r*q*x+r*a*p/b*ln(b*x+a)+r*q*c/d*ln(
d*x+c)
```

Maxima [A] time = 1.15706, size = 101, normalized size = 1.66

$$x\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)-\frac{\left(bfp\left(\frac{x}{b}-\frac{a\log(bx+a)}{b^{2}}\right)+dfq\left(\frac{x}{d}-\frac{c\log(dx+c)}{d^{2}}\right)\right)r}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="maxima")
```

```
[Out] x*log(((b*x + a)^p*(d*x + c)^q*f)^r*e) - (b*f*p*(x/b - a*log(b*x + a)/b^2)
+ d*f*q*(x/d - c*log(d*x + c)/d^2))*r/f
```

Fricas [A] time = 1.08855, size = 182, normalized size = 2.98

$$\frac{bdrx\log\left(f\right)+bdx\log\left(e\right)-\left(bdp+bdq\right)rx+\left(bdprx+adpr\right)\log\left(bx+a\right)+\left(bdqrx+bcqr\right)\log\left(dx+c\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="fricas")
```

```
[Out] (b*d*r*x*log(f) + b*d*x*log(e) - (b*d*p + b*d*q)*r*x + (b*d*p*r*x + a*d*p*r
)*log(b*x + a) + (b*d*q*r*x + b*c*q*r)*log(d*x + c))/(b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23847, size = 242, normalized size = 3.97

$$prx\log(bx+a)+qrx\log(dx+c)-\left(pr+qr-r\log\left(f\right)-1\right)x+\frac{\left(adpr+bcqr\right)\log\left(\left|bdx^2+bcx+adx+ac\right|\right)}{2\,bd}+\frac{\left(abcdpr-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

```
[Out] p*r*x*log(b*x + a) + q*r*x*log(d*x + c) - (p*r + q*r - r*log(f) - 1)*x + 1/
2*(a*d*p*r + b*c*q*r)*log(abs(b*d*x^2 + b*c*x + a*d*x + a*c))/(b*d) + 1/2*(
a*b*c*d*p*r - a^2*d^2*p*r - b^2*c^2*q*r + a*b*c*d*q*r)*log(abs((2*b*d*x + b
*c + a*d - abs(-b*c + a*d))/(2*b*d*x + b*c + a*d + abs(-b*c + a*d))))/(b*d*
abs(-b*c + a*d))
```

3.30 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}\,dx$

Optimal. Leaf size=148

$$-\frac{pr\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{h}-\frac{qr\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{h}+\frac{\log(g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h}-\frac{pr\log(g+hx)\log\left(\right.}{h}$$

```
[Out] -((p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g + h*x])/h) - (q*r*Log[-((h*(
c + d*x))/(d*g - c*h))]*Log[g + h*x])/h + (Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r]*Log[g + h*x])/h - (p*r*PolyLog[2, (b*(g + h*x))/(b*g - a*h)])/h - (q*r
*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/h
```

Rubi [A] time = 0.120007, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2494, 2394, 2393, 2391}

$$-\frac{pr\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{h}-\frac{qr\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{h}+\frac{\log(g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h}-\frac{pr\log(g+hx)\log\left(\right.}{h}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x),x]
```


```
[Out] -((p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g + h*x])/h) - (q*r*Log[-((h*(
c + d*x))/(d*g - c*h))]*Log[g + h*x])/h + (Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r]*Log[g + h*x])/h - (p*r*PolyLog[2, (b*(g + h*x))/(b*g - a*h)])/h - (q*r
*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/h
```


Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}dx = \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{h} - \frac{(bpr)\int \frac{\log(g+hx)}{a+bx}dx}{h} - \frac{(dqr)\int \frac{\log(g+hx)}{c+dx}}{h}$$
$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} + \log\left(e\left(f(a+bx)^{p}\right.\right.$$
$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} + \log\left(e\left(f(a+bx)^{p}\right.\right.$$
$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(g+hx)}{h} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(g+hx)}{h} + \log\left(e\left(f(a+bx)^{p}\right.\right.$$

Mathematica [A] time = 0.0834048, size = 163, normalized size = 1.1

$$\frac{pr\text{PolyLog}\left(2,\frac{h(a+bx)}{ah-bg}\right)+qr\text{PolyLog}\left(2,\frac{h(c+dx)}{ch-dg}\right)+\log(g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)-pr\log(a+bx)\log(g+hx)}{h}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x),x]
```

```
[Out] (-(p*r*Log[a + b*x]*Log[g + h*x]) - q*r*Log[c + d*x]*Log[g + h*x] + Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] + p*r*Log[a + b*x]*Log[(b*(g + h
*x))/(b*g - a*h)] + q*r*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] + p*r*P
olyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] + q*r*PolyLog[2, (h*(c + d*x))/(-(d
*g) + c*h)])/h
```

Maple [F] time = 0.622, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x)
```

Maxima [A] time = 1.21998, size = 251, normalized size = 1.7

$$\frac{\left(\frac{\left(\log(bx+a)\log\left(\frac{bhx+ah}{bg-ah}+1\right)+\text{Li}_2\left(-\frac{bhx+ah}{bg-ah}\right)\right)fp}{h}+\frac{\left(\log(dx+c)\log\left(\frac{dhx+ch}{dg-ch}+1\right)+\text{Li}_2\left(-\frac{dhx+ch}{dg-ch}\right)\right)fq}{h}\right)r}{f} - \frac{\left(fp\log\left(bx+a\right)+fq\log\left(dx+c\right)\right)r\log\left(h\right.}{fh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x, algorithm="maxima")
```

```
[Out] ((log(b*x + a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b
*g - a*h)))*f*p/h + (log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + dilo
g(-(d*h*x + c*h)/(d*g - c*h)))*f*q/h)*r/f - (f*p*log(b*x + a) + f*q*log(d*x
 + c))*r*log(h*x + g)/(f*h) + log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(h*x
+ g)/h
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hx+g}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*x + g), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*x + g), x)
```

3.31 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^2}\,dx$

Optimal. Leaf size=128

$$-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)}+\frac{bpr\log(a+bx)}{h(bg-ah)}-\frac{bpr\log(g+hx)}{h(bg-ah)}+\frac{dqr\log(c+dx)}{h(dg-ch)}-\frac{dqr\log(g+hx)}{h(dg-ch)}$$

```
[Out] (b*p*r*Log[a + b*x])/(h*(b*g - a*h)) + (d*q*r*Log[c + d*x])/(h*(d*g - c*h))
 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(g + h*x)) - (b*p*r*Log[g + h*x]
)/(h*(b*g - a*h)) - (d*q*r*Log[g + h*x])/(h*(d*g - c*h))
```

Rubi [A] time = 0.0524397, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2495, 36, 31}

$$-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)}+\frac{bpr\log(a+bx)}{h(bg-ah)}-\frac{bpr\log(g+hx)}{h(bg-ah)}+\frac{dqr\log(c+dx)}{h(dg-ch)}-\frac{dqr\log(g+hx)}{h(dg-ch)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^2,x]
```

```
[Out] (b*p*r*Log[a + b*x])/(h*(b*g - a*h)) + (d*q*r*Log[c + d*x])/(h*(d*g - c*h))
 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(g + h*x)) - (b*p*r*Log[g + h*x]
)/(h*(b*g - a*h)) - (d*q*r*Log[g + h*x])/(h*(d*g - c*h))
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^2}dx = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)} + \frac{(bpr)\int \frac{1}{(a+bx)(g+hx)}dx}{h} + \frac{(dqr)\int \frac{1}{(c+dx)(g+hx)}d}{h}$$
$$= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)} - \frac{(bpr)\int \frac{1}{g+hx}dx}{bg-ah} + \frac{\left(b^2pr\right)\int \frac{1}{a+bx}dx}{h(bg-ah)} - \frac{(dqr)\int}{dg}$$
$$= \frac{bpr\log(a+bx)}{h(bg-ah)} + \frac{dqr\log(c+dx)}{h(dg-ch)} - \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)} - \frac{bpr\log(g}{h(bg-}$$

Mathematica [A] time = 0.232556, size = 93, normalized size = 0.73

$$\frac{-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx} + \frac{bpr(\log(a+bx)-\log(g+hx))}{bg-ah} + \frac{dqr(\log(c+dx)-\log(g+hx))}{dg-ch}}{h}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^2,x]
```

```
[Out] (-(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)) + (b*p*r*(Log[a + b*x] -
 Log[g + h*x]))/(b*g - a*h) + (d*q*r*(Log[c + d*x] - Log[g + h*x]))/(d*g -
c*h))/h
```

Maple [F] time = 0.493, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)}{\left(hx+g\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x)
```

Maxima [A] time = 1.25269, size = 166, normalized size = 1.3

$$\frac{\left(bfp\left(\frac{\log(bx+a)}{bg-ah} - \frac{\log(hx+g)}{bg-ah}\right) + dfq\left(\frac{\log(dx+c)}{dg-ch} - \frac{\log(hx+g)}{dg-ch}\right)\right)r}{fh} - \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{\left(hx+g\right)h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="maxima")
```

```
[Out] (b*f*p*(log(b*x + a)/(b*g - a*h) - log(h*x + g)/(b*g - a*h)) + d*f*q*(log(d
*x + c)/(d*g - c*h) - log(h*x + g)/(d*g - c*h)))*r/(f*h) - log(((b*x + a)^p
*(d*x + c)^q*f)^r*e)/((h*x + g)*h)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.46868, size = 257, normalized size = 2.01

$$\frac{b^2pr\log(|-bx-a|)}{b^2gh-abh^2}+\frac{d^2qr\log(|dx+c|)}{d^2gh-cdh^2}-\frac{pr\log(bx+a)}{h^2x+gh}-\frac{qr\log(dx+c)}{h^2x+gh}-\frac{\left(bdgpr-bchpr+bdgqr-adhqr\right)\log\left(hx+\right.}{bdg^2h-bcgh^2-adgh^2+ach^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] b^2*p*r*log(abs(-b*x - a))/(b^2*g*h - a*b*h^2) + d^2*q*r*log(abs(d*x + c))/
(d^2*g*h - c*d*h^2) - p*r*log(b*x + a)/(h^2*x + g*h) - q*r*log(d*x + c)/(h^
2*x + g*h) - (b*d*g*p*r - b*c*h*p*r + b*d*g*q*r - a*d*h*q*r)*log(h*x + g)/(
b*d*g^2*h - b*c*g*h^2 - a*d*g*h^2 + a*c*h^3) - (r*log(f) + 1)/(h^2*x + g*h)
```

3.32 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^3}\,dx$

Optimal. Leaf size=202

$$\frac{b^2pr\log(a+bx)}{2h(bg-ah)^2}-\frac{b^2pr\log(g+hx)}{2h(bg-ah)^2}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2}+\frac{bpr}{2h(g+hx)(bg-ah)}+\frac{d^2qr\log(c+dx)}{2h(dg-ch)^2}-\frac{d^2q}{2h}$$

```
[Out] (b*p*r)/(2*h*(b*g - a*h)*(g + h*x)) + (d*q*r)/(2*h*(d*g - c*h)*(g + h*x)) +
 (b^2*p*r*Log[a + b*x])/(2*h*(b*g - a*h)^2) + (d^2*q*r*Log[c + d*x])/(2*h*(
d*g - c*h)^2) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(2*h*(g + h*x)^2) - (b
^2*p*r*Log[g + h*x])/(2*h*(b*g - a*h)^2) - (d^2*q*r*Log[g + h*x])/(2*h*(d*g
 - c*h)^2)
```

Rubi [A] time = 0.111538, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2495, 44}

$$\frac{b^2pr\log(a+bx)}{2h(bg-ah)^2}-\frac{b^2pr\log(g+hx)}{2h(bg-ah)^2}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2}+\frac{bpr}{2h(g+hx)(bg-ah)}+\frac{d^2qr\log(c+dx)}{2h(dg-ch)^2}-\frac{d^2q}{2h}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^3,x]
```

```
[Out] (b*p*r)/(2*h*(b*g - a*h)*(g + h*x)) + (d*q*r)/(2*h*(d*g - c*h)*(g + h*x)) +
 (b^2*p*r*Log[a + b*x])/(2*h*(b*g - a*h)^2) + (d^2*q*r*Log[c + d*x])/(2*h*(
d*g - c*h)^2) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(2*h*(g + h*x)^2) - (b
^2*p*r*Log[g + h*x])/(2*h*(b*g - a*h)^2) - (d^2*q*r*Log[g + h*x])/(2*h*(d*g
 - c*h)^2)
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^3}\,dx = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2} + \frac{(bpr)\int\frac{1}{(a+bx)(g+hx)^2}dx}{2h} + \frac{(dqr)\int\frac{1}{(c+dx)(g+hx)^2}dx}{2h}$$

$$= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2} + \frac{(bpr)\int\left(\frac{b^2}{(bg-ah)^2(a+bx)} - \frac{h}{(bg-ah)(g+hx)^2} - \frac{bh}{(bg-ah)^2(g\cdots}\right.}{2h}$$

$$= \frac{bpr}{2h(bg-ah)(g+hx)} + \frac{dqr}{2h(dg-ch)(g+hx)} + \frac{b^2pr\log(a+bx)}{2h(bg-ah)^2} + \frac{d^2qr\log(c+dx)}{2h(dg-ch)^2}$$

Mathematica [A] time = 0.549866, size = 206, normalized size = 1.02

$$\frac{\frac{r(g+hx)\left((bc-ad)(bg-ah)(dg-ch)(bdg(p+q)-h(adq+bcp))-(g+hx)\left(d^2q(ad-bc)(bg-ah)^2(\log(c+dx)-\log(g+hx))-b^2p(bc-ad)(dg-ch)^2(\log(a+bx)-\log(g+hx))\right)\right)}{(bc-ad)(bg-ah)^2(dg-ch)^2} - \log\cdots}{2h(g+hx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^3,x]
```

```
[Out] (-Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (r*(g + h*x)*((b*c - a*d)*(b*g - a
*h)*(d*g - c*h)*(b*d*g*(p + q) - h*(b*c*p + a*d*q)) - (g + h*x)*(-(b^2*(b*c
 - a*d)*(d*g - c*h)^2*p*(Log[a + b*x] - Log[g + h*x])) + d^2*(-(b*c) + a*d)
*(b*g - a*h)^2*q*(Log[c + d*x] - Log[g + h*x]))))/((b*c - a*d)*(b*g - a*h)^
2*(d*g - c*h)^2))/(2*h*(g + h*x)^2)
```

Maple [F] time = 0.496, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^p\left(dx+c\right)^q\right)^r\right)}{\left(hx+g\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x)
```

Maxima [A] time = 1.25929, size = 313, normalized size = 1.55

$$\frac{\left(bfp\left(\frac{b\log(bx+a)}{b^2g^2-2\,abgh+a^2h^2} - \frac{b\log\left(hx+g\right)}{b^2g^2-2\,abgh+a^2h^2} + \frac{1}{bg^2-agh+\left(bgh-ah^2\right)x}\right) + dfq\left(\frac{d\log(dx+c)}{d^2g^2-2\,cdgh+c^2h^2} - \frac{d\log\left(hx+g\right)}{d^2g^2-2\,cdgh+c^2h^2} + \frac{1}{dg^2-cgh+\left(dgh-ch^2\right)x}\right)\right)r}{2\,fh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b*f*p*(b*log(b*x + a)/(b^2*g^2 - 2*a*b*g*h + a^2*h^2) - b*log(h*x + g)
/(b^2*g^2 - 2*a*b*g*h + a^2*h^2) + 1/(b*g^2 - a*g*h + (b*g*h - a*h^2)*x)) +
 d*f*q*(d*log(d*x + c)/(d^2*g^2 - 2*c*d*g*h + c^2*h^2) - d*log(h*x + g)/(d^
```

```
2*g^2 - 2*c*d*g*h + c^2*h^2) + 1/(d*g^2 - c*g*h + (d*g*h - c*h^2)*x)))*r/(f
*h) - 1/2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*x + g)^2*h)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**3,x)

[Out] Timed out
```

Giac [B] time = 1.57893, size = 1413, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^3,x, algorithm="giac")

[Out] -1/2*p*r*log(b*x + a)/(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1/2*q*r*log(d*x + c)/
(h^3*x^2 + 2*g*h^2*x + g^2*h) - 1/2*(b^2*d^2*g^2*p*r - 2*b^2*c*d*g*h*p*r +
b^2*c^2*h^2*p*r + b^2*d^2*g^2*q*r - 2*a*b*d^2*g*h*q*r + a^2*d^2*h^2*q*r)*lo
g(h*x + g)/(b^2*d^2*g^4*h - 2*b^2*c*d*g^3*h^2 - 2*a*b*d^2*g^3*h^2 + b^2*c^2
*g^2*h^3 + 4*a*b*c*d*g^2*h^3 + a^2*d^2*g^2*h^3 - 2*a*b*c^2*g*h^4 - 2*a^2*c*
d*g*h^4 + a^2*c^2*h^5) + 1/4*(b^2*d^2*g^2*p*r - 2*b^2*c*d*g*h*p*r + b^2*c^2
*h^2*p*r + b^2*d^2*g^2*q*r - 2*a*b*d^2*g*h*q*r + a^2*d^2*h^2*q*r)*log(abs(b
*d*x^2 + b*c*x + a*d*x + a*c))/(b^2*d^2*g^4*h - 2*b^2*c*d*g^3*h^2 - 2*a*b*d
^2*g^3*h^2 + b^2*c^2*g^2*h^3 + 4*a*b*c*d*g^2*h^3 + a^2*d^2*g^2*h^3 - 2*a*b*
c^2*g*h^4 - 2*a^2*c*d*g*h^4 + a^2*c^2*h^5) + 1/2*(b*d*g*h*p*r*x - b*c*h^2*p
*r*x + b*d*g*h*q*r*x - a*d*h^2*q*r*x + b*d*g^2*p*r - b*c*g*h*p*r + b*d*g^2*
q*r - a*d*g*h*q*r - b*d*g^2*r*log(f) + b*c*g*h*r*log(f) + a*d*g*h*r*log(f)
- a*c*h^2*r*log(f) - b*d*g^2 + b*c*g*h + a*d*g*h - a*c*h^2)/(b*d*g^2*h^3*x^
2 - b*c*g*h^4*x^2 - a*d*g*h^4*x^2 + a*c*h^5*x^2 + 2*b*d*g^3*h^2*x - 2*b*c*g
^2*h^3*x - 2*a*d*g^2*h^3*x + 2*a*c*g*h^4*x + b*d*g^4*h - b*c*g^3*h^2 - a*d*
g^3*h^2 + a*c*g^2*h^3) + 1/4*(b^3*c*d^2*g^2*p*r - a*b^2*d^3*g^2*p*r - 2*b^3
*c^2*d*g*h*p*r + 2*a*b^2*c*d^2*g*h*p*r + b^3*c^3*h^2*p*r - a*b^2*c^2*d*h^2*
p*r - b^3*c*d^2*g^2*q*r + a*b^2*d^3*g^2*q*r + 2*a*b^2*c*d^2*g*h*q*r - 2*a^2
*b*d^3*g*h*q*r - a^2*b*c*d^2*h^2*q*r + a^3*d^3*h^2*q*r)*log(abs((2*b*d*x +
b*c + a*d - abs(-b*c + a*d))/(2*b*d*x + b*c + a*d + abs(-b*c + a*d))))/((b^
2*d^2*g^4*h - 2*b^2*c*d*g^3*h^2 - 2*a*b*d^2*g^3*h^2 + b^2*c^2*g^2*h^3 + 4*a
```

```
*b*c*d*g^2*h^3 + a^2*d^2*g^2*h^3 - 2*a*b*c^2*g*h^4 - 2*a^2*c*d*g*h^4 + a^2*
c^2*h^5)*abs(-b*c + a*d))
```

3.33 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^4}\,dx$

Optimal. Leaf size=260

$$\frac{b^2pr}{3h(g+hx)(bg-ah)^2}+\frac{b^3pr\log(a+bx)}{3h(bg-ah)^3}-\frac{b^3pr\log(g+hx)}{3h(bg-ah)^3}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3}+\frac{bpr}{6h(g+hx)^2(bg-ah)}$$

```
[Out] (b*p*r)/(6*h*(b*g - a*h)*(g + h*x)^2) + (d*q*r)/(6*h*(d*g - c*h)*(g + h*x)^
2) + (b^2*p*r)/(3*h*(b*g - a*h)^2*(g + h*x)) + (d^2*q*r)/(3*h*(d*g - c*h)^2
*(g + h*x)) + (b^3*p*r*Log[a + b*x])/(3*h*(b*g - a*h)^3) + (d^3*q*r*Log[c +
 d*x])/(3*h*(d*g - c*h)^3) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(3*h*(g +
 h*x)^3) - (b^3*p*r*Log[g + h*x])/(3*h*(b*g - a*h)^3) - (d^3*q*r*Log[g + h*
x])/(3*h*(d*g - c*h)^3)
```

Rubi [A] time = 0.151959, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2495, 44}

$$\frac{b^2pr}{3h(g+hx)(bg-ah)^2}+\frac{b^3pr\log(a+bx)}{3h(bg-ah)^3}-\frac{b^3pr\log(g+hx)}{3h(bg-ah)^3}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3}+\frac{bpr}{6h(g+hx)^2(bg-ah)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^4,x]
```


```
[Out] (b*p*r)/(6*h*(b*g - a*h)*(g + h*x)^2) + (d*q*r)/(6*h*(d*g - c*h)*(g + h*x)^
2) + (b^2*p*r)/(3*h*(b*g - a*h)^2*(g + h*x)) + (d^2*q*r)/(3*h*(d*g - c*h)^2
*(g + h*x)) + (b^3*p*r*Log[a + b*x])/(3*h*(b*g - a*h)^3) + (d^3*q*r*Log[c +
 d*x])/(3*h*(d*g - c*h)^3) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(3*h*(g +
 h*x)^3) - (b^3*p*r*Log[g + h*x])/(3*h*(b*g - a*h)^3) - (d^3*q*r*Log[g + h*
x])/(3*h*(d*g - c*h)^3)
```


Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^4}\,dx = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3} + \frac{(bpr)\int \frac{1}{(a+bx)(g+hx)^3}\,dx}{3h} + \frac{(dqr)\int \frac{1}{(c+dx)(g+hx)^3}\,dx}{3h}$$

$$= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3} + \frac{(bpr)\int\left(\frac{b^3}{(bg-ah)^3(a+bx)} - \frac{h}{(bg-ah)(g+hx)^3} - \frac{bh}{(bg-ah)^2(g-}\right.}{3h}$$

$$= \frac{bpr}{6h(bg-ah)(g+hx)^2} + \frac{dqr}{6h(dg-ch)(g+hx)^2} + \frac{b^2pr}{3h(bg-ah)^2(g+hx)} + \frac{d}{3h(dg-c}$$

Mathematica [A] time = 0.959729, size = 254, normalized size = 0.98

$$\frac{\frac{r(g+hx)\left((bg-ah)^2(dg-ch)^2(bdg(p+q)-h(adq+bcp))-(g+hx)\left((bg-ah)(dg-ch)\left(-2a^2d^2h^2q+4abd^2ghq-2b^2\left(c^2h^2p-2cdghp+d^2g^2(p+q)\right)\right)-2(g+hx)\left(b^3p(dg-ch)^3(\log(a\right.\right.\right.}{(bg-ah)^3(dg-ch)^3}}{6h(g+hx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^4,x]
```

```
[Out] (-2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (r*(g + h*x)*((b*g - a*h)^2*(d*g
 - c*h)^2*(b*d*g*(p + q) - h*(b*c*p + a*d*q)) - (g + h*x)*((b*g - a*h)*(d*g
 - c*h)*(4*a*b*d^2*g*h*q - 2*a^2*d^2*h^2*q - 2*b^2*(-2*c*d*g*h*p + c^2*h^2*
p + d^2*g^2*(p + q))) - 2*(g + h*x)*(b^3*(d*g - c*h)^3*p*(Log[a + b*x] - Lo
g[g + h*x]) + d^3*(b*g - a*h)^3*q*(Log[c + d*x] - Log[g + h*x])))))/((b*g -
 a*h)^3*(d*g - c*h)^3))/(6*h*(g + h*x)^3)
```

Maple [F] time = 0.519, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)}{(hx+g)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x)
```

Maxima [A] time = 1.29783, size = 616, normalized size = 2.37

$$\frac{\left(\left(\frac{2b^2\log(bx+a)}{b^3g^3-3ab^2g^2h+3a^2bgh^2-a^3h^3} - \frac{2b^2\log(hx+g)}{b^3g^3-3ab^2g^2h+3a^2bgh^2-a^3h^3} + \frac{2bhx+3bg-ah}{b^2g^4-2abg^3h+a^2g^2h^2+\left(b^2g^2h^2-2abgh^3+a^2h^4\right)x^2+2\left(b^2g^3h-2abg^2h^2+a^2gh^3\right)x}\right)bfp\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x, algorithm="maxima")
```

```
[Out] 1/6*((2*b^2*log(b*x + a)/(b^3*g^3 - 3*a*b^2*g^2*h + 3*a^2*b*g*h^2 - a^3*h^3
) - 2*b^2*log(h*x + g)/(b^3*g^3 - 3*a*b^2*g^2*h + 3*a^2*b*g*h^2 - a^3*h^3)
```

```
+ (2*b*h*x + 3*b*g - a*h)/(b^2*g^4 - 2*a*b*g^3*h + a^2*g^2*h^2 + (b^2*g^2*h
^2 - 2*a*b*g*h^3 + a^2*h^4)*x^2 + 2*(b^2*g^3*h - 2*a*b*g^2*h^2 + a^2*g*h^3)
*x))*b*f*p + (2*d^2*log(d*x + c)/(d^3*g^3 - 3*c*d^2*g^2*h + 3*c^2*d*g*h^2 -
 c^3*h^3) - 2*d^2*log(h*x + g)/(d^3*g^3 - 3*c*d^2*g^2*h + 3*c^2*d*g*h^2 - c
^3*h^3) + (2*d*h*x + 3*d*g - c*h)/(d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2 + (d
^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*x^2 + 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^
2*g*h^3)*x))*d*f*q)*r/(f*h) - 1/3*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*
x + g)^3*h)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.86354, size = 2383, normalized size = 9.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/3*b^4*p*r*log(abs(b*x + a))/(b^4*g^3*h - 3*a*b^3*g^2*h^2 + 3*a^2*b^2*g*h^
3 - a^3*b*h^4) + 1/3*d^4*q*r*log(abs(d*x + c))/(d^4*g^3*h - 3*c*d^3*g^2*h^2
 + 3*c^2*d^2*g*h^3 - c^3*d*h^4) - 1/3*p*r*log(b*x + a)/(h^4*x^3 + 3*g*h^3*x
^2 + 3*g^2*h^2*x + g^3*h) - 1/3*q*r*log(d*x + c)/(h^4*x^3 + 3*g*h^3*x^2 + 3
*g^2*h^2*x + g^3*h) - 1/3*(b^3*d^3*g^3*p*r - 3*b^3*c*d^2*g^2*h*p*r + 3*b^3*
c^2*d*g*h^2*p*r - b^3*c^3*h^3*p*r + b^3*d^3*g^3*q*r - 3*a*b^2*d^3*g^2*h*q*r
 + 3*a^2*b*d^3*g*h^2*q*r - a^3*d^3*h^3*q*r)*log(h*x + g)/(b^3*d^3*g^6*h - 3
*b^3*c*d^2*g^5*h^2 - 3*a*b^2*d^3*g^5*h^2 + 3*b^3*c^2*d*g^4*h^3 + 9*a*b^2*c*
d^2*g^4*h^3 + 3*a^2*b*d^3*g^4*h^3 - b^3*c^3*g^3*h^4 - 9*a*b^2*c^2*d*g^3*h^4
 - 9*a^2*b*c*d^2*g^3*h^4 - a^3*d^3*g^3*h^4 + 3*a*b^2*c^3*g^2*h^5 + 9*a^2*b*
c^2*d*g^2*h^5 + 3*a^3*c*d^2*g^2*h^5 - 3*a^2*b*c^3*g*h^6 - 3*a^3*c^2*d*g*h^6
 + a^3*c^3*h^7) + 1/6*(2*b^2*d^2*g^2*h^2*p*r*x^2 - 4*b^2*c*d*g*h^3*p*r*x^2
+ 2*b^2*c^2*h^4*p*r*x^2 + 2*b^2*d^2*g^2*h^2*q*r*x^2 - 4*a*b*d^2*g*h^3*q*r*x
^2 + 2*a^2*d^2*h^4*q*r*x^2 + 5*b^2*d^2*g^3*h*p*r*x - 10*b^2*c*d*g^2*h^2*p*r
*x - a*b*d^2*g^2*h^2*p*r*x + 5*b^2*c^2*g*h^3*p*r*x + 2*a*b*c*d*g*h^3*p*r*x
```

```
- a*b*c^2*h^4*p*r*x + 5*b^2*d^2*g^3*h*q*r*x - b^2*c*d*g^2*h^2*q*r*x - 10*a*
b*d^2*g^2*h^2*q*r*x + 2*a*b*c*d*g*h^3*q*r*x + 5*a^2*d^2*g*h^3*q*r*x - a^2*c
*d*h^4*q*r*x + 3*b^2*d^2*g^4*p*r - 6*b^2*c*d*g^3*h*p*r - a*b*d^2*g^3*h*p*r
+ 3*b^2*c^2*g^2*h^2*p*r + 2*a*b*c*d*g^2*h^2*p*r - a*b*c^2*g*h^3*p*r + 3*b^2
*d^2*g^4*q*r - b^2*c*d*g^3*h*q*r - 6*a*b*d^2*g^3*h*q*r + 2*a*b*c*d*g^2*h^2*
q*r + 3*a^2*d^2*g^2*h^2*q*r - a^2*c*d*g*h^3*q*r - 2*b^2*d^2*g^4*r*log(f) +
4*b^2*c*d*g^3*h*r*log(f) + 4*a*b*d^2*g^3*h*r*log(f) - 2*b^2*c^2*g^2*h^2*r*l
og(f) - 8*a*b*c*d*g^2*h^2*r*log(f) - 2*a^2*d^2*g^2*h^2*r*log(f) + 4*a*b*c^2
*g*h^3*r*log(f) + 4*a^2*c*d*g*h^3*r*log(f) - 2*a^2*c^2*h^4*r*log(f) - 2*b^2
*d^2*g^4 + 4*b^2*c*d*g^3*h + 4*a*b*d^2*g^3*h - 2*b^2*c^2*g^2*h^2 - 8*a*b*c*
d*g^2*h^2 - 2*a^2*d^2*g^2*h^2 + 4*a*b*c^2*g*h^3 + 4*a^2*c*d*g*h^3 - 2*a^2*c
^2*h^4)/(b^2*d^2*g^4*h^4*x^3 - 2*b^2*c*d*g^3*h^5*x^3 - 2*a*b*d^2*g^3*h^5*x^
3 + b^2*c^2*g^2*h^6*x^3 + 4*a*b*c*d*g^2*h^6*x^3 + a^2*d^2*g^2*h^6*x^3 - 2*a
*b*c^2*g*h^7*x^3 - 2*a^2*c*d*g*h^7*x^3 + a^2*c^2*h^8*x^3 + 3*b^2*d^2*g^5*h^
3*x^2 - 6*b^2*c*d*g^4*h^4*x^2 - 6*a*b*d^2*g^4*h^4*x^2 + 3*b^2*c^2*g^3*h^5*x
^2 + 12*a*b*c*d*g^3*h^5*x^2 + 3*a^2*d^2*g^3*h^5*x^2 - 6*a*b*c^2*g^2*h^6*x^2
 - 6*a^2*c*d*g^2*h^6*x^2 + 3*a^2*c^2*g*h^7*x^2 + 3*b^2*d^2*g^6*h^2*x - 6*b^
2*c*d*g^5*h^3*x - 6*a*b*d^2*g^5*h^3*x + 3*b^2*c^2*g^4*h^4*x + 12*a*b*c*d*g^
4*h^4*x + 3*a^2*d^2*g^4*h^4*x - 6*a*b*c^2*g^3*h^5*x - 6*a^2*c*d*g^3*h^5*x +
 3*a^2*c^2*g^2*h^6*x + b^2*d^2*g^7*h - 2*b^2*c*d*g^6*h^2 - 2*a*b*d^2*g^6*h^
2 + b^2*c^2*g^5*h^3 + 4*a*b*c*d*g^5*h^3 + a^2*d^2*g^5*h^3 - 2*a*b*c^2*g^4*h
^4 - 2*a^2*c*d*g^4*h^4 + a^2*c^2*g^3*h^5)
```

3.34 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^5}\,dx$

Optimal. Leaf size=318

$$\frac{b^3pr}{4h(g+hx)(bg-ah)^3}+\frac{b^2pr}{8h(g+hx)^2(bg-ah)^2}+\frac{b^4pr\log(a+bx)}{4h(bg-ah)^4}-\frac{b^4pr\log(g+hx)}{4h(bg-ah)^4}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h(g+hx)^4}$$

```
[Out] (b*p*r)/(12*h*(b*g - a*h)*(g + h*x)^3) + (d*q*r)/(12*h*(d*g - c*h)*(g + h*x
)^3) + (b^2*p*r)/(8*h*(b*g - a*h)^2*(g + h*x)^2) + (d^2*q*r)/(8*h*(d*g - c*
h)^2*(g + h*x)^2) + (b^3*p*r)/(4*h*(b*g - a*h)^3*(g + h*x)) + (d^3*q*r)/(4*
h*(d*g - c*h)^3*(g + h*x)) + (b^4*p*r*Log[a + b*x])/(4*h*(b*g - a*h)^4) + (
d^4*q*r*Log[c + d*x])/(4*h*(d*g - c*h)^4) - Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]/(4*h*(g + h*x)^4) - (b^4*p*r*Log[g + h*x])/(4*h*(b*g - a*h)^4) - (d^4
*q*r*Log[g + h*x])/(4*h*(d*g - c*h)^4)
```

Rubi [A] time = 0.194835, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2495, 44}

$$\frac{b^3pr}{4h(g+hx)(bg-ah)^3}+\frac{b^2pr}{8h(g+hx)^2(bg-ah)^2}+\frac{b^4pr\log(a+bx)}{4h(bg-ah)^4}-\frac{b^4pr\log(g+hx)}{4h(bg-ah)^4}-\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h(g+hx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^5,x]
```

```
[Out] (b*p*r)/(12*h*(b*g - a*h)*(g + h*x)^3) + (d*q*r)/(12*h*(d*g - c*h)*(g + h*x
)^3) + (b^2*p*r)/(8*h*(b*g - a*h)^2*(g + h*x)^2) + (d^2*q*r)/(8*h*(d*g - c*
h)^2*(g + h*x)^2) + (b^3*p*r)/(4*h*(b*g - a*h)^3*(g + h*x)) + (d^3*q*r)/(4*
h*(d*g - c*h)^3*(g + h*x)) + (b^4*p*r*Log[a + b*x])/(4*h*(b*g - a*h)^4) + (
d^4*q*r*Log[c + d*x])/(4*h*(d*g - c*h)^4) - Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]/(4*h*(g + h*x)^4) - (b^4*p*r*Log[g + h*x])/(4*h*(b*g - a*h)^4) - (d^4
*q*r*Log[g + h*x])/(4*h*(d*g - c*h)^4)
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^5}dx = -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h(g+hx)^4} + \frac{(bpr)\int \frac{1}{(a+bx)(g+hx)^4}dx}{4h} + \frac{(dqr)\int \frac{1}{(c+dx)(g+hx)^4}dx}{4h}$$

$$= -\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h(g+hx)^4} + \frac{(bpr)\int\left(\frac{b^4}{(bg-ah)^4(a+bx)} - \frac{h}{(bg-ah)(g+hx)^4} - \frac{bh}{(bg-ah)^2(g-}\right.}{4h}$$

$$= \frac{bpr}{12h(bg-ah)(g+hx)^3} + \frac{dqr}{12h(dg-ch)(g+hx)^3} + \frac{b^2pr}{8h(bg-ah)^2(g+hx)^2} + \frac{}{8h(dg}$$

Mathematica [A] time = 1.82202, size = 480, normalized size = 1.51

$$\frac{r(g+hx)\left(2(bg-ah)^3(dg-ch)^3(bdg(p+q)-h(adq+bcp))-(g+hx)\left(6(g+hx)\left(-(g+hx)\left(-\log(g+hx)\left(6a^2b^2d^4g^2h^2q-4a^3bd^4gh^3q+a^4d^4h^4q-4ab^3d^4g^3hq+b^4\left(6c^2d^2g^2h^2p-4\right.\right.\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g + h*x)^5,x]
```

```
[Out] (-6*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (r*(g + h*x)*(2*(b*g - a*h)^3*(d
*g - c*h)^3*(b*d*g*(p + q) - h*(b*c*p + a*d*q)) - (g + h*x)*((b*g - a*h)^2*
(d*g - c*h)^2*(6*a*b*d^2*g*h*q - 3*a^2*d^2*h^2*q - 3*b^2*(-2*c*d*g*h*p + c^
2*h^2*p + d^2*g^2*(p + q))) + 6*(g + h*x)*(-((b*g - a*h)*(-(d*g) + c*h)*(3*
a*b^2*d^3*g^2*h*q - 3*a^2*b*d^3*g*h^2*q + a^3*d^3*h^3*q - b^3*(-3*c*d^2*g^2
*h*p + 3*c^2*d*g*h^2*p - c^3*h^3*p + d^3*g^3*(p + q)))) - (g + h*x)*(b^4*(d
*g - c*h)^4*p*Log[a + b*x] + d^4*(b*g - a*h)^4*q*Log[c + d*x] - (-4*a*b^3*d
^4*g^3*h*q + 6*a^2*b^2*d^4*g^2*h^2*q - 4*a^3*b*d^4*g*h^3*q + a^4*d^4*h^4*q
+ b^4*(-4*c*d^3*g^3*h*p + 6*c^2*d^2*g^2*h^2*p - 4*c^3*d*g*h^3*p + c^4*h^4*p
 + d^4*g^4*(p + q)))*Log[g + h*x])))))/((b*g - a*h)^4*(d*g - c*h)^4))/(24*h
*(g + h*x)^4)
```

Maple [F] time = 0.509, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^p\left(dx+c\right)^q\right)^r\right)}{\left(hx+g\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x)
```

Maxima [B] time = 1.41247, size = 1048, normalized size = 3.3

$$\frac{\left(\left(\frac{6\,b^3\log(bx+a)}{b^4g^4-4\,ab^3g^3h+6\,a^2b^2g^2h^2-4\,a^3bgh^3+a^4h^4} - \frac{6\,b^3\log(hx+g)}{b^4g^4-4\,ab^3g^3h+6\,a^2b^2g^2h^2-4\,a^3bgh^3+a^4h^4} + \frac{6\,b^2h^2x^2}{b^3g^6-3\,ab^2g^5h+3\,a^2bg^4h^2-a^3g^3h^3+\left(b^3g^3h^3-3\,ab^2g^2h^4+3\,a^2bg\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x, algorithm="maxima")
```

```
[Out] 1/24*((6*b^3*log(b*x + a)/(b^4*g^4 - 4*a*b^3*g^3*h + 6*a^2*b^2*g^2*h^2 - 4*
a^3*b*g*h^3 + a^4*h^4) - 6*b^3*log(h*x + g)/(b^4*g^4 - 4*a*b^3*g^3*h + 6*a^
2*b^2*g^2*h^2 - 4*a^3*b*g*h^3 + a^4*h^4) + (6*b^2*h^2*x^2 + 11*b^2*g^2 - 7*
a*b*g*h + 2*a^2*h^2 + 3*(5*b^2*g*h - a*b*h^2)*x)/(b^3*g^6 - 3*a*b^2*g^5*h +
 3*a^2*b*g^4*h^2 - a^3*g^3*h^3 + (b^3*g^3*h^3 - 3*a*b^2*g^2*h^4 + 3*a^2*b*g
*h^5 - a^3*h^6)*x^3 + 3*(b^3*g^4*h^2 - 3*a*b^2*g^3*h^3 + 3*a^2*b*g^2*h^4 -
a^3*g*h^5)*x^2 + 3*(b^3*g^5*h - 3*a*b^2*g^4*h^2 + 3*a^2*b*g^3*h^3 - a^3*g^2
*h^4)*x))*b*f*p + (6*d^3*log(d*x + c)/(d^4*g^4 - 4*c*d^3*g^3*h + 6*c^2*d^2*
g^2*h^2 - 4*c^3*d*g*h^3 + c^4*h^4) - 6*d^3*log(h*x + g)/(d^4*g^4 - 4*c*d^3*
g^3*h + 6*c^2*d^2*g^2*h^2 - 4*c^3*d*g*h^3 + c^4*h^4) + (6*d^2*h^2*x^2 + 11*
d^2*g^2 - 7*c*d*g*h + 2*c^2*h^2 + 3*(5*d^2*g*h - c*d*h^2)*x)/(d^3*g^6 - 3*c
*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3 + (d^3*g^3*h^3 - 3*c*d^2*g^2*h^4
 + 3*c^2*d*g*h^5 - c^3*h^6)*x^3 + 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*
d*g^2*h^4 - c^3*g*h^5)*x^2 + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h
^3 - c^3*g^2*h^4)*x))*d*f*q)*r/(f*h) - 1/4*log(((b*x + a)^p*(d*x + c)^q*f)^
r*e)/((h*x + g)^4*h)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*x+g)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.77682, size = 6884, normalized size = 21.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] -1/4*p*r*log(b*x + a)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*x
+ g^4*h) - 1/4*q*r*log(d*x + c)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*
g^3*h^2*x + g^4*h) - 1/4*(b^4*d^4*g^4*p*r - 4*b^4*c*d^3*g^3*h*p*r + 6*b^4*c
^2*d^2*g^2*h^2*p*r - 4*b^4*c^3*d*g*h^3*p*r + b^4*c^4*h^4*p*r + b^4*d^4*g^4*
```

q*r - 4*a*b^3*d^4*g^3*h*q*r + 6*a^2*b^2*d^4*g^2*h^2*q*r - 4*a^3*b*d^4*g*h^3 *q*r + a^4*d^4*h^4*q*r)*log(h*x + g)/(b^4*d^4*g^8*h - 4*b^4*c*d^3*g^7*h^2 - 4*a*b^3*d^4*g^7*h^2 + 6*b^4*c^2*d^2*g^6*h^3 + 16*a*b^3*c*d^3*g^6*h^3 + 6*a ^2*b^2*d^4*g^6*h^3 - 4*b^4*c^3*d*g^5*h^4 - 24*a*b^3*c^2*d^2*g^5*h^4 - 24*a^ 2*b^2*c*d^3*g^5*h^4 - 4*a^3*b*d^4*g^5*h^4 + b^4*c^4*g^4*h^5 + 16*a*b^3*c^3* d*g^4*h^5 + 36*a^2*b^2*c^2*d^2*g^4*h^5 + 16*a^3*b*c*d^3*g^4*h^5 + a^4*d^4*g ^4*h^5 - 4*a*b^3*c^4*g^3*h^6 - 24*a^2*b^2*c^3*d*g^3*h^6 - 24*a^3*b*c^2*d^2* g^3*h^6 - 4*a^4*c*d^3*g^3*h^6 + 6*a^2*b^2*c^4*g^2*h^7 + 16*a^3*b*c^3*d*g^2* h^7 + 6*a^4*c^2*d^2*g^2*h^7 - 4*a^3*b*c^4*g*h^8 - 4*a^4*c^3*d*g*h^8 + a^4*c ^4*h^9) + 1/8*(b^4*d^4*g^4*p*r - 4*b^4*c*d^3*g^3*h*p*r + 6*b^4*c^2*d^2*g^2* h^2*p*r - 4*b^4*c^3*d*g*h^3*p*r + b^4*c^4*h^4*p*r + b^4*d^4*g^4*q*r - 4*a*b ^3*d^4*g^3*h*q*r + 6*a^2*b^2*d^4*g^2*h^2*q*r - 4*a^3*b*d^4*g*h^3*q*r + a^4* d^4*h^4*q*r)*log(abs(b*d*x^2 + b*c*x + a*d*x + a*c))/(b^4*d^4*g^8*h - 4*b^4 *c*d^3*g^7*h^2 - 4*a*b^3*d^4*g^7*h^2 + 6*b^4*c^2*d^2*g^6*h^3 + 16*a*b^3*c*d ^3*g^6*h^3 + 6*a^2*b^2*d^4*g^6*h^3 - 4*b^4*c^3*d*g^5*h^4 - 24*a*b^3*c^2*d^2 *g^5*h^4 - 24*a^2*b^2*c*d^3*g^5*h^4 - 4*a^3*b*d^4*g^5*h^4 + b^4*c^4*g^4*h^5 + 16*a*b^3*c^3*d*g^4*h^5 + 36*a^2*b^2*c^2*d^2*g^4*h^5 + 16*a^3*b*c*d^3*g^4 *h^5 + a^4*d^4*g^4*h^5 - 4*a*b^3*c^4*g^3*h^6 - 24*a^2*b^2*c^3*d*g^3*h^6 - 2 4*a^3*b*c^2*d^2*g^3*h^6 - 4*a^4*c*d^3*g^3*h^6 + 6*a^2*b^2*c^4*g^2*h^7 + 16* a^3*b*c^3*d*g^2*h^7 + 6*a^4*c^2*d^2*g^2*h^7 - 4*a^3*b*c^4*g*h^8 - 4*a^4*c^3 *d*g*h^8 + a^4*c^4*h^9) + 1/24*(6*b^3*d^3*g^3*h^3*p*r*x^3 - 18*b^3*c*d^2*g^ 2*h^4*p*r*x^3 + 18*b^3*c^2*d*g*h^5*p*r*x^3 - 6*b^3*c^3*h^6*p*r*x^3 + 6*b^3* d^3*g^3*h^3*q*r*x^3 - 18*a*b^2*d^3*g^2*h^4*q*r*x^3 + 18*a^2*b*d^3*g*h^5*q*r *x^3 - 6*a^3*d^3*h^6*q*r*x^3 + 21*b^3*d^3*g^4*h^2*p*r*x^2 - 63*b^3*c*d^2*g^ 3*h^3*p*r*x^2 - 3*a*b^2*d^3*g^3*h^3*p*r*x^2 + 63*b^3*c^2*d*g^2*h^4*p*r*x^2 + 9*a*b^2*c*d^2*g^2*h^4*p*r*x^2 - 21*b^3*c^3*g*h^5*p*r*x^2 - 9*a*b^2*c^2*d* g*h^5*p*r*x^2 + 3*a*b^2*c^3*h^6*p*r*x^2 + 21*b^3*d^3*g^4*h^2*q*r*x^2 - 3*b^ 3*c*d^2*g^3*h^3*q*r*x^2 - 63*a*b^2*d^3*g^3*h^3*q*r*x^2 + 9*a*b^2*c*d^2*g^2* h^4*q*r*x^2 + 63*a^2*b*d^3*g^2*h^4*q*r*x^2 - 9*a^2*b*c*d^2*g*h^5*q*r*x^2 - 21*a^3*d^3*g*h^5*q*r*x^2 + 3*a^3*c*d^2*h^6*q*r*x^2 + 26*b^3*d^3*g^5*h*p*r*x - 78*b^3*c*d^2*g^4*h^2*p*r*x - 10*a*b^2*d^3*g^4*h^2*p*r*x + 78*b^3*c^2*d*g ^3*h^3*p*r*x + 30*a*b^2*c*d^2*g^3*h^3*p*r*x + 2*a^2*b*d^3*g^3*h^3*p*r*x - 2 6*b^3*c^3*g^2*h^4*p*r*x - 30*a*b^2*c^2*d*g^2*h^4*p*r*x - 6*a^2*b*c*d^2*g^2* h^4*p*r*x + 10*a*b^2*c^3*g*h^5*p*r*x + 6*a^2*b*c^2*d*g*h^5*p*r*x - 2*a^2*b* c^3*h^6*p*r*x + 26*b^3*d^3*g^5*h*q*r*x - 10*b^3*c*d^2*g^4*h^2*q*r*x - 78*a* b^2*d^3*g^4*h^2*q*r*x + 2*b^3*c^2*d*g^3*h^3*q*r*x + 30*a*b^2*c*d^2*g^3*h^3* q*r*x + 78*a^2*b*d^3*g^3*h^3*q*r*x - 6*a*b^2*c^2*d*g^2*h^4*q*r*x - 30*a^2*b *c*d^2*g^2*h^4*q*r*x - 26*a^3*d^3*g^2*h^4*q*r*x + 6*a^2*b*c^2*d*g*h^5*q*r*x + 10*a^3*c*d^2*g*h^5*q*r*x - 2*a^3*c^2*d*h^6*q*r*x + 11*b^3*d^3*g^6*p*r - 33*b^3*c*d^2*g^5*h*p*r - 7*a*b^2*d^3*g^5*h*p*r + 33*b^3*c^2*d*g^4*h^2*p*r + 21*a*b^2*c*d^2*g^4*h^2*p*r + 2*a^2*b*d^3*g^4*h^2*p*r - 11*b^3*c^3*g^3*h^3* p*r - 21*a*b^2*c^2*d*g^3*h^3*p*r - 6*a^2*b*c*d^2*g^3*h^3*p*r + 7*a*b^2*c^3* g^2*h^4*p*r + 6*a^2*b*c^2*d*g^2*h^4*p*r - 2*a^2*b*c^3*g*h^5*p*r + 11*b^3*d^ 3*g^6*q*r - 7*b^3*c*d^2*g^5*h*q*r - 33*a*b^2*d^3*g^5*h*q*r + 2*b^3*c^2*d*g^ 4*h^2*q*r + 21*a*b^2*c*d^2*g^4*h^2*q*r + 33*a^2*b*d^3*g^4*h^2*q*r - 6*a*b^2 *c^2*d*g^3*h^3*q*r - 21*a^2*b*c*d^2*g^3*h^3*q*r - 11*a^3*d^3*g^3*h^3*q*r + 6*a^2*b*c^2*d*g^2*h^4*q*r + 7*a^3*c*d^2*g^2*h^4*q*r - 2*a^3*c^2*d*g*h^5*q*r - 6*b^3*d^3*g^6*r*log(f) + 18*b^3*c*d^2*g^5*h*r*log(f) + 18*a*b^2*d^3*g^5* h*r*log(f) - 18*b^3*c^2*d*g^4*h^2*r*log(f) - 54*a*b^2*c*d^2*g^4*h^2*r*log(f) - 18*a^2*b*d^3*g^4*h^2*r*log(f) + 6*b^3*c^3*g^3*h^3*r*log(f) + 54*a*b^2*c ^2*d*g^3*h^3*r*log(f) + 54*a^2*b*c*d^2*g^3*h^3*r*log(f) + 6*a^3*d^3*g^3*h^3 *r*log(f) - 18*a*b^2*c^3*g^2*h^4*r*log(f) - 54*a^2*b*c^2*d*g^2*h^4*r*log(f) - 18*a^3*c*d^2*g^2*h^4*r*log(f) + 18*a^2*b*c^3*g*h^5*r*log(f) + 18*a^3*c^2 *d*g*h^5*r*log(f) - 6*a^3*c^3*h^6*r*log(f) - 6*b^3*d^3*g^6 + 18*b^3*c*d^2*g ^5*h + 18*a*b^2*d^3*g^5*h - 18*b^3*c^2*d*g^4*h^2 - 54*a*b^2*c*d^2*g^4*h^2 - 18*a^2*b*d^3*g^4*h^2 + 6*b^3*c^3*g^3*h^3 + 54*a*b^2*c^2*d*g^3*h^3 + 54*a^2 *b*c*d^2*g^3*h^3 + 6*a^3*d^3*g^3*h^3 - 18*a*b^2*c^3*g^2*h^4 - 54*a^2*b*c^2* d*g^2*h^4 - 18*a^3*c*d^2*g^2*h^4 + 18*a^2*b*c^3*g*h^5 + 18*a^3*c^2*d*g*h^5 - 6*a^3*c^3*h^6)/(b^3*d^3*g^6*h^5*x^4 - 3*b^3*c*d^2*g^5*h^6*x^4 - 3*a*b^2*d

```
^3*g^5*h^6*x^4 + 3*b^3*c^2*d*g^4*h^7*x^4 + 9*a*b^2*c*d^2*g^4*h^7*x^4 + 3*a^
2*b*d^3*g^4*h^7*x^4 - b^3*c^3*g^3*h^8*x^4 - 9*a*b^2*c^2*d*g^3*h^8*x^4 - 9*a
^2*b*c*d^2*g^3*h^8*x^4 - a^3*d^3*g^3*h^8*x^4 + 3*a*b^2*c^3*g^2*h^9*x^4 + 9*
a^2*b*c^2*d*g^2*h^9*x^4 + 3*a^3*c*d^2*g^2*h^9*x^4 - 3*a^2*b*c^3*g*h^10*x^4
 - 3*a^3*c^2*d*g*h^10*x^4 + a^3*c^3*h^11*x^4 + 4*b^3*d^3*g^7*h^4*x^3 - 12*b^
3*c*d^2*g^6*h^5*x^3 - 12*a*b^2*d^3*g^6*h^5*x^3 + 12*b^3*c^2*d*g^5*h^6*x^3 +
 36*a*b^2*c*d^2*g^5*h^6*x^3 + 12*a^2*b*d^3*g^5*h^6*x^3 - 4*b^3*c^3*g^4*h^7*
x^3 - 36*a*b^2*c^2*d*g^4*h^7*x^3 - 36*a^2*b*c*d^2*g^4*h^7*x^3 - 4*a^3*d^3*g
^4*h^7*x^3 + 12*a*b^2*c^3*g^3*h^8*x^3 + 36*a^2*b*c^2*d*g^3*h^8*x^3 + 12*a^3
*c*d^2*g^3*h^8*x^3 - 12*a^2*b*c^3*g^2*h^9*x^3 - 12*a^3*c^2*d*g^2*h^9*x^3 +
4*a^3*c^3*g*h^10*x^3 + 6*b^3*d^3*g^8*h^3*x^2 - 18*b^3*c*d^2*g^7*h^4*x^2 - 1
8*a*b^2*d^3*g^7*h^4*x^2 + 18*b^3*c^2*d*g^6*h^5*x^2 + 54*a*b^2*c*d^2*g^6*h^5
*x^2 + 18*a^2*b*d^3*g^6*h^5*x^2 - 6*b^3*c^3*g^5*h^6*x^2 - 54*a*b^2*c^2*d*g^
5*h^6*x^2 - 54*a^2*b*c*d^2*g^5*h^6*x^2 - 6*a^3*d^3*g^5*h^6*x^2 + 18*a*b^2*c
^3*g^4*h^7*x^2 + 54*a^2*b*c^2*d*g^4*h^7*x^2 + 18*a^3*c*d^2*g^4*h^7*x^2 - 18
*a^2*b*c^3*g^3*h^8*x^2 - 18*a^3*c^2*d*g^3*h^8*x^2 + 6*a^3*c^3*g^2*h^9*x^2 +
 4*b^3*d^3*g^9*h^2*x - 12*b^3*c*d^2*g^8*h^3*x - 12*a*b^2*d^3*g^8*h^3*x + 12
*b^3*c^2*d*g^7*h^4*x + 36*a*b^2*c*d^2*g^7*h^4*x + 12*a^2*b*d^3*g^7*h^4*x -
4*b^3*c^3*g^6*h^5*x - 36*a*b^2*c^2*d*g^6*h^5*x - 36*a^2*b*c*d^2*g^6*h^5*x -
 4*a^3*d^3*g^6*h^5*x + 12*a*b^2*c^3*g^5*h^6*x + 36*a^2*b*c^2*d*g^5*h^6*x +
12*a^3*c*d^2*g^5*h^6*x - 12*a^2*b*c^3*g^4*h^7*x - 12*a^3*c^2*d*g^4*h^7*x +
4*a^3*c^3*g^3*h^8*x + b^3*d^3*g^10*h - 3*b^3*c*d^2*g^9*h^2 - 3*a*b^2*d^3*g^
9*h^2 + 3*b^3*c^2*d*g^8*h^3 + 9*a*b^2*c*d^2*g^8*h^3 + 3*a^2*b*d^3*g^8*h^3 -
 b^3*c^3*g^7*h^4 - 9*a*b^2*c^2*d*g^7*h^4 - 9*a^2*b*c*d^2*g^7*h^4 - a^3*d^3*
g^7*h^4 + 3*a*b^2*c^3*g^6*h^5 + 9*a^2*b*c^2*d*g^6*h^5 + 3*a^3*c*d^2*g^6*h^5
 - 3*a^2*b*c^3*g^5*h^6 - 3*a^3*c^2*d*g^5*h^6 + a^3*c^3*g^4*h^7) + 1/8*(b^5*
c*d^4*g^4*p*r - a*b^4*d^5*g^4*p*r - 4*b^5*c^2*d^3*g^3*h*p*r + 4*a*b^4*c*d^4
*g^3*h*p*r + 6*b^5*c^3*d^2*g^2*h^2*p*r - 6*a*b^4*c^2*d^3*g^2*h^2*p*r - 4*b^
5*c^4*d*g*h^3*p*r + 4*a*b^4*c^3*d^2*g*h^3*p*r + b^5*c^5*h^4*p*r - a*b^4*c^4
*d*h^4*p*r - b^5*c*d^4*g^4*q*r + a*b^4*d^5*g^4*q*r + 4*a*b^4*c*d^4*g^3*h*q*
r - 4*a^2*b^3*d^5*g^3*h*q*r - 6*a^2*b^3*c*d^4*g^2*h^2*q*r + 6*a^3*b^2*d^5*g
^2*h^2*q*r + 4*a^3*b^2*c*d^4*g*h^3*q*r - 4*a^4*b*d^5*g*h^3*q*r - a^4*b*c*d^
4*h^4*q*r + a^5*d^5*h^4*q*r)*log(abs((2*b*d*x + b*c + a*d - abs(-b*c + a*d)
)/(2*b*d*x + b*c + a*d + abs(-b*c + a*d))))/((b^4*d^4*g^8*h - 4*b^4*c*d^3*g
^7*h^2 - 4*a*b^3*d^4*g^7*h^2 + 6*b^4*c^2*d^2*g^6*h^3 + 16*a*b^3*c*d^3*g^6*h
^3 + 6*a^2*b^2*d^4*g^6*h^3 - 4*b^4*c^3*d*g^5*h^4 - 24*a*b^3*c^2*d^2*g^5*h^4
 - 24*a^2*b^2*c*d^3*g^5*h^4 - 4*a^3*b*d^4*g^5*h^4 + b^4*c^4*g^4*h^5 + 16*a*
b^3*c^3*d*g^4*h^5 + 36*a^2*b^2*c^2*d^2*g^4*h^5 + 16*a^3*b*c*d^3*g^4*h^5 + a
^4*d^4*g^4*h^5 - 4*a*b^3*c^4*g^3*h^6 - 24*a^2*b^2*c^3*d*g^3*h^6 - 24*a^3*b*
c^2*d^2*g^3*h^6 - 4*a^4*c*d^3*g^3*h^6 + 6*a^2*b^2*c^4*g^2*h^7 + 16*a^3*b*c^
3*d*g^2*h^7 + 6*a^4*c^2*d^2*g^2*h^7 - 4*a^3*b*c^4*g*h^8 - 4*a^4*c^3*d*g*h^8
 + a^4*c^4*h^9)*abs(-b*c + a*d))
```

3.35 $\int (g+hx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx$

Optimal. Leaf size=2240

result too large to display

```
[Out] (2*(b*g - a*h)^3*p^2*r^2*x)/b^3 + (5*(b*g - a*h)^3*p*q*r^2*x)/(8*b^3) + (5*
(b*g - a*h)^2*(d*g - c*h)*p*q*r^2*x)/(12*b^2*d) + (5*(b*g - a*h)*(d*g - c*h
)^2*p*q*r^2*x)/(12*b*d^2) + (5*(d*g - c*h)^3*p*q*r^2*x)/(8*d^3) + (2*(d*g -
 c*h)^3*q^2*r^2*x)/d^3 + (3*h*(b*g - a*h)^2*p^2*r^2*(a + b*x)^2)/(4*b^4) +
(2*h^2*(b*g - a*h)*p^2*r^2*(a + b*x)^3)/(9*b^4) + (h^3*p^2*r^2*(a + b*x)^4)
/(32*b^4) + (3*h*(d*g - c*h)^2*q^2*r^2*(c + d*x)^2)/(4*d^4) + (2*h^2*(d*g -
 c*h)*q^2*r^2*(c + d*x)^3)/(9*d^4) + (h^3*q^2*r^2*(c + d*x)^4)/(32*d^4) + (
3*(b*g - a*h)^2*p*q*r^2*(g + h*x)^2)/(16*b^2*h) + ((b*g - a*h)*(d*g - c*h)*
p*q*r^2*(g + h*x)^2)/(6*b*d*h) + (3*(d*g - c*h)^2*p*q*r^2*(g + h*x)^2)/(16*
d^2*h) + (7*(b*g - a*h)*p*q*r^2*(g + h*x)^3)/(72*b*h) + (7*(d*g - c*h)*p*q*
r^2*(g + h*x)^3)/(72*d*h) + (p*q*r^2*(g + h*x)^4)/(16*h) + ((b*g - a*h)^4*p
*q*r^2*Log[a + b*x])/(8*b^4*h) + ((b*g - a*h)^3*(d*g - c*h)*p*q*r^2*Log[a +
 b*x])/(6*b^3*d*h) + ((b*g - a*h)^2*(d*g - c*h)^2*p*q*r^2*Log[a + b*x])/(4*
b^2*d^2*h) - (2*(b*g - a*h)^3*p^2*r^2*(a + b*x)*Log[a + b*x])/b^4 - ((d*g -
 c*h)^3*p*q*r^2*(a + b*x)*Log[a + b*x])/(2*b*d^3) - (3*h*(b*g - a*h)^2*p^2*
r^2*(a + b*x)^2*Log[a + b*x])/(2*b^4) - (2*h^2*(b*g - a*h)*p^2*r^2*(a + b*x
)^3*Log[a + b*x])/(3*b^4) - (h^3*p^2*r^2*(a + b*x)^4*Log[a + b*x])/(8*b^4)
- ((d*g - c*h)^2*p*q*r^2*(g + h*x)^2*Log[a + b*x])/(4*d^2*h) - ((d*g - c*h)
*p*q*r^2*(g + h*x)^3*Log[a + b*x])/(6*d*h) - (p*q*r^2*(g + h*x)^4*Log[a + b
*x])/(8*h) - ((b*g - a*h)^4*p^2*r^2*Log[a + b*x]^2)/(4*b^4*h) + ((b*g - a*h
)^2*(d*g - c*h)^2*p*q*r^2*Log[c + d*x])/(4*b^2*d^2*h) + ((b*g - a*h)*(d*g -
 c*h)^3*p*q*r^2*Log[c + d*x])/(6*b*d^3*h) + ((d*g - c*h)^4*p*q*r^2*Log[c +
d*x])/(8*d^4*h) - ((b*g - a*h)^3*p*q*r^2*(c + d*x)*Log[c + d*x])/(2*b^3*d)
- (2*(d*g - c*h)^3*q^2*r^2*(c + d*x)*Log[c + d*x])/d^4 - (3*h*(d*g - c*h)^2
*q^2*r^2*(c + d*x)^2*Log[c + d*x])/(2*d^4) - (2*h^2*(d*g - c*h)*q^2*r^2*(c
+ d*x)^3*Log[c + d*x])/(3*d^4) - (h^3*q^2*r^2*(c + d*x)^4*Log[c + d*x])/(8*
d^4) - ((b*g - a*h)^2*p*q*r^2*(g + h*x)^2*Log[c + d*x])/(4*b^2*h) - ((b*g -
 a*h)*p*q*r^2*(g + h*x)^3*Log[c + d*x])/(6*b*h) - (p*q*r^2*(g + h*x)^4*Log[
c + d*x])/(8*h) - ((b*g - a*h)^4*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*
Log[c + d*x])/(2*b^4*h) - ((d*g - c*h)^4*q^2*r^2*Log[c + d*x]^2)/(4*d^4*h)
- ((d*g - c*h)^4*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(2*d^
4*h) + ((b*g - a*h)^3*p*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]))/(2*b^3) + ((d*g - c*h)^3*q*r*x*(p*r*Log[a +
b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*d^3) +
((b*g - a*h)^2*p*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(4*b^2*h) + ((d*g - c*h)^2*q*r*(g + h*x)^
2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^
r]))/(4*d^2*h) + ((b*g - a*h)*p*r*(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c
 + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(6*b*h) + ((d*g - c*h)*q*r
*(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r]))/(6*d*h) + (p*r*(g + h*x)^4*(p*r*Log[a + b*x] + q*r*Log[c +
d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(8*h) + (q*r*(g + h*x)^4*(p*r
*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(
8*h) + ((b*g - a*h)^4*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x]
 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*b^4*h) + ((d*g - c*h)^4*q*r*Lo
g[c + d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]))/(2*d^4*h) + ((g + h*x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^2)/(4*h) - ((d*g - c*h)^4*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))]
)/(2*d^4*h) - ((b*g - a*h)^4*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
/(2*b^4*h)
```

Rubi [A] time = 2.43569, antiderivative size = 2220, normalized size of antiderivative = 0.99, number of steps used = 49, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {2498, 2513, 2411, 43, 2334, 12, 2301, 2418, 2389, 2295, 2394, 2393, 2391, 2395}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*(b*g - a*h)^3*p^2*r^2*x)/b^3 + (5*(b*g - a*h)^3*p*q*r^2*x)/(8*b^3) + (5*
(b*g - a*h)^2*(d*g - c*h)*p*q*r^2*x)/(12*b^2*d) + (5*(b*g - a*h)*(d*g - c*h
)^2*p*q*r^2*x)/(12*b*d^2) + (5*(d*g - c*h)^3*p*q*r^2*x)/(8*d^3) + (2*(d*g -
 c*h)^3*q^2*r^2*x)/d^3 + (3*h*(b*g - a*h)^2*p^2*r^2*(a + b*x)^2)/(4*b^4) +
(2*h^2*(b*g - a*h)*p^2*r^2*(a + b*x)^3)/(9*b^4) + (h^3*p^2*r^2*(a + b*x)^4)
/(32*b^4) + (3*h*(d*g - c*h)^2*q^2*r^2*(c + d*x)^2)/(4*d^4) + (2*h^2*(d*g -
 c*h)*q^2*r^2*(c + d*x)^3)/(9*d^4) + (h^3*q^2*r^2*(c + d*x)^4)/(32*d^4) + (
3*(b*g - a*h)^2*p*q*r^2*(g + h*x)^2)/(16*b^2*h) + ((b*g - a*h)*(d*g - c*h)*
p*q*r^2*(g + h*x)^2)/(6*b*d*h) + (3*(d*g - c*h)^2*p*q*r^2*(g + h*x)^2)/(16*
d^2*h) + (7*(b*g - a*h)*p*q*r^2*(g + h*x)^3)/(72*b*h) + (7*(d*g - c*h)*p*q*
r^2*(g + h*x)^3)/(72*d*h) + (p*q*r^2*(g + h*x)^4)/(16*h) + ((b*g - a*h)^4*p
*q*r^2*Log[a + b*x])/(8*b^4*h) + ((b*g - a*h)^3*(d*g - c*h)*p*q*r^2*Log[a +
 b*x])/(6*b^3*d*h) + ((b*g - a*h)^2*(d*g - c*h)^2*p*q*r^2*Log[a + b*x])/(4*
b^2*d^2*h) - ((d*g - c*h)^3*p*q*r^2*(a + b*x)*Log[a + b*x])/(2*b*d^3) - ((d
*g - c*h)^2*p*q*r^2*(g + h*x)^2*Log[a + b*x])/(4*d^2*h) - ((d*g - c*h)*p*q*
r^2*(g + h*x)^3*Log[a + b*x])/(6*d*h) - (p*q*r^2*(g + h*x)^4*Log[a + b*x])/
(8*h) + ((b*g - a*h)^4*p^2*r^2*Log[a + b*x]^2)/(4*b^4*h) - (p^2*r^2*Log[a +
 b*x]*((48*h*(b*g - a*h)^3*(a + b*x))/b^4 + (36*h^2*(b*g - a*h)^2*(a + b*x)
^2)/b^4 + (16*h^3*(b*g - a*h)*(a + b*x)^3)/b^4 + (3*h^4*(a + b*x)^4)/b^4 +
(12*(b*g - a*h)^4*Log[a + b*x])/b^4))/(24*h) + ((b*g - a*h)^2*(d*g - c*h)^2
*p*q*r^2*Log[c + d*x])/(4*b^2*d^2*h) + ((b*g - a*h)*(d*g - c*h)^3*p*q*r^2*L
og[c + d*x])/(6*b*d^3*h) + ((d*g - c*h)^4*p*q*r^2*Log[c + d*x])/(8*d^4*h) -
 ((b*g - a*h)^3*p*q*r^2*(c + d*x)*Log[c + d*x])/(2*b^3*d) - ((b*g - a*h)^2*
p*q*r^2*(g + h*x)^2*Log[c + d*x])/(4*b^2*h) - ((b*g - a*h)*p*q*r^2*(g + h*x
)^3*Log[c + d*x])/(6*b*h) - (p*q*r^2*(g + h*x)^4*Log[c + d*x])/(8*h) - ((b*
g - a*h)^4*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(2*b^4*h
) + ((d*g - c*h)^4*q^2*r^2*Log[c + d*x]^2)/(4*d^4*h) - (q^2*r^2*Log[c + d*x
]*((48*h*(d*g - c*h)^3*(c + d*x))/d^4 + (36*h^2*(d*g - c*h)^2*(c + d*x)^2)/
d^4 + (16*h^3*(d*g - c*h)*(c + d*x)^3)/d^4 + (3*h^4*(c + d*x)^4)/d^4 + (12*
(d*g - c*h)^4*Log[c + d*x])/d^4))/(24*h) - ((d*g - c*h)^4*p*q*r^2*Log[a + b
*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(2*d^4*h) + ((b*g - a*h)^3*p*r*x*(p*r*L
og[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*
b^3) + ((d*g - c*h)^3*q*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]))/(2*d^3) + ((b*g - a*h)^2*p*r*(g + h*x)^2*(p*
r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/
(4*b^2*h) + ((d*g - c*h)^2*q*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c +
d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(4*d^2*h) + ((b*g - a*h)*p*r*
(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r]))/(6*b*h) + ((d*g - c*h)*q*r*(g + h*x)^3*(p*r*Log[a + b*x] + q
*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(6*d*h) + (p*r*(g
+ h*x)^4*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]))/(8*h) + (q*r*(g + h*x)^4*(p*r*Log[a + b*x] + q*r*Log[c + d*x] -
 Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(8*h) + ((b*g - a*h)^4*p*r*Log[a +
b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]))/(2*b^4*h) + ((d*g - c*h)^4*q*r*Log[c + d*x]*(p*r*Log[a + b*x] + q*r
*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*d^4*h) + ((g + h*
x)^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(4*h) - ((d*g - c*h)^4*p*q*r^2
*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(2*d^4*h) - ((b*g - a*h)^4*p*q*r
```

```
^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(2*b^4*h)
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
```

```
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int (g+hx)^3 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx = \frac{(g+hx)^4 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h} - \frac{(bpr)\int \frac{(g+hx)^4 \log\left(e\left(f(a+bx)^p(c+\right.\right.}{a+bx}}{2h}$$

$$= \frac{(g+hx)^4 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h} - \frac{\left(bp^2r^2\right)\int \frac{(g+hx)^4\log(a+bx)}{a+bx}dx}{2h}$$

$$= \frac{(g+hx)^4 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4h} - \frac{\left(p^2r^2\right)\text{Subst}\left(\int \frac{\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)^4 \text{lo}}{x}\right.}{2h}$$

$$= -\frac{p^2r^2\log(a+bx)\left(\frac{48h(bg-ah)^3(a+bx)}{b^4}+\frac{36h^2(bg-ah)^2(a+bx)^2}{b^4}+\frac{16h^3(bg-ah)(a+bx)^3}{b^4}\right.}{24h}$$

$$= -\frac{(dg-ch)^2pqr^2(g+hx)^2\log(a+bx)}{4d^2h} - \frac{(dg-ch)pqr^2(g+hx)^3\log(a+b}{6dh}$$

$$= \frac{2(bg-ah)^3p^2r^2x}{b^3} + \frac{(bg-ah)^3pqr^2x}{2b^3} + \frac{(dg-ch)^3pqr^2x}{2d^3} + \frac{2(dg-ch)^3q^2}{d^3}$$

$$= \frac{2(bg-ah)^3p^2r^2x}{b^3} + \frac{5(bg-ah)^3pqr^2x}{8b^3} + \frac{5(bg-ah)^2(dg-ch)pqr^2x}{12b^2d} + 5($$

Mathematica [A] time = 3.01291, size = 1386, normalized size = 0.62

$$\frac{72a\left(-4b^3g^3+6ab^2hg^2-4a^2bh^2g+a^3h^3\right)p^2r^2\log^2(a+bx)d^4+12pr\log(a+bx)\left(12c\left(-4d^3g^3+6cd^2hg^2-4c^2dh^2g+c^3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (72*a*d^4*(-4*b^3*g^3 + 6*a*b^2*g^2*h - 4*a^2*b*g*h^2 + a^3*h^3)*p^2*r^2*Lo
g[a + b*x]^2 + 12*p*r*Log[a + b*x]*(12*b^4*c*(-4*d^3*g^3 + 6*c*d^2*g^2*h -
4*c^2*d*g*h^2 + c^3*h^3)*q*r*Log[c + d*x] - 12*(4*a*b^3*d^4*g^3 - 6*a^2*b^2
*d^4*g^2*h + 4*a^3*b*d^4*g*h^2 - a^4*d^4*h^3 + b^4*c*(-4*d^3*g^3 + 6*c*d^2*
g^2*h - 4*c^2*d*g*h^2 + c^3*h^3))*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*
((12*b^3*(-4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3)*q + a^3*d^3
*h^3*(25*p + 3*q) - 4*a^2*b*d^2*h^2*(22*d*g*p + 4*d*g*q - c*h*q) + 6*a*b^2*
d*h*(-4*c*d*g*h*q + c^2*h^2*q + 6*d^2*g^2*(3*p + q)))*r + 12*d^3*(4*b^3*g^3
 - 6*a*b^2*g^2*h + 4*a^2*b*g*h^2 - a^3*h^3)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r])) + b*(72*b^3*c*(-4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3
)*q^2*r^2*Log[c + d*x]^2 + 12*q*r*Log[c + d*x]*((12*a^3*c*d^3*h^3*p + 6*a^2
*b*c*d^2*h^2*(-8*d*g + c*h)*p + 4*a*b^2*d*(12*d^3*g^3 + 18*c*d^2*g^2*h - 6*
c^2*d*g*h^2 + c^3*h^3)*p + b^3*c*(-48*d^3*g^3*(p + q) + 36*c*d^2*g^2*h*(p +
 3*q) - 8*c^2*d*g*h^2*(2*p + 11*q) + c^3*h^3*(3*p + 25*q)))*r - 12*b^3*c*(-
4*d^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3)*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r]) + d*(r^2*(-60*a^3*d^3*h^3*p*(5*p + 3*q)*x + 6*a^2*b*d^2*h^2
*p*x*(-20*c*h*q + 16*d*g*(11*p + 8*q) + d*h*(13*p + 9*q)*x) + b^3*x*(-60*c^
3*h^3*q*(3*p + 5*q) + 6*c^2*d*h^2*q*(16*g*(8*p + 11*q) + h*(9*p + 13*q)*x)
- 4*c*d^2*h*q*(p + q)*(324*g^2 + 60*g*h*x + 7*h^2*x^2) + d^3*(p + q)^2*(576
*g^3 + 216*g^2*h*x + 64*g*h^2*x^2 + 9*h^3*x^3)) - 4*a*b^2*p*(36*c^3*h^3*q +
 6*c^2*d*h^2*q*(-24*g + 5*h*x) - 12*c*d^2*h*q*(-18*g^2 + 12*g*h*x + h^2*x^2
) + d^3*(-144*g^3*q + 324*g^2*h*(p + q)*x + 60*g*h^2*(p + q)*x^2 + 7*h^3*(p
 + q)*x^3))) + 12*r*(12*a^3*d^3*h^3*p*x - 6*a^2*b*d^3*h^2*p*x*(8*g + h*x) +
```

```
4*a*b^2*d^3*p*(-12*g^3 + 18*g^2*h*x + 6*g*h^2*x^2 + h^3*x^3) - b^3*x*(-12*
c^3*h^3*q + 6*c^2*d*h^2*q*(8*g + h*x) - 4*c*d^2*h*q*(18*g^2 + 6*g*h*x + h^2
*x^2) + d^3*(p + q)*(48*g^3 + 36*g^2*h*x + 16*g*h^2*x^2 + 3*h^3*x^3)))*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 72*b^3*d^3*x*(4*g^3 + 6*g^2*h*x + 4*g*h^
2*x^2 + h^3*x^3)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)) - 144*(4*a*b^3*d^
4*g^3 - 6*a^2*b^2*d^4*g^2*h + 4*a^3*b*d^4*g*h^2 - a^4*d^4*h^3 + b^4*c*(-4*d
^3*g^3 + 6*c*d^2*g^2*h - 4*c^2*d*g*h^2 + c^3*h^3))*p*q*r^2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)])/(288*b^4*d^4)
```

Maple [F] time = 0.392, size = 0, normalized size = 0.

$$\int (hx+g)^3 \left(\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((h*x+g)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Maxima [A] time = 1.45213, size = 2429, normalized size = 1.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
```

```
[Out] 1/4*(h^3*x^4 + 4*g*h^2*x^3 + 6*g^2*h*x^2 + 4*g^3*x)*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)^2 + 1/24*r*(12*(4*a*b^3*f*g^3*p - 6*a^2*b^2*f*g^2*h*p + 4*a^3
*b*f*g*h^2*p - a^4*f*h^3*p)*log(b*x + a)/b^4 + 12*(4*c*d^3*f*g^3*q - 6*c^2*
d^2*f*g^2*h*q + 4*c^3*d*f*g*h^2*q - c^4*f*h^3*q)*log(d*x + c)/d^4 - (3*b^3*
d^3*f*h^3*(p + q)*x^4 - 4*(a*b^2*d^3*f*h^3*p - (4*d^3*f*g*h^2*(p + q) - c*d
^2*f*h^3*q)*b^3)*x^3 - 6*(4*a*b^2*d^3*f*g*h^2*p - a^2*b*d^3*f*h^3*p - (6*d^
3*f*g^2*h*(p + q) - 4*c*d^2*f*g*h^2*q + c^2*d*f*h^3*q)*b^3)*x^2 - 12*(6*a*b
^2*d^3*f*g^2*h*p - 4*a^2*b*d^3*f*g*h^2*p + a^3*d^3*f*h^3*p - (4*d^3*f*g^3*(
p + q) - 6*c*d^2*f*g^2*h*q + 4*c^2*d*f*g*h^2*q - c^3*f*h^3*q)*b^3)*x)/(b^3*
d^3))*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f + 1/288*r^2*(12*(12*a^3*c*d^3*
f^2*h^3*p*q - 6*(8*c*d^3*f^2*g*h^2*p*q - c^2*d^2*f^2*h^3*p*q)*a^2*b + 4*(18
*c*d^3*f^2*g^2*h*p*q - 6*c^2*d^2*f^2*g*h^2*p*q + c^3*d*f^2*h^3*p*q)*a*b^2 -
 (48*(p*q + q^2)*c*d^3*f^2*g^3 - 36*(p*q + 3*q^2)*c^2*d^2*f^2*g^2*h + 8*(2*
p*q + 11*q^2)*c^3*d*f^2*g*h^2 - (3*p*q + 25*q^2)*c^4*f^2*h^3)*b^3)*log(d*x
+ c)/(b^3*d^4) - 144*(4*a*b^3*d^4*f^2*g^3*p*q - 6*a^2*b^2*d^4*f^2*g^2*h*p*q
 + 4*a^3*b*d^4*f^2*g*h^2*p*q - a^4*d^4*f^2*h^3*p*q - (4*c*d^3*f^2*g^3*p*q -
 6*c^2*d^2*f^2*g^2*h*p*q + 4*c^3*d*f^2*g*h^2*p*q - c^4*f^2*h^3*p*q)*b^4)*(l
og(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
- a*d)))/(b^4*d^4) + (9*(p^2 + 2*p*q + q^2)*b^4*d^4*f^2*h^3*x^4 - 144*(4*c*
d^3*f^2*g^3*p*q - 6*c^2*d^2*f^2*g^2*h*p*q + 4*c^3*d*f^2*g*h^2*p*q - c^4*f^2
*h^3*p*q)*b^4*log(b*x + a)*log(d*x + c) - 72*(4*c*d^3*f^2*g^3*q^2 - 6*c^2*d
^2*f^2*g^2*h*q^2 + 4*c^3*d*f^2*g*h^2*q^2 - c^4*f^2*h^3*q^2)*b^4*log(d*x + c
)^2 - 4*(7*(p^2 + p*q)*a*b^3*d^4*f^2*h^3 - (16*(p^2 + 2*p*q + q^2)*d^4*f^2*
g*h^2 - 7*(p*q + q^2)*c*d^3*f^2*h^3)*b^4)*x^3 + 6*((13*p^2 + 9*p*q)*a^2*b^2
*d^4*f^2*h^3 + 8*(c*d^3*f^2*h^3*p*q - 5*(p^2 + p*q)*d^4*f^2*g*h^2)*a*b^3 +
(36*(p^2 + 2*p*q + q^2)*d^4*f^2*g^2*h - 40*(p*q + q^2)*c*d^3*f^2*g*h^2 + (9
```

```
*p*q + 13*q^2)*c^2*d^2*f^2*h^3)*b^4)*x^2 - 72*(4*a*b^3*d^4*f^2*g^3*p^2 - 6*
a^2*b^2*d^4*f^2*g^2*h*p^2 + 4*a^3*b*d^4*f^2*g*h^2*p^2 - a^4*d^4*f^2*h^3*p^2
)*log(b*x + a)^2 - 12*(5*(5*p^2 + 3*p*q)*a^3*b*d^4*f^2*h^3 + 2*(5*c*d^3*f^2
*h^3*p*q - 4*(11*p^2 + 8*p*q)*d^4*f^2*g*h^2)*a^2*b^2 - 2*(24*c*d^3*f^2*g*h^
2*p*q - 5*c^2*d^2*f^2*h^3*p*q - 54*(p^2 + p*q)*d^4*f^2*g^2*h)*a*b^3 - (48*(
p^2 + 2*p*q + q^2)*d^4*f^2*g^3 - 108*(p*q + q^2)*c*d^3*f^2*g^2*h + 8*(8*p*q
 + 11*q^2)*c^2*d^2*f^2*g*h^2 - 5*(3*p*q + 5*q^2)*c^3*d*f^2*h^3)*b^4)*x + 12
*((25*p^2 + 3*p*q)*a^4*d^4*f^2*h^3 + 4*(c*d^3*f^2*h^3*p*q - 2*(11*p^2 + 2*p
*q)*d^4*f^2*g*h^2)*a^3*b - 6*(4*c*d^3*f^2*g*h^2*p*q - c^2*d^2*f^2*h^3*p*q -
 6*(3*p^2 + p*q)*d^4*f^2*g^2*h)*a^2*b^2 + 12*(6*c*d^3*f^2*g^2*h*p*q - 4*c^2
*d^2*f^2*g*h^2*p*q + c^3*d*f^2*h^3*p*q - 4*(p^2 + p*q)*d^4*f^2*g^3)*a*b^3)*
log(b*x + a))/(b^4*d^4))/f^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(h^3x^3+3\,gh^2x^2+3\,g^2hx+g^3\right)\log\left(\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
```

```
[Out] integral((h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3)*log(((b*x + a)^p*(d*x +
c)^q*f)^r*e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(hx+g\right)^3\log\left(\left((bx+a)^p(dx+c)^qf\right)^re\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate((h*x + g)^3*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.36 $\int (g+hx)^2 \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$

Optimal. Leaf size=1645

result too large to display

```
[Out] (2*(b*g - a*h)^2*p^2*r^2*x)/b^2 + (8*(b*g - a*h)^2*p*q*r^2*x)/(9*b^2) + (2*
(b*g - a*h)*(d*g - c*h)*p*q*r^2*x)/(3*b*d) + (8*(d*g - c*h)^2*p*q*r^2*x)/(9
*d^2) + (2*(d*g - c*h)^2*q^2*r^2*x)/d^2 + (h*(b*g - a*h)*p^2*r^2*(a + b*x)^
2)/(2*b^3) + (2*h^2*p^2*r^2*(a + b*x)^3)/(27*b^3) + (h*(d*g - c*h)*q^2*r^2*
(c + d*x)^2)/(2*d^3) + (2*h^2*q^2*r^2*(c + d*x)^3)/(27*d^3) + (5*(b*g - a*h
)*p*q*r^2*(g + h*x)^2)/(18*b*h) + (5*(d*g - c*h)*p*q*r^2*(g + h*x)^2)/(18*d
*h) + (4*p*q*r^2*(g + h*x)^3)/(27*h) + (2*(b*g - a*h)^3*p*q*r^2*Log[a + b*x
])/(9*b^3*h) + ((b*g - a*h)^2*(d*g - c*h)*p*q*r^2*Log[a + b*x])/(3*b^2*d*h)
 - (2*(b*g - a*h)^2*p^2*r^2*(a + b*x)*Log[a + b*x])/b^3 - (2*(d*g - c*h)^2*
p*q*r^2*(a + b*x)*Log[a + b*x])/(3*b*d^2) - (h*(b*g - a*h)*p^2*r^2*(a + b*x
)^2*Log[a + b*x])/b^3 - (2*h^2*p^2*r^2*(a + b*x)^3*Log[a + b*x])/(9*b^3) -
((d*g - c*h)*p*q*r^2*(g + h*x)^2*Log[a + b*x])/(3*d*h) - (2*p*q*r^2*(g + h*
x)^3*Log[a + b*x])/(9*h) - ((b*g - a*h)^3*p^2*r^2*Log[a + b*x]^2)/(3*b^3*h)
 + ((b*g - a*h)*(d*g - c*h)^2*p*q*r^2*Log[c + d*x])/(3*b*d^2*h) + (2*(d*g -
 c*h)^3*p*q*r^2*Log[c + d*x])/(9*d^3*h) - (2*(b*g - a*h)^2*p*q*r^2*(c + d*x
)*Log[c + d*x])/(3*b^2*d) - (2*(d*g - c*h)^2*q^2*r^2*(c + d*x)*Log[c + d*x]
)/d^3 - (h*(d*g - c*h)*q^2*r^2*(c + d*x)^2*Log[c + d*x])/d^3 - (2*h^2*q^2*r
^2*(c + d*x)^3*Log[c + d*x])/(9*d^3) - ((b*g - a*h)*p*q*r^2*(g + h*x)^2*Log
[c + d*x])/(3*b*h) - (2*p*q*r^2*(g + h*x)^3*Log[c + d*x])/(9*h) - (2*(b*g -
 a*h)^3*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b^3*h) -
 ((d*g - c*h)^3*q^2*r^2*Log[c + d*x]^2)/(3*d^3*h) - (2*(d*g - c*h)^3*p*q*r^
2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(3*d^3*h) + (2*(b*g - a*h)^2
*p*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x
)^q)^r]))/(3*b^2) + (2*(d*g - c*h)^2*q*r*x*(p*r*Log[a + b*x] + q*r*Log[c +
d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*d^2) + ((b*g - a*h)*p*r*(g
 + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]))/(3*b*h) + ((d*g - c*h)*q*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r
*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*d*h) + (2*p*r*(g
+ h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]))/(9*h) + (2*q*r*(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x]
 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(9*h) + (2*(b*g - a*h)^3*p*r*Log[
a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]))/(3*b^3*h) + (2*(d*g - c*h)^3*q*r*Log[c + d*x]*(p*r*Log[a + b*x]
 + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*d^3*h) + ((
g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(3*h) - (2*(d*g - c*h)^3
*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(3*d^3*h) - (2*(b*g - a*
h)^3*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*b^3*h)
```

Rubi [A] time = 1.75844, antiderivative size = 1657, normalized size of antiderivative = 1.01, number of steps used = 47, number of rules used = 15, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.484, Rules used = {2498, 2513, 2411, 43, 2334, 12, 14, 2301, 2418, 2389, 2295, 2394, 2393, 2391, 2395}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*(b*g - a*h)^2*p^2*r^2*x)/b^2 + (8*(b*g - a*h)^2*p*q*r^2*x)/(9*b^2) + (2*
(b*g - a*h)*(d*g - c*h)*p*q*r^2*x)/(3*b*d) + (8*(d*g - c*h)^2*p*q*r^2*x)/(9
*d^2) + (2*(d*g - c*h)^2*q^2*r^2*x)/d^2 + (h*(b*g - a*h)*p^2*r^2*(a + b*x)^
2)/(2*b^3) + (2*h^2*p^2*r^2*(a + b*x)^3)/(27*b^3) + (h*(d*g - c*h)*q^2*r^2*
(c + d*x)^2)/(2*d^3) + (2*h^2*q^2*r^2*(c + d*x)^3)/(27*d^3) + (5*(b*g - a*h
)*p*q*r^2*(g + h*x)^2)/(18*b*h) + (5*(d*g - c*h)*p*q*r^2*(g + h*x)^2)/(18*d
*h) + (4*p*q*r^2*(g + h*x)^3)/(27*h) + (2*(b*g - a*h)^3*p*q*r^2*Log[a + b*x
])/(9*b^3*h) + ((b*g - a*h)^2*(d*g - c*h)*p*q*r^2*Log[a + b*x])/(3*b^2*d*h)
 - (2*(d*g - c*h)^2*p*q*r^2*(a + b*x)*Log[a + b*x])/(3*b*d^2) - ((d*g - c*h
)*p*q*r^2*(g + h*x)^2*Log[a + b*x])/(3*d*h) - (2*p*q*r^2*(g + h*x)^3*Log[a
+ b*x])/(9*h) + ((b*g - a*h)^3*p^2*r^2*Log[a + b*x]^2)/(3*b^3*h) - (p^2*r^2
*Log[a + b*x]*((18*h*(b*g - a*h)^2*(a + b*x))/b^3 + (9*h^2*(b*g - a*h)*(a +
 b*x)^2)/b^3 + (2*h^3*(a + b*x)^3)/b^3 + (6*(b*g - a*h)^3*Log[a + b*x])/b^3
))/(9*h) + ((b*g - a*h)*(d*g - c*h)^2*p*q*r^2*Log[c + d*x])/(3*b*d^2*h) + (
2*(d*g - c*h)^3*p*q*r^2*Log[c + d*x])/(9*d^3*h) - (2*(b*g - a*h)^2*p*q*r^2*
(c + d*x)*Log[c + d*x])/(3*b^2*d) - ((b*g - a*h)*p*q*r^2*(g + h*x)^2*Log[c
+ d*x])/(3*b*h) - (2*p*q*r^2*(g + h*x)^3*Log[c + d*x])/(9*h) - (2*(b*g - a*
h)^3*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b^3*h) + ((
d*g - c*h)^3*q^2*r^2*Log[c + d*x]^2)/(3*d^3*h) - (q^2*r^2*Log[c + d*x]*((18
*h*(d*g - c*h)^2*(c + d*x))/d^3 + (9*h^2*(d*g - c*h)*(c + d*x)^2)/d^3 + (2*
h^3*(c + d*x)^3)/d^3 + (6*(d*g - c*h)^3*Log[c + d*x])/d^3))/(9*h) - (2*(d*g
 - c*h)^3*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(3*d^3*h) +
(2*(b*g - a*h)^2*p*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]))/(3*b^2) + (2*(d*g - c*h)^2*q*r*x*(p*r*Log[a + b*x
] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*d^2) + ((b
*g - a*h)*p*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r]))/(3*b*h) + ((d*g - c*h)*q*r*(g + h*x)^2*(p*r*Lo
g[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*d
*h) + (2*p*r*(g + h*x)^3*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]))/(9*h) + (2*q*r*(g + h*x)^3*(p*r*Log[a + b*x] +
q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(9*h) + (2*(b*g -
 a*h)^3*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]))/(3*b^3*h) + (2*(d*g - c*h)^3*q*r*Log[c + d*x]*(
p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])
)/(3*d^3*h) + ((g + h*x)^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(3*h) -
(2*(d*g - c*h)^3*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(3*d^3*h
) - (2*(b*g - a*h)^3*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*b^3*
h)
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int (g+hx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx=\frac{(g+hx)^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h}-\frac{(2bpr)\int\frac{(g+hx)^3\log\left(e\left(f(a+bx)^p(c\right.\right.}{a+bx}}{3h}$$

$$=\frac{(g+hx)^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h}-\frac{\left(2bp^2r^2\right)\int\frac{(g+hx)^3\log(a+bx)}{a+bx}dx}{3h}$$

$$=\frac{(g+hx)^3\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h}-\frac{\left(2p^2r^2\right)\text{Subst}\left(\int\frac{\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)^3}{x}\right.}{3h}$$

$$=-\frac{p^2r^2\log(a+bx)\left(\frac{18h(bg-ah)^2(a+bx)}{b^3}+\frac{9h^2(bg-ah)(a+bx)^2}{b^3}+\frac{2h^3(a+bx)^3}{b^3}+\frac{6(bg-a}{}\right.}{9h}$$

$$=-\frac{(dg-ch)pqr^2(g+hx)^2\log(a+bx)}{3dh}-\frac{2pqr^2(g+hx)^3\log(a+bx)}{9h}-\frac{p^2r}{}$$

$$=\frac{2(bg-ah)^2pqr^2x}{3b^2}+\frac{2(dg-ch)^2pqr^2x}{3d^2}-\frac{2(dg-ch)^2pqr^2(a+bx)\log(a+}{3bd^2}$$

$$=\frac{8(bg-ah)^2pqr^2x}{9b^2}+\frac{2(bg-ah)(dg-ch)pqr^2x}{3bd}+\frac{8(dg-ch)^2pqr^2x}{9d^2}+\frac{5(b}{}$$

$$=\frac{2(bg-ah)^2p^2r^2x}{b^2}+\frac{8(bg-ah)^2pqr^2x}{9b^2}+\frac{2(bg-ah)(dg-ch)pqr^2x}{3bd}+\frac{8(a}{}$$

Mathematica [A] time = 1.89012, size = 899, normalized size = 0.55

$$\frac{-18a\left(3b^2g^2-3abhg+a^2h^2\right)p^2r^2\log^2(a+bx)d^3-6pr\log(a+bx)\left(6c\left(3d^2g^2-3cdhg+c^2h^2\right)qr\log(c+dx)b^3-6(bc-a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (-18*a*d^3*(3*b^2*g^2 - 3*a*b*g*h + a^2*h^2)*p^2*r^2*Log[a + b*x]^2 - 6*p*r
*Log[a + b*x]*(6*b^3*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)*q*r*Log[c + d*x] -
 6*(b*c - a*d)*(a^2*d^2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2 - 3*c
*d*g*h + c^2*h^2))*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*((6*b^2*(3*d^2*
g^2 - 3*c*d*g*h + c^2*h^2)*q + a^2*d^2*h^2*(11*p + 2*q) - 3*a*b*d*h*(-(c*h*
q) + 3*d*g*(3*p + q)))*r - 6*d^2*(3*b^2*g^2 - 3*a*b*g*h + a^2*h^2)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])) + b*(-18*b^2*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*
h^2)*q^2*r^2*Log[c + d*x]^2 - 6*q*r*Log[c + d*x]*((6*a^2*c*d^2*h^2*p - 3*a*
b*d*(6*d^2*g^2 + 6*c*d*g*h - c^2*h^2)*p + b^2*c*(18*d^2*g^2*(p + q) - 9*c*d
*g*h*(p + 3*q) + c^2*h^2*(2*p + 11*q)))*r - 6*b^2*c*(3*d^2*g^2 - 3*c*d*g*h
+ c^2*h^2)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + d*(r^2*(6*a^2*d^2*h^2*p*
(11*p + 8*q)*x + b^2*x*(6*c^2*h^2*q*(8*p + 11*q) - 3*c*d*h*q*(p + q)*(54*g
+ 5*h*x) + d^2*(p + q)^2*(108*g^2 + 27*g*h*x + 4*h^2*x^2)) - 3*a*b*p*(-12*c
^2*h^2*q - 12*c*d*h*q*(-3*g + h*x) + d^2*(-36*g^2*q + 54*g*h*(p + q)*x + 5*
h^2*(p + q)*x^2))) - 6*r*(6*a^2*d^2*h^2*p*x + 3*a*b*d^2*p*(6*g^2 - 6*g*h*x
- h^2*x^2) + b^2*x*(6*c^2*h^2*q - 3*c*d*h*q*(6*g + h*x) + d^2*(p + q)*(18*g
^2 + 9*g*h*x + 2*h^2*x^2)))*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 18*b^2*d
^2*x*(3*g^2 + 3*g*h*x + h^2*x^2)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)) +
 36*(b*c - a*d)*(a^2*d^2*h^2 + a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2 - 3*
c*d*g*h + c^2*h^2))*p*q*r^2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(54*b
^3*d^3)
```

Maple [F] time = 0.385, size = 0, normalized size = 0.

$$\int (hx+g)^2 \left(\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((h*x+g)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Maxima [A] time = 1.37896, size = 1516, normalized size = 0.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(h^2*x^3 + 3*g*h*x^2 + 3*g^2*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2
+ 1/9*r*(6*(3*a*b^2*f*g^2*p - 3*a^2*b*f*g*h*p + a^3*f*h^2*p)*log(b*x + a)/b
^3 + 6*(3*c*d^2*f*g^2*q - 3*c^2*d*f*g*h*q + c^3*f*h^2*q)*log(d*x + c)/d^3 -
 (2*b^2*d^2*f*h^2*(p + q)*x^3 - 3*(a*b*d^2*f*h^2*p - (3*d^2*f*g*h*(p + q) -
 c*d*f*h^2*q)*b^2)*x^2 - 6*(3*a*b*d^2*f*g*h*p - a^2*d^2*f*h^2*p - (3*d^2*f*
g^2*(p + q) - 3*c*d*f*g*h*q + c^2*f*h^2*q)*b^2)*x)/(b^2*d^2))*log(((b*x + a
)^p*(d*x + c)^q*f)^r*e)/f - 1/54*r^2*(6*(6*a^2*c*d^2*f^2*h^2*p*q - 3*(6*c*d
^2*f^2*g*h*p*q - c^2*d*f^2*h^2*p*q)*a*b + (18*(p*q + q^2)*c*d^2*f^2*g^2 - 9
```

```
*(p*q + 3*q^2)*c^2*d*f^2*g*h + (2*p*q + 11*q^2)*c^3*f^2*h^2)*b^2)*log(d*x +
 c)/(b^2*d^3) + 36*(3*a*b^2*d^3*f^2*g^2*p*q - 3*a^2*b*d^3*f^2*g*h*p*q + a^3
*d^3*f^2*h^2*p*q - (3*c*d^2*f^2*g^2*p*q - 3*c^2*d*f^2*g*h*p*q + c^3*f^2*h^2
*p*q)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x
 + a*d)/(b*c - a*d)))/(b^3*d^3) - (4*(p^2 + 2*p*q + q^2)*b^3*d^3*f^2*h^2*x^
3 - 36*(3*c*d^2*f^2*g^2*p*q - 3*c^2*d*f^2*g*h*p*q + c^3*f^2*h^2*p*q)*b^3*lo
g(b*x + a)*log(d*x + c) - 18*(3*c*d^2*f^2*g^2*q^2 - 3*c^2*d*f^2*g*h*q^2 + c
^3*f^2*h^2*q^2)*b^3*log(d*x + c)^2 - 3*(5*(p^2 + p*q)*a*b^2*d^3*f^2*h^2 - (
9*(p^2 + 2*p*q + q^2)*d^3*f^2*g*h - 5*(p*q + q^2)*c*d^2*f^2*h^2)*b^3)*x^2 -
 18*(3*a*b^2*d^3*f^2*g^2*p^2 - 3*a^2*b*d^3*f^2*g*h*p^2 + a^3*d^3*f^2*h^2*p^
2)*log(b*x + a)^2 + 6*((11*p^2 + 8*p*q)*a^2*b*d^3*f^2*h^2 + 3*(2*c*d^2*f^2*
h^2*p*q - 9*(p^2 + p*q)*d^3*f^2*g*h)*a*b^2 + (18*(p^2 + 2*p*q + q^2)*d^3*f^
2*g^2 - 27*(p*q + q^2)*c*d^2*f^2*g*h + (8*p*q + 11*q^2)*c^2*d*f^2*h^2)*b^3)
*x - 6*((11*p^2 + 2*p*q)*a^3*d^3*f^2*h^2 + 3*(c*d^2*f^2*h^2*p*q - 3*(3*p^2
+ p*q)*d^3*f^2*g*h)*a^2*b - 6*(3*c*d^2*f^2*g*h*p*q - c^2*d*f^2*h^2*p*q - 3*
(p^2 + p*q)*d^3*f^2*g^2)*a*b^2)*log(b*x + a))/(b^3*d^3))/f^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(h^2x^2 + 2\,ghx + g^2\right)\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2, x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas
")
```

```
[Out] integral((h^2*x^2 + 2*g*h*x + g^2)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (hx+g)^2 \log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate((h*x + g)^2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.37 $\int (g+hx) \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) dx$

Optimal. Leaf size=1063

result too large to display

```
[Out] (3*(b*g - a*h)*p*q*r^2*x)/(2*b) + (3*(d*g - c*h)*p*q*r^2*x)/(2*d) + (p*q*r^
2*(g + h*x)^2)/(2*h) + (p^2*r^2*(4*b*g - 3*a*h + b*h*x)^2)/(4*b^2*h) + (q^2
*r^2*(4*d*g - 3*c*h + d*h*x)^2)/(4*d^2*h) + ((b*g - a*h)^2*p*q*r^2*Log[a +
b*x])/(2*b^2*h) - (2*(b*g - a*h)*p^2*r^2*(a + b*x)*Log[a + b*x])/b^2 - ((d*
g - c*h)*p*q*r^2*(a + b*x)*Log[a + b*x])/(b*d) - (h*p^2*r^2*(a + b*x)^2*Log
[a + b*x])/(2*b^2) - (p*q*r^2*(g + h*x)^2*Log[a + b*x])/(2*h) - ((b*g - a*h
)^2*p^2*r^2*Log[a + b*x]^2)/(2*b^2*h) + ((d*g - c*h)^2*p*q*r^2*Log[c + d*x]
)/(2*d^2*h) - ((b*g - a*h)*p*q*r^2*(c + d*x)*Log[c + d*x])/(b*d) - (2*(d*g
- c*h)*q^2*r^2*(c + d*x)*Log[c + d*x])/d^2 - (h*q^2*r^2*(c + d*x)^2*Log[c +
 d*x])/(2*d^2) - (p*q*r^2*(g + h*x)^2*Log[c + d*x])/(2*h) - ((b*g - a*h)^2*
p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b^2*h) - ((d*g - c
*h)^2*q^2*r^2*Log[c + d*x]^2)/(2*d^2*h) - ((d*g - c*h)^2*p*q*r^2*Log[a + b*
x]*Log[(b*(c + d*x))/(b*c - a*d)])/(d^2*h) + ((b*g - a*h)*p*r*x*(p*r*Log[a
+ b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/b + ((d*
g - c*h)*q*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r]))/d + (p*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x]
 - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*h) + (q*r*(g + h*x)^2*(p*r*Log
[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*h)
 + ((b*g - a*h)^2*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(b^2*h) + ((d*g - c*h)^2*q*r*Log[c +
d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]))/(d^2*h) + ((g + h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(2*h
) - ((d*g - c*h)^2*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(d^2*h
) - ((b*g - a*h)^2*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b^2*h)
```

Rubi [A] time = 1.16285, antiderivative size = 1097, normalized size of antiderivative = 1.03, number of steps used = 39, number of rules used = 15, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.517, Rules used = {2498, 2513, 2411, 43, 2334, 12, 14, 2301, 2418, 2389, 2295, 2394, 2393, 2391, 2395}

$$\frac{p^2r^2\log^2(a+bx)(bg-ah)^2}{2b^2h}+\frac{pqr^2\log(a+bx)(bg-ah)^2}{2b^2h}-\frac{pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)(bg-ah)^2}{b^2h}+\frac{pr\log(a+bx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (3*(b*g - a*h)*p*q*r^2*x)/(2*b) + (3*(d*g - c*h)*p*q*r^2*x)/(2*d) + (p*q*r^
2*(g + h*x)^2)/(2*h) + (p^2*r^2*(4*b*g - 3*a*h + b*h*x)^2)/(4*b^2*h) + (q^2
*r^2*(4*d*g - 3*c*h + d*h*x)^2)/(4*d^2*h) + ((b*g - a*h)^2*p*q*r^2*Log[a +
b*x])/(2*b^2*h) - ((d*g - c*h)*p*q*r^2*(a + b*x)*Log[a + b*x])/(b*d) - (p*q
*r^2*(g + h*x)^2*Log[a + b*x])/(2*h) + ((b*g - a*h)^2*p^2*r^2*Log[a + b*x]^
2)/(2*b^2*h) - (p^2*r^2*Log[a + b*x]*((4*h*(b*g - a*h)*(a + b*x))/b^2 + (h^
2*(a + b*x)^2)/b^2 + (2*(b*g - a*h)^2*Log[a + b*x])/b^2))/(2*h) + ((d*g - c
*h)^2*p*q*r^2*Log[c + d*x])/(2*d^2*h) - ((b*g - a*h)*p*q*r^2*(c + d*x)*Log[
c + d*x])/(b*d) - (p*q*r^2*(g + h*x)^2*Log[c + d*x])/(2*h) - ((b*g - a*h)^2
*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b^2*h) + ((d*g -
c*h)^2*q^2*r^2*Log[c + d*x]^2)/(2*d^2*h) - (q^2*r^2*Log[c + d*x]*((4*h*(d*g
 - c*h)*(c + d*x))/d^2 + (h^2*(c + d*x)^2)/d^2 + (2*(d*g - c*h)^2*Log[c + d
```

```
*x])/d^2))/(2*h) - ((d*g - c*h)^2*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b
*c - a*d)])/(d^2*h) + ((b*g - a*h)*p*r*x*(p*r*Log[a + b*x] + q*r*Log[c + d*
x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/b + ((d*g - c*h)*q*r*x*(p*r*Log
[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/d + (
p*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]))/(2*h) + (q*r*(g + h*x)^2*(p*r*Log[a + b*x] + q*r*Log[c +
 d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(2*h) + ((b*g - a*h)^2*p*r*L
og[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c
+ d*x)^q)^r]))/(b^2*h) + ((d*g - c*h)^2*q*r*Log[c + d*x]*(p*r*Log[a + b*x]
+ q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(d^2*h) + ((g +
 h*x)^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2)/(2*h) - ((d*g - c*h)^2*p*q*
r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(d^2*h) - ((b*g - a*h)^2*p*q*
r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b^2*h)
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int (g+hx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx = \frac{(g+hx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h} - \frac{(bpr)\int \frac{(g+hx)^2\log\left(e\left(f(a+bx)^p(c+d\right.\right.}{a+bx}}{h}$$

$$= \frac{(g+hx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h} - \frac{\left(bp^2r^2\right)\int \frac{(g+hx)^2\log(a+bx)}{a+bx}dx}{h} -$$

$$= \frac{(g+hx)^2\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h} - \frac{\left(p^2r^2\right)\text{Subst}\left(\int \frac{\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)^2\log}{x}\right.}{h}$$

$$= -\frac{p^2r^2\log(a+bx)\left(\frac{4h(bg-ah)(a+bx)}{b^2}+\frac{h^2(a+bx)^2}{b^2}+\frac{2(bg-ah)^2\log(a+bx)}{b^2}\right)}{2h} - \frac{q^2r^2\log}{}$$

$$= -\frac{pqr^2(g+hx)^2\log(a+bx)}{2h} - \frac{p^2r^2\log(a+bx)\left(\frac{4h(bg-ah)(a+bx)}{b^2}+\frac{h^2(a+bx)^2}{b^2}\right.}{2h}$$

$$= \frac{(bg-ah)pqr^2x}{b} + \frac{(dg-ch)pqr^2x}{d} - \frac{(dg-ch)pqr^2(a+bx)\log(a+bx)}{bd} -$$

$$= \frac{3(bg-ah)pqr^2x}{2b} + \frac{3(dg-ch)pqr^2x}{2d} + \frac{pqr^2(g+hx)^2}{2h} + \frac{p^2r^2(4bg-3ah+}{4b^2h}$$

$$= \frac{3(bg-ah)pqr^2x}{2b} + \frac{3(dg-ch)pqr^2x}{2d} + \frac{pqr^2(g+hx)^2}{2h} + \frac{p^2r^2(4bg-3ah+}{4b^2h}$$

Mathematica [A] time = 1.12735, size = 480, normalized size = 0.45

$$\frac{-4pqr^2(bc-ad)(adh+bch-2bdg)\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)+2pr\log(a+bx)\left(ad\left((4bdg-2adh)\log\left(e\left(f(a+bx)^p(c+dx)^q\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (2*a*d^2*(-2*b*g + a*h)*p^2*r^2*Log[a + b*x]^2 + 2*p*r*Log[a + b*x]*(2*b^2*
c*(-2*d*g + c*h)*q*r*Log[c + d*x] - 2*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h
)*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(2*b*(-2*d*g + c*h)*q*r + a*d*h*
(3*p + q)*r + (4*b*d*g - 2*a*d*h)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])) +
b*(2*b*c*(-2*d*g + c*h)*q^2*r^2*Log[c + d*x]^2 + 2*q*r*Log[c + d*x]*(2*a*d*
(2*d*g + c*h)*p*r + b*c*(-4*d*g*(p + q) + c*h*(p + 3*q))*r - 2*b*c*(-2*d*g
+ c*h)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]) + d*(r^2*(-2*a*p*(-4*d*g*q + 2
*c*h*q + 3*d*h*(p + q)*x) + b*(p + q)*x*(-6*c*h*q + d*(p + q)*(8*g + h*x)))
 - 2*r*(2*a*d*p*(2*g - h*x) + b*x*(-2*c*h*q + d*(p + q)*(4*g + h*x)))*Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*d*x*(2*g + h*x)*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r]^2)) - 4*(b*c - a*d)*(-2*b*d*g + b*c*h + a*d*h)*p*q*r^2*Poly
Log[2, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b^2*d^2)
```

Maple [F] time = 0.176, size = 0, normalized size = 0.

$$\int (hx+g)\left(\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int((h*x+g)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Maxima [A] time = 1.37604, size = 841, normalized size = 0.79

$$\frac{1}{2}\left(hx^2+2gx\right)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2+\frac{r\left(\frac{2\left(2abfgp-a^2fhp\right)\log(bx+a)}{b^2}+\frac{2\left(2cdfgq-c^2fhq\right)\log(dx+c)}{d^2}-\frac{bdfh(p+q)x^2-2\left(adfh\cdots\right.}{\cdots}\right.}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(h*x^2 + 2*g*x)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2 + 1/2*r*(2*(2*a*
b*f*g*p - a^2*f*h*p)*log(b*x + a)/b^2 + 2*(2*c*d*f*g*q - c^2*f*h*q)*log(d*x
 + c)/d^2 - (b*d*f*h*(p + q)*x^2 - 2*(a*d*f*h*p - (2*d*f*g*(p + q) - c*f*h*
q)*b)*x)/(b*d))*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f + 1/4*r^2*(2*(2*a*c*
d*f^2*h*p*q - (4*(p*q + q^2)*c*d*f^2*g - (p*q + 3*q^2)*c^2*f^2*h)*b)*log(d*
x + c)/(b*d^2) - 4*(2*a*b*d^2*f^2*g*p*q - a^2*d^2*f^2*h*p*q - (2*c*d*f^2*g*
p*q - c^2*f^2*h*p*q)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1)
+ dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b^2*d^2) + ((p^2 + 2*p*q + q^2)*b^2*d
^2*f^2*h*x^2 - 4*(2*c*d*f^2*g*p*q - c^2*f^2*h*p*q)*b^2*log(b*x + a)*log(d*x
 + c) - 2*(2*c*d*f^2*g*q^2 - c^2*f^2*h*q^2)*b^2*log(d*x + c)^2 - 2*(2*a*b*d
^2*f^2*g*p^2 - a^2*d^2*f^2*h*p^2)*log(b*x + a)^2 - 2*(3*(p^2 + p*q)*a*b*d^2
*f^2*h - (4*(p^2 + 2*p*q + q^2)*d^2*f^2*g - 3*(p*q + q^2)*c*d*f^2*h)*b^2)*x
 + 2*((3*p^2 + p*q)*a^2*d^2*f^2*h + 2*(c*d*f^2*h*p*q - 2*(p^2 + p*q)*d^2*f^
2*g)*a*b)*log(b*x + a))/(b^2*d^2))/f^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(hx+g\right)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")
```

```
[Out] integral((h*x + g)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (hx+g)\log \left(\left((bx+a)^p (dx+c)^q f \right)^r e \right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate((h*x + g)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.38 $\int \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\,dx$

Optimal. Leaf size=269

$$-\frac{2pqr^2(bc-ad)\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd}+\frac{(a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}-\frac{2r(p+q)(a+bx)\log\left(e\left(f(a+bx)^p\right.\right.}{b}$$

```
[Out] 2*(p + q)^2*r^2*x - (2*(b*c - a*d)*q*(p + q)*r^2*Log[c + d*x])/(b*d) - (2*(
b*c - a*d)*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*d) -
((b*c - a*d)*q^2*r^2*Log[c + d*x]^2)/(b*d) - (2*(p + q)*r*(a + b*x)*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/b + (2*(b*c - a*d)*q*r*Log[c + d*x]*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/(b*d) + ((a + b*x)*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^2)/b - (2*(b*c - a*d)*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*
d)])/(b*d)
```

Rubi [A] time = 0.153009, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2487, 2494, 2394, 2393, 2391, 2390, 2301, 31, 8}

$$-\frac{2pqr^2(bc-ad)\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{bd}+\frac{(a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}-\frac{2r(p+q)(a+bx)\log\left(e\left(f(a+bx)^p\right.\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```


```
[Out] 2*(p + q)^2*r^2*x - (2*(b*c - a*d)*q*(p + q)*r^2*Log[c + d*x])/(b*d) - (2*(
b*c - a*d)*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*d) -
((b*c - a*d)*q^2*r^2*Log[c + d*x]^2)/(b*d) - (2*(p + q)*r*(a + b*x)*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])/b + (2*(b*c - a*d)*q*r*Log[c + d*x]*Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r])/(b*d) + ((a + b*x)*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^2)/b - (2*(b*c - a*d)*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*
d)])/(b*d)
```


Rule 2487

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + (Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]^(s - 1)/(c + d*x), x], x] - Dist[r*s*(p + q), Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, p, q,
r, s}, x] && NeQ[b*c - a*d, 0] && NeQ[p + q, 0] && IGtQ[s, 0] && LtQ[s, 4]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)dx=\frac{(a+bx)\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}+\frac{(2(bc-ad)qr)\int\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{c+dx}\,d}{b}$$

$$=-\frac{2(p+q)r(a+bx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{b}+\frac{2(bc-ad)qr\log(c+dx)\log\left(\right.}{bd}$$

$$=2(p+q)^2r^2x-\frac{2(bc-ad)q(p+q)r^2\log(c+dx)}{bd}-\frac{2(bc-ad)pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\mathrm{lo}}{bd}$$

$$=2(p+q)^2r^2x-\frac{2(bc-ad)q(p+q)r^2\log(c+dx)}{bd}-\frac{2(bc-ad)pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\mathrm{lo}}{bd}$$

$$=2(p+q)^2r^2x-\frac{2(bc-ad)q(p+q)r^2\log(c+dx)}{bd}-\frac{2(bc-ad)pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\mathrm{lo}}{bd}$$

Mathematica [A] time = 0.215741, size = 437, normalized size = 1.62

$$\frac{2pqr^2(bc-ad)\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)+bdx\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)-2adpr\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)-2bdprx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2,x]
```

```
[Out] (p^2*r^2*(2*b*x - 2*(a + b*x)*Log[a + b*x] + (a + b*x)*Log[a + b*x]^2))/b +
 (2*a*d*p*q*r^2 + 4*b*d*p*q*r^2*x + 2*b*d*q^2*r^2*x - d*p^2*r^2*(2*a + b*x)
*Log[a + b*x]^2 - 2*b*c*p*q*r^2*Log[c + d*x] + 2*a*d*p*q*r^2*Log[c + d*x] -
 2*b*c*q^2*r^2*Log[c + d*x] - b*c*q^2*r^2*Log[c + d*x]^2 - 2*a*d*p*r*Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b*d*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)
^q)^r] - 2*b*d*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*c*q*r*Log[c
 + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + b*d*x*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r]^2 + 2*p*r*Log[a + b*x]*(-(b*c*q*r*Log[c + d*x]) + (b*c - a*d
)*q*r*Log[(b*(c + d*x))/(b*c - a*d)] + d*(a*(p - q)*r + b*p*r*x + a*Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])) + 2*(b*c - a*d)*p*q*r^2*PolyLog[2, (d*(a +
b*x))/(-(b*c) + a*d)])/(b*d)
```

Maple [F] time = 0.131, size = 0, normalized size = 0.

$$\int \left(\ln \left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x)
```

Maxima [A] time = 1.36595, size = 402, normalized size = 1.49

$$x\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)^{2}-\frac{2\left(f(p+q)x-\frac{afp\log(bx+a)}{b}-\frac{cfq\log(dx+c)}{d}\right)r\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)}{f}-\frac{\left(\frac{2\left(pq+q^{2}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="maxima")
```

```
[Out] x*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2 - 2*(f*(p + q)*x - a*f*p*log(b*x +
 a)/b - c*f*q*log(d*x + c)/d)*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/f - (2
*(p*q + q^2)*c*f^2*log(d*x + c)/d - 2*(b*c*f^2*p*q - a*d*f^2*p*q)*(log(b*x
+ a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d))
)/(b*d) + (a*d*f^2*p^2*log(b*x + a)^2 + 2*b*c*f^2*p*q*log(b*x + a)*log(d*x
+ c) + b*c*f^2*q^2*log(d*x + c)^2 - 2*(p^2 + 2*p*q + q^2)*b*d*f^2*x + 2*(p^
2 + p*q)*a*d*f^2*log(b*x + a))/(b*d))*r^2/f^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2, x)
```

3.39 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}\,dx$

Optimal. Leaf size=1471

result too large to display

```
[Out] (p*q*r^2*Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[((b*g - a*h)*(c + d*x))/((d*
g - c*h)*(a + b*x))]^2)/h + (p^2*r^2*Log[a + b*x]^2*Log[g + h*x])/h + (2*p*
q*r^2*Log[a + b*x]*Log[c + d*x]*Log[g + h*x])/h + (q^2*r^2*Log[c + d*x]^2*L
og[g + h*x])/h - (2*p*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*L
og[g + h*x])/h - (2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*L
og[g + h*x])/h + (Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2*Log[g + h*x])/h -
(p^2*r^2*Log[a + b*x]^2*Log[(b*(g + h*x))/(b*g - a*h)])/h - (2*p*q*r^2*Log[
a + b*x]*Log[-((h*(c + d*x))/(d*g - c*h))]*Log[(b*(g + h*x))/(b*g - a*h)])/
h + (p*q*r^2*Log[-((h*(c + d*x))/(d*g - c*h))]^2*Log[(b*(g + h*x))/(b*g - a
*h)])/h - (2*p*q*r^2*Log[-((h*(c + d*x))/(d*g - c*h))]*Log[((b*g - a*h)*(c
+ d*x))/((d*g - c*h)*(a + b*x))]*Log[(b*(g + h*x))/(b*g - a*h)])/h + (p*q*r
^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]^2*Log[(b*(g + h*x))
/(b*g - a*h)])/h + (2*p*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
*Log[(b*(g + h*x))/(b*g - a*h)])/h - (2*p*q*r^2*Log[a + b*x]*Log[c + d*x]*L
og[(d*(g + h*x))/(d*g - c*h)])/h - (q^2*r^2*Log[c + d*x]^2*Log[(d*(g + h*x)
)/(d*g - c*h)])/h + (2*p*q*r^2*Log[a + b*x]*Log[-((h*(c + d*x))/(d*g - c*h)
)]*Log[(d*(g + h*x))/(d*g - c*h)])/h - (p*q*r^2*Log[-((h*(c + d*x))/(d*g -
c*h))]^2*Log[(d*(g + h*x))/(d*g - c*h)])/h + (2*p*q*r^2*Log[-((h*(c + d*x))
/(d*g - c*h))]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Log[(d*
(g + h*x))/(d*g - c*h)])/h + (2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)])/h - (p*q*r^2*Log[((b*g - a*h)*(c
 + d*x))/((d*g - c*h)*(a + b*x))]^2*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c
*h)*(a + b*x)))])/h - (2*p*r*(q*r*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, -((h*(a + b*
x))/(b*g - a*h))])/h + (2*q*r*(p*r*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)
*(a + b*x))] + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, -((h*(c + d
*x))/(d*g - c*h))])/h + (2*p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)
*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/h - (2*p*q*r^2*Log[((
b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*PolyLog[2, ((b*g - a*h)*(c +
 d*x))/((d*g - c*h)*(a + b*x))])/h - (2*p^2*r^2*PolyLog[3, -((h*(a + b*x))/
(b*g - a*h))])/h - (2*p*q*r^2*PolyLog[3, -((h*(a + b*x))/(b*g - a*h))])/h -
 (2*p*q*r^2*PolyLog[3, -((h*(c + d*x))/(d*g - c*h))])/h - (2*q^2*r^2*PolyLo
g[3, -((h*(c + d*x))/(d*g - c*h))])/h - (2*p*q*r^2*PolyLog[3, (b*(c + d*x))
/(d*(a + b*x))])/h + (2*p*q*r^2*PolyLog[3, ((b*g - a*h)*(c + d*x))/((d*g -
c*h)*(a + b*x))])/h
```

Rubi [A] time = 1.93148, antiderivative size = 2096, normalized size of antiderivative = 1.42, number of steps used = 29, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {2497, 2500, 2394, 2393, 2391, 2433, 2375, 2317, 2374, 6589, 2440, 2437, 2435, 2315}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x),x]
```

```
[Out] -((Log[(a + b*x)^(p*r)]^2*Log[g + h*x])/h) - (2*p*q*r^2*Log[-((d*(a + b*x))
/(b*c - a*d))]*Log[c + d*x]*Log[g + h*x])/h - (2*p*q*r^2*Log[a + b*x]*Log[(
```

```
b*(c + d*x))/(b*c - a*d)]*Log[g + h*x]])/h + (2*q*r*(p*r*Log[a + b*x] - Log[
(a + b*x)^(p*r)])*Log[-((h*(c + d*x))/(d*g - c*h))]*Log[g + h*x]])/h + (2*p*
r*Log[-((h*(a + b*x))/(b*g - a*h))]*(q*r*Log[c + d*x] - Log[(c + d*x)^(q*r)
])*Log[g + h*x]])/h - (Log[(c + d*x)^(q*r)]^2*Log[g + h*x]])/h + (2*p*r*Log[-
((h*(a + b*x))/(b*g - a*h))]*(Log[(a + b*x)^(p*r)] + Log[(c + d*x)^(q*r)] -
 Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*x]])/h + (2*q*r*Log[-((h*(c
 + d*x))/(d*g - c*h))]*(Log[(a + b*x)^(p*r)] + Log[(c + d*x)^(q*r)] - Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*x]])/h + (Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r]^2*Log[g + h*x]])/h + (Log[(a + b*x)^(p*r)]^2*Log[(b*(g + h*x))
/(b*g - a*h)]])/h + (Log[(c + d*x)^(q*r)]^2*Log[(d*(g + h*x))/(d*g - c*h)]])/
h - (p*q*r^2*(Log[(b*(c + d*x))/(b*c - a*d)] + Log[(b*g - a*h)/(b*(g + h*x)
)] - Log[((b*g - a*h)*(c + d*x))/((b*c - a*d)*(g + h*x))]])*Log[-(((b*c - a*
d)*(g + h*x))/((d*g - c*h)*(a + b*x)))]^2)/h + (p*q*r^2*(Log[(b*(c + d*x))/
(b*c - a*d)] - Log[-((h*(c + d*x))/(d*g - c*h))]])*(Log[a + b*x] + Log[-(((b
*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x)))]])^2)/h - (p*q*r^2*(Log[-((d*(
a + b*x))/(b*c - a*d))] + Log[(d*g - c*h)/(d*(g + h*x))] - Log[-(((d*g - c*
h)*(a + b*x))/((b*c - a*d)*(g + h*x)))]])*Log[((b*c - a*d)*(g + h*x))/((b*g
 - a*h)*(c + d*x))]^2)/h + (p*q*r^2*(Log[-((d*(a + b*x))/(b*c - a*d))] - Log
[-((h*(a + b*x))/(b*g - a*h))]])*(Log[c + d*x] + Log[((b*c - a*d)*(g + h*x))
/((b*g - a*h)*(c + d*x))]])^2)/h - (2*p*q*r^2*(Log[g + h*x] - Log[-(((b*c -
a*d)*(g + h*x))/((d*g - c*h)*(a + b*x)))]])*PolyLog[2, -((d*(a + b*x))/(b*c
 - a*d))]])/h + (2*p*r*Log[(a + b*x)^(p*r)]*PolyLog[2, -((h*(a + b*x))/(b*g -
 a*h))]])/h - (2*p*q*r^2*(Log[g + h*x] - Log[((b*c - a*d)*(g + h*x))/((b*g -
 a*h)*(c + d*x))]])*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]])/h + (2*q*r*Log[(c
 + d*x)^(q*r)]*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))]])/h + (2*p*q*r^2*Log
[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x)))]*PolyLog[2, (h*(a + b*x
))/(b*(g + h*x))]])/h - (2*p*q*r^2*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h
)*(a + b*x)))]*PolyLog[2, -(((d*g - c*h)*(a + b*x))/((b*c - a*d)*(g + h*x))
)]])/h + (2*p*q*r^2*Log[((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))]*Pol
yLog[2, (h*(c + d*x))/(d*(g + h*x))]])/h - (2*p*q*r^2*Log[((b*c - a*d)*(g +
h*x))/((b*g - a*h)*(c + d*x))]*PolyLog[2, ((b*g - a*h)*(c + d*x))/((b*c - a
*d)*(g + h*x))]])/h + (2*p*r*(q*r*Log[c + d*x] - Log[(c + d*x)^(q*r)])*PolyL
og[2, (b*(g + h*x))/(b*g - a*h)]])/h + (2*p*r*(Log[(a + b*x)^(p*r)] + Log[(c
 + d*x)^(q*r)] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (b*(g + h
*x))/(b*g - a*h)]])/h - (2*p*q*r^2*(Log[c + d*x] + Log[((b*c - a*d)*(g + h*x
))/((b*g - a*h)*(c + d*x))]])*PolyLog[2, (b*(g + h*x))/(b*g - a*h)]])/h + (2*
q*r*(p*r*Log[a + b*x] - Log[(a + b*x)^(p*r)])*PolyLog[2, (d*(g + h*x))/(d*g
 - c*h)]])/h + (2*q*r*(Log[(a + b*x)^(p*r)] + Log[(c + d*x)^(q*r)] - Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (d*(g + h*x))/(d*g - c*h)]])/h - (
2*p*q*r^2*(Log[a + b*x] + Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b
*x)))]])*PolyLog[2, (d*(g + h*x))/(d*g - c*h)]])/h + (2*p*q*r^2*PolyLog[3, -(
(d*(a + b*x))/(b*c - a*d))]])/h - (2*p^2*r^2*PolyLog[3, -((h*(a + b*x))/(b*g
 - a*h))]])/h + (2*p*q*r^2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)]])/h - (2*q^2
*r^2*PolyLog[3, -((h*(c + d*x))/(d*g - c*h))]])/h + (2*p*q*r^2*PolyLog[3, (h
*(a + b*x))/(b*(g + h*x))]])/h - (2*p*q*r^2*PolyLog[3, -(((d*g - c*h)*(a + b
*x))/((b*c - a*d)*(g + h*x)))]])/h + (2*p*q*r^2*PolyLog[3, (h*(c + d*x))/(d*
(g + h*x))]])/h - (2*p*q*r^2*PolyLog[3, ((b*g - a*h)*(c + d*x))/((b*c - a*d)
*(g + h*x))]])/h + (2*p*q*r^2*PolyLog[3, (b*(g + h*x))/(b*g - a*h)]])/h + (2*
p*q*r^2*PolyLog[3, (d*(g + h*x))/(d*g - c*h)]])/h
```

Rule 2497

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^2/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a
 + b*x)^p*(c + d*x)^q)^r]^2)/h, x] + (-Dist[(2*b*p*r)/h, Int[(Log[g + h*x]*
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(a + b*x), x], x] - Dist[(2*d*q*r)/h,
 Int[(Log[g + h*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(c + d*x), x], x])
 /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2500

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.)))/((j_.) + (k
_.)*(x_)), x_Symbol] :> Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
```

```
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2437

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[(Log[i + j*x]*Log[c*(d + e*x)^n])/x, x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-(((b*c - a*d)*x)/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))]
, x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{g+hx}\,dx = \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{h} - \frac{(2bpr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(\cdots}{a+bx}}{h}$$

$$= \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(g+hx)}{h} - \frac{(2bpr)\int \frac{\log((a+bx)^{pr})\log(g+hx)}{a+bx}\,dx}{h} \cdots$$

$$= \frac{2pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(\log\left((a+bx)^{pr}\right)+\log\left((c+dx)^{qr}\right)-\log\left(e\left(f(a+bx)^p(c+\cdots\right.\right.\right.}{h}$$

$$= -\frac{\log^2\left((a+bx)^{pr}\right)\log(g+hx)}{h} - \frac{\log^2\left((c+dx)^{qr}\right)\log(g+hx)}{h} + \frac{2pr\log\left(-\frac{h(a+\cdots}{bg-\cdots}\right.}{\cdots}$$

$$= -\frac{\log^2\left((a+bx)^{pr}\right)\log(g+hx)}{h} - \frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)\log(g+hx)}{h} \cdots$$

$$= -\frac{\log^2\left((a+bx)^{pr}\right)\log(g+hx)}{h} - \frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)\log(g+hx)}{h} \cdots$$

$$= -\frac{\log^2\left((a+bx)^{pr}\right)\log(g+hx)}{h} - \frac{2pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)\log(g+hx)}{h} \cdots$$

Mathematica [A] time = 0.280696, size = 1370, normalized size = 0.93

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x),x]
```

```
[Out] (p*q*r^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*
g - c*h)*(a + b*x))]^2 + p^2*r^2*Log[a + b*x]^2*Log[g + h*x] + 2*p*q*r^2*Lo
g[a + b*x]*Log[c + d*x]*Log[g + h*x] + q^2*r^2*Log[c + d*x]^2*Log[g + h*x]
- 2*p*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] - 2*
q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] + Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r]^2*Log[g + h*x] - p^2*r^2*Log[a + b*x]^2*Log[
(b*(g + h*x))/(b*g - a*h)] - 2*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*
g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] + p*q*r^2*Log[(h*(c + d*x))/(-(d*
g) + c*h)]^2*Log[(b*(g + h*x))/(b*g - a*h)] - 2*p*q*r^2*Log[(h*(c + d*x))/(
-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Log[(b*
(g + h*x))/(b*g - a*h)] + p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))]^2*Log[(b*(g + h*x))/(b*g - a*h)] + 2*p*r*Log[a + b*x]*Log[e*(f*
(a + b*x)^p*(c + d*x)^q)^r]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*p*q*r^2*Log[
a + b*x]*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] - q^2*r^2*Log[c + d*x]
^2*Log[(d*(g + h*x))/(d*g - c*h)] + 2*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x)
)/(-(d*g) + c*h)]*Log[(d*(g + h*x))/(d*g - c*h)] - p*q*r^2*Log[(h*(c + d*x)
)/(-(d*g) + c*h)]^2*Log[(d*(g + h*x))/(d*g - c*h)] + 2*p*q*r^2*Log[(h*(c +
d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*
Log[(d*(g + h*x))/(d*g - c*h)] + 2*q*r*Log[c + d*x]*Log[e*(f*(a + b*x)^p*(c
 + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] - p*q*r^2*Log[((b*g - a*h)*(c
+ d*x))/((d*g - c*h)*(a + b*x))]^2*Log[((-(b*c) + a*d)*(g + h*x))/((d*g - c
*h)*(a + b*x))] + 2*p*r*(-(q*r*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a
+ b*x))]) + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (h*(a + b*x))/
```

```
(-(b*g) + a*h)] + 2*q*r*(p*r*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a +
b*x))] + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*PolyLog[2, (h*(c + d*x))/(-(
d*g) + c*h)] + 2*p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x)
)]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] - 2*p*q*r^2*Log[((b*g - a*h)*(c
+ d*x))/((d*g - c*h)*(a + b*x))]*PolyLog[2, ((b*g - a*h)*(c + d*x))/((d*g -
 c*h)*(a + b*x))] - 2*p^2*r^2*PolyLog[3, (h*(a + b*x))/(-(b*g) + a*h)] - 2*
p*q*r^2*PolyLog[3, (h*(a + b*x))/(-(b*g) + a*h)] - 2*p*q*r^2*PolyLog[3, (h*
(c + d*x))/(-(d*g) + c*h)] - 2*q^2*r^2*PolyLog[3, (h*(c + d*x))/(-(d*g) + c
*h)] - 2*p*q*r^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))] + 2*p*q*r^2*PolyLo
g[3, ((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))])/h
```

Maple [F] time = 0.578, size = 0, normalized size = 0.

$$\int \frac{\left(\ln \left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)\right)^{2}}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)^{2}}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)^{2}}{hx+g},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g), x)
```

3.40 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^2}\,dx$

Optimal. Leaf size=832

$$\frac{2bpq\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)r^2}{h(bg-ah)}+\frac{2dpq\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)r^2}{h(dg-ch)}-\frac{2dpq\log(a+bx)\log\left(\frac{b(g+hx)}{bg-ah}\right)r^2}{h(dg-ch)}-\frac{2bpq\log(c+d}{h(bg}$$

```
[Out] (2*b*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(h*(b*g - a*h)
) + (2*d*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(h*(d*g - c*h
)) - (2*b*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*
(a + b*x)^p*(c + d*x)^q)^r]))/(h*(b*g - a*h)) - (2*d*q*r*Log[c + d*x]*(p*r*
Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(h
*(d*g - c*h)) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(h*(g + h*x)) + (2*b
*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r])*Log[g + h*x])/(h*(b*g - a*h)) + (2*d*q*r*(p*r*Log[a + b*x] + q*r*Log
[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/(h*(d*g - c
*h)) - (2*d*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)])/(h*(d*g -
c*h)) - (2*b*p*q*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])/(h*(b*g -
 a*h)) - (2*b*p^2*r^2*Log[a + b*x]*Log[1 + (b*g - a*h)/(h*(a + b*x))])/(h*(
b*g - a*h)) - (2*d*q^2*r^2*Log[c + d*x]*Log[1 + (d*g - c*h)/(h*(c + d*x))])
/(h*(d*g - c*h)) + (2*b*p^2*r^2*PolyLog[2, -((b*g - a*h)/(h*(a + b*x)))])/(
h*(b*g - a*h)) + (2*d*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(h*
(d*g - c*h)) - (2*d*p*q*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))])/(h*(d
*g - c*h)) + (2*d*q^2*r^2*PolyLog[2, -((d*g - c*h)/(h*(c + d*x)))])/(h*(d*g
 - c*h)) + (2*b*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(h*(b*g - a*
h)) - (2*b*p*q*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(h*(b*g - a*h)
)
```

Rubi [A] time = 0.93174, antiderivative size = 878, normalized size of antiderivative = 1.06, number of steps used = 35, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {2498, 2513, 2411, 2344, 2301, 2317, 2391, 2418, 2394, 2393, 36, 31}

$$\frac{bp^2\log^2(a+bx)r^2}{h(bg-ah)}+\frac{dq^2\log^2(c+dx)r^2}{h(dg-ch)}+\frac{2bpq\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)r^2}{h(bg-ah)}+\frac{2dpq\log(a+bx)\log\left(\frac{b(c+dx)}{bc-ad}\right)r^2}{h(dg-ch)}-\frac{2bp^2\text{lo}}{h(b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^2,x]
```

```
[Out] (b*p^2*r^2*Log[a + b*x]^2)/(h*(b*g - a*h)) + (2*b*p*q*r^2*Log[-((d*(a + b*x
))/(b*c - a*d))]*Log[c + d*x])/(h*(b*g - a*h)) + (d*q^2*r^2*Log[c + d*x]^2)
/(h*(d*g - c*h)) + (2*d*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]
)/(h*(d*g - c*h)) - (2*b*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(h*(b*g - a*h)) - (2*d*q*r*Log
[c + d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]))/(h*(d*g - c*h)) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(h*(g
 + h*x)) + (2*b*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*
x)^p*(c + d*x)^q)^r])*Log[g + h*x])/(h*(b*g - a*h)) + (2*d*q*r*(p*r*Log[a +
 b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*
x])/(h*(d*g - c*h)) - (2*b*p^2*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*
h)])/(h*(b*g - a*h)) - (2*d*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a
*h)])/(h*(d*g - c*h)) - (2*b*p*q*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g -
c*h)])/(h*(b*g - a*h)) - (2*d*q^2*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g -
 c*h)])/(h*(d*g - c*h)) + (2*d*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*
```

```
d))])/(h*(d*g - c*h)) - (2*b*p^2*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h)
)])/(h*(b*g - a*h)) - (2*d*p*q*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))]
)/(h*(d*g - c*h)) + (2*b*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(h*
(b*g - a*h)) - (2*b*p*q*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(h*(b
*g - a*h)) - (2*d*q^2*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(h*(d*g
 - c*h))
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^2}dx = -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)} + \frac{(2bpr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)(g+hx)}dx}{h} + \frac{(2dqr)}{}$$

$$= -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)} + \frac{\left(2bp^2r^2\right)\int \frac{\log(a+bx)}{(a+bx)(g+hx)}dx}{h} + \frac{\left(2bpqr^2\right)\int \frac{\log}{(a+}}{h}$$

$$= -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{h(g+hx)} + \frac{\left(2p^2r^2\right)\text{Subst}\left(\int \frac{\log(x)}{x\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)}dx, x, a+bx\right)}{h}$$

$$= -\frac{2bpr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx\right.\right.\right.}{h(bg-ah)}$$

$$= \frac{bp^2r^2\log^2(a+bx)}{h(bg-ah)} + \frac{2bpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)} + \frac{dq^2r^2\log^2(c+dx)}{h(dg-ch)} +$$

$$= \frac{bp^2r^2\log^2(a+bx)}{h(bg-ah)} + \frac{2bpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)} + \frac{dq^2r^2\log^2(c+dx)}{h(dg-ch)} +$$

$$= \frac{bp^2r^2\log^2(a+bx)}{h(bg-ah)} + \frac{2bpqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{h(bg-ah)} + \frac{dq^2r^2\log^2(c+dx)}{h(dg-ch)} +$$

Mathematica [B] time = 2.67099, size = 2930, normalized size = 3.52

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^2,x]
```

```
[Out] (-(b*d*g^2*p^2*r^2*Log[a + b*x]^2) + b*c*g*h*p^2*r^2*Log[a + b*x]^2 - b*d*g
*h*p^2*r^2*x*Log[a + b*x]^2 + b*c*h^2*p^2*r^2*x*Log[a + b*x]^2 - 2*b*d*g^2*
p*q*r^2*Log[a + b*x]*Log[c + d*x] + 2*a*d*g*h*p*q*r^2*Log[a + b*x]*Log[c +
d*x] - 2*b*d*g*h*p*q*r^2*x*Log[a + b*x]*Log[c + d*x] + 2*a*d*h^2*p*q*r^2*x*
Log[a + b*x]*Log[c + d*x] - b*d*g^2*q^2*r^2*Log[c + d*x]^2 + a*d*g*h*q^2*r^
2*Log[c + d*x]^2 - b*d*g*h*q^2*r^2*x*Log[c + d*x]^2 + a*d*h^2*q^2*r^2*x*Log
[c + d*x]^2 + 2*b*c*g*h*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*
h)] - 2*a*d*g*h*p*q*r^2*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)] + 2*
b*c*h^2*p*q*r^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)] - 2*a*d*h^
2*p*q*r^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-(d*g) + c*h)] - b*c*g*h*p*q*r^
2*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 + a*d*g*h*p*q*r^2*Log[(h*(c + d*x))/(
-(d*g) + c*h)]^2 - b*c*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 +
a*d*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]^2 + 2*b*c*g*h*p*q*r^2*L
og[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(
a + b*x))] - 2*a*d*g*h*p*q*r^2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g
- a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*b*c*h^2*p*q*r^2*x*Log[(-(b*c
) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))
] - 2*a*d*h^2*p*q*r^2*x*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*g - a*h)*
(c + d*x))/((d*g - c*h)*(a + b*x))] + 2*b*c*g*h*p*q*r^2*Log[(h*(c + d*x))/(
-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*a*d
*g*h*p*q*r^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/
((d*g - c*h)*(a + b*x))] + 2*b*c*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) +
c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*a*d*h^2*p*q*
```

```
r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[((b*g - a*h)*(c + d*x))/((d*g -
 c*h)*(a + b*x))] - b*c*g*h*p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h
)*(a + b*x))]^2 + a*d*g*h*p*q*r^2*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))]^2 - b*c*h^2*p*q*r^2*x*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))]^2 + a*d*h^2*p*q*r^2*x*Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*
(a + b*x))]^2 + 2*b*d*g^2*p*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r] - 2*b*c*g*h*p*r*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*
b*d*g*h*p*r*x*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*b*c*h^2
*p*r*x*Log[a + b*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*d*g^2*q*r*Lo
g[c + d*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*a*d*g*h*q*r*Log[c + d*x
]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + 2*b*d*g*h*q*r*x*Log[c + d*x]*Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*a*d*h^2*q*r*x*Log[c + d*x]*Log[e*(f*(a
+ b*x)^p*(c + d*x)^q)^r] - b*d*g^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 +
 b*c*g*h*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 + a*d*g*h*Log[e*(f*(a + b*x
)^p*(c + d*x)^q)^r]^2 - a*c*h^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2 - 2*
b*d*g^2*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*a*d*g*h*p*q
*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*d*g*h*p*q*r^2*x*Log[
a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*a*d*h^2*p*q*r^2*x*Log[a + b*x]*
Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*d*g^2*p*q*r^2*Log[(h*(c + d*x))/(-(d*g
) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*c*g*h*p*q*r^2*Log[(h*(c + d*
x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*d*g*h*p*q*r^2*x*Lo
g[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*c*h^2*
p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(b*(g + h*x))/(b*g - a*h)]
- 2*b*d*g^2*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(b*(g + h*x))/(b*g
 - a*h)] + 2*b*c*g*h*p*r*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(b*(g + h
*x))/(b*g - a*h)] - 2*b*d*g*h*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Lo
g[(b*(g + h*x))/(b*g - a*h)] + 2*b*c*h^2*p*r*x*Log[e*(f*(a + b*x)^p*(c + d*
x)^q)^r]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*d*g^2*p*q*r^2*Log[a + b*x]*Lo
g[(d*(g + h*x))/(d*g - c*h)] - 2*a*d*g*h*p*q*r^2*Log[a + b*x]*Log[(d*(g + h
*x))/(d*g - c*h)] + 2*b*d*g*h*p*q*r^2*x*Log[a + b*x]*Log[(d*(g + h*x))/(d*g
 - c*h)] - 2*a*d*h^2*p*q*r^2*x*Log[a + b*x]*Log[(d*(g + h*x))/(d*g - c*h)]
- 2*b*d*g^2*p*q*r^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g + h*x))/(d*
g - c*h)] + 2*b*c*g*h*p*q*r^2*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[(d*(g +
 h*x))/(d*g - c*h)] - 2*b*d*g*h*p*q*r^2*x*Log[(h*(c + d*x))/(-(d*g) + c*h)]
*Log[(d*(g + h*x))/(d*g - c*h)] + 2*b*c*h^2*p*q*r^2*x*Log[(h*(c + d*x))/(-(
d*g) + c*h)]*Log[(d*(g + h*x))/(d*g - c*h)] - 2*b*d*g^2*q*r*Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] + 2*a*d*g*h*q*r*Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] - 2*b*d*g*h*q
*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g - c*h)] +
2*a*d*h^2*q*r*x*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[(d*(g + h*x))/(d*g
 - c*h)] + 2*p*(b*c*h*p + a*d*h*q - b*d*g*(p + q))*r^2*(g + h*x)*PolyLog[2,
 (h*(a + b*x))/(-(b*g) + a*h)] + 2*q*(b*c*h*p + a*d*h*q - b*d*g*(p + q))*r^
2*(g + h*x)*PolyLog[2, (h*(c + d*x))/(-(d*g) + c*h)] + 2*b*c*g*h*p*q*r^2*Po
lyLog[2, (b*(c + d*x))/(d*(a + b*x))] - 2*a*d*g*h*p*q*r^2*PolyLog[2, (b*(c
+ d*x))/(d*(a + b*x))] + 2*b*c*h^2*p*q*r^2*x*PolyLog[2, (b*(c + d*x))/(d*(a
 + b*x))] - 2*a*d*h^2*p*q*r^2*x*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(h
*(-(b*g) + a*h)*(-(d*g) + c*h)*(g + h*x))
```

Maple [F] time = 0.498, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)\right)^{2}}{\left(hx+g\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x)
```

Maxima [A] time = 1.57674, size = 1006, normalized size = 1.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x, algorithm="maxima
")
```

```
[Out] 2*(b*f*p*log(b*x + a)/(b*g - a*h) + d*f*q*log(d*x + c)/(d*g - c*h) - (a*d*f
*h*q - (d*f*g*(p + q) - c*f*h*p)*b)*log(h*x + g)/((d*g*h - c*h^2)*a - (d*g^
2 - c*g*h)*b))*r*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(f*h) - (2*(b*c*f^2*h
*p*q - a*d*f^2*h*p*q)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + di
log(-(b*d*x + a*d)/(b*c - a*d)))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b) +
2*(a*d*f^2*h*p*q + (c*f^2*h*p^2 - (p^2 + p*q)*d*f^2*g)*b)*(log(b*x + a)*log
((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b*g - a*h)))/((d*g*
h - c*h^2)*a - (d*g^2 - c*g*h)*b) + 2*(a*d*f^2*h*q^2 + (c*f^2*h*p*q - (p*q
+ q^2)*d*f^2*g)*b)*(log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + dilog
(-(d*h*x + c*h)/(d*g - c*h)))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b) - ((d
*f^2*g*p^2 - c*f^2*h*p^2)*b*log(b*x + a)^2 + 2*(b*d*f^2*g*p*q - a*d*f^2*h*p
*q)*log(b*x + a)*log(d*x + c) + (b*d*f^2*g*q^2 - a*d*f^2*h*q^2)*log(d*x + c
)^2 + 2*((a*d*f^2*h*p*q + (c*f^2*h*p^2 - (p^2 + p*q)*d*f^2*g)*b)*log(b*x +
a) + (a*d*f^2*h*q^2 + (c*f^2*h*p*q - (p*q + q^2)*d*f^2*g)*b)*log(d*x + c))*
log(h*x + g))/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*r^2/(f^2*h) - log(((
b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((h*x + g)*h)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{h^2x^2+2\,ghx+g^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h^2*x^2 + 2*g*h*x + g^2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{\left(hx+g\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g)^2, x)
```

3.41 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^3}\,dx$

Optimal. Leaf size=1304

result too large to display

```
[Out] -((b*d*p*q*r^2*Log[a + b*x])/(h*(b*g - a*h)*(d*g - c*h))) + (d*p*q*r^2*Log[
a + b*x])/(h*(d*g - c*h)*(g + h*x)) - (b*p^2*r^2*(a + b*x)*Log[a + b*x])/((
b*g - a*h)^2*(g + h*x)) - (b*d*p*q*r^2*Log[c + d*x])/(h*(b*g - a*h)*(d*g -
c*h)) + (b*p*q*r^2*Log[c + d*x])/(h*(b*g - a*h)*(g + h*x)) - (d*q^2*r^2*(c
+ d*x)*Log[c + d*x])/((d*g - c*h)^2*(g + h*x)) + (b^2*p*q*r^2*Log[-((d*(a +
 b*x))/(b*c - a*d))]*Log[c + d*x])/(h*(b*g - a*h)^2) + (d^2*p*q*r^2*Log[a +
 b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(h*(d*g - c*h)^2) - (b*p*r*(p*r*Log[a
 + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(h*(b*g
 - a*h)*(g + h*x)) - (d*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]))/(h*(d*g - c*h)*(g + h*x)) - (b^2*p*r*Log[a +
 b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)
^q)^r]))/(h*(b*g - a*h)^2) - (d^2*q*r*Log[c + d*x]*(p*r*Log[a + b*x] + q*r*
Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(h*(d*g - c*h)^2) - L
og[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(2*h*(g + h*x)^2) + (b^2*p^2*r^2*Log[
g + h*x])/(h*(b*g - a*h)^2) + (2*b*d*p*q*r^2*Log[g + h*x])/(h*(b*g - a*h)*(
d*g - c*h)) + (d^2*q^2*r^2*Log[g + h*x])/(h*(d*g - c*h)^2) + (b^2*p*r*(p*r*
Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log
[g + h*x])/(h*(b*g - a*h)^2) + (d^2*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x
] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/(h*(d*g - c*h)^2) -
 (d^2*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)])/(h*(d*g - c*h)^2
) - (b^2*p*q*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])/(h*(b*g - a*h
)^2) - (b^2*p^2*r^2*Log[a + b*x]*Log[1 + (b*g - a*h)/(h*(a + b*x))])/(h*(b*
g - a*h)^2) - (d^2*q^2*r^2*Log[c + d*x]*Log[1 + (d*g - c*h)/(h*(c + d*x))])
/(h*(d*g - c*h)^2) + (b^2*p^2*r^2*PolyLog[2, -((b*g - a*h)/(h*(a + b*x)))])
/(h*(b*g - a*h)^2) + (d^2*p*q*r^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])
/(h*(d*g - c*h)^2) - (d^2*p*q*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))])
/(h*(d*g - c*h)^2) + (d^2*q^2*r^2*PolyLog[2, -((d*g - c*h)/(h*(c + d*x)))])
/(h*(d*g - c*h)^2) + (b^2*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(h
*(b*g - a*h)^2) - (b^2*p*q*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(h
*(b*g - a*h)^2)
```

Rubi [A] time = 1.40757, antiderivative size = 1362, normalized size of antiderivative = 1.04, number of steps used = 47, number of rules used = 16, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.516, Rules used = {2498, 2513, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2418, 2394, 2393, 2395, 36, 44}

$$\frac{p^2r^2\log^2(a+bx)b^2}{2h(bg-ah)^2}+\frac{pqr^2\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)b^2}{h(bg-ah)^2}-\frac{pr\log(a+bx)\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a\right.\right.}{h(bg-ah)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^3,x]
```

```
[Out] -((b*d*p*q*r^2*Log[a + b*x])/(h*(b*g - a*h)*(d*g - c*h))) + (d*p*q*r^2*Log[
a + b*x])/(h*(d*g - c*h)*(g + h*x)) - (b*p^2*r^2*(a + b*x)*Log[a + b*x])/((
b*g - a*h)^2*(g + h*x)) + (b^2*p^2*r^2*Log[a + b*x]^2)/(2*h*(b*g - a*h)^2)
- (b*d*p*q*r^2*Log[c + d*x])/(h*(b*g - a*h)*(d*g - c*h)) + (b*p*q*r^2*Log[c
 + d*x])/(h*(b*g - a*h)*(g + h*x)) - (d*q^2*r^2*(c + d*x)*Log[c + d*x])/((d
```

```
*g - c*h)^2*(g + h*x)) + (b^2*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log
[c + d*x])/(h*(b*g - a*h)^2) + (d^2*q^2*r^2*Log[c + d*x]^2)/(2*h*(d*g - c*h
)^2) + (d^2*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(h*(d*g -
c*h)^2) - (b*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^
p*(c + d*x)^q)^r]))/(h*(b*g - a*h)*(g + h*x)) - (d*q*r*(p*r*Log[a + b*x] +
q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(h*(d*g - c*h)*(g
 + h*x)) - (b^2*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log
[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(h*(b*g - a*h)^2) - (d^2*q*r*Log[c + d*
x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)
^r]))/(h*(d*g - c*h)^2) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(2*h*(g +
h*x)^2) + (b^2*p^2*r^2*Log[g + h*x])/(h*(b*g - a*h)^2) + (2*b*d*p*q*r^2*Log
[g + h*x])/(h*(b*g - a*h)*(d*g - c*h)) + (d^2*q^2*r^2*Log[g + h*x])/(h*(d*g
 - c*h)^2) + (b^2*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/(h*(b*g - a*h)^2) + (d^2*q*r*(p*r*Log
[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g
+ h*x])/(h*(d*g - c*h)^2) - (b^2*p^2*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*
g - a*h)])/(h*(b*g - a*h)^2) - (d^2*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/
(b*g - a*h)])/(h*(d*g - c*h)^2) - (b^2*p*q*r^2*Log[c + d*x]*Log[(d*(g + h*x
))/(d*g - c*h)])/(h*(b*g - a*h)^2) - (d^2*q^2*r^2*Log[c + d*x]*Log[(d*(g +
h*x))/(d*g - c*h)])/(h*(d*g - c*h)^2) + (d^2*p*q*r^2*PolyLog[2, -((d*(a + b
*x))/(b*c - a*d))])/(h*(d*g - c*h)^2) - (b^2*p^2*r^2*PolyLog[2, -((h*(a + b
*x))/(b*g - a*h))])/(h*(b*g - a*h)^2) - (d^2*p*q*r^2*PolyLog[2, -((h*(a + b
*x))/(b*g - a*h))])/(h*(d*g - c*h)^2) + (b^2*p*q*r^2*PolyLog[2, (b*(c + d*x
))/(b*c - a*d)])/(h*(b*g - a*h)^2) - (b^2*p*q*r^2*PolyLog[2, -((h*(c + d*x)
)/(d*g - c*h))])/(h*(b*g - a*h)^2) - (d^2*q^2*r^2*PolyLog[2, -((h*(c + d*x)
)/(d*g - c*h))])/(h*(d*g - c*h)^2)
```

Rule 2498

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
tQ[s, 0] && NeQ[m, -1]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
```

```
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^3}\,dx = -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2} + \frac{(bpr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)(g+hx)^2}\,dx}{h} + \frac{(dqr)\int \frac{\log(\ldots}{\ldots}}{\ldots}$$

$$= -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2} + \frac{\left(bp^2r^2\right)\int \frac{\log(a+bx)}{(a+bx)(g+hx)^2}\,dx}{h} + \frac{\left(bpqr^2\right)\int \frac{\log(c+\ldots}{(a+bx)(g+\ldots}}{h}$$

$$= -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2h(g+hx)^2} + \frac{\left(p^2r^2\right)\text{Subst}\left(\int \frac{\log(x)}{x\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)^2}\,dx, x, a+bx\right)}{h} + \ldots$$

$$= -\frac{bpr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)}{h(bg-ah)(g+hx)} - \frac{dqr\left(pr\ldots\right.}{\ldots}$$

$$= \frac{dpqr^2\log(a+bx)}{h(dg-ch)(g+hx)} - \frac{bp^2r^2(a+bx)\log(a+bx)}{(bg-ah)^2(g+hx)} + \frac{bpqr^2\log(c+dx)}{h(bg-ah)(g+hx)} - \frac{dq^2r^2(c+\ldots}{(dg-\ldots}$$

$$= \frac{dpqr^2\log(a+bx)}{h(dg-ch)(g+hx)} - \frac{bp^2r^2(a+bx)\log(a+bx)}{(bg-ah)^2(g+hx)} + \frac{b^2p^2r^2\log^2(a+bx)}{2h(bg-ah)^2} + \frac{bpqr^2\log\ldots}{h(bg-a\ldots}$$

$$= -\frac{bdpqr^2\log(a+bx)}{h(bg-ah)(dg-ch)} + \frac{dpqr^2\log(a+bx)}{h(dg-ch)(g+hx)} - \frac{bp^2r^2(a+bx)\log(a+bx)}{(bg-ah)^2(g+hx)} + \frac{b^2p^2r^2\ldots}{2h(\ldots}$$

Mathematica [B] time = 6.32159, size = 15976, normalized size = 12.25

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^3,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.5, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)\right)^{2}}{\left(hx+g\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x)
```

Maxima [A] time = 2.44984, size = 2507, normalized size = 1.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x, algorithm="maxima
")
```

```
[Out] (b^2*f*p*log(b*x + a)/(b^2*g^2 - 2*a*b*g*h + a^2*h^2) + d^2*f*q*log(d*x + c
)/(d^2*g^2 - 2*c*d*g*h + c^2*h^2) + (2*a*b*d^2*f*g*h*q - a^2*d^2*f*h^2*q -
(d^2*f*g^2*(p + q) - 2*c*d*f*g*h*p + c^2*f*h^2*p)*b^2)*log(h*x + g)/((d^2*g
^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*
h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2) + (a*d*f*h*q - (d*f*g
*(p + q) - c*f*h*p)*b)/((d*g^2*h - c*g*h^2)*a - (d*g^3 - c*g^2*h)*b + ((d*g
*h^2 - c*h^3)*a - (d*g^2*h - c*g*h^2)*b)*x))*r*log(((b*x + a)^p*(d*x + c)^q
*f)^r*e)/(f*h) + 1/2*(2*(2*a*b*d^2*f^2*g*h*p*q - a^2*d^2*f^2*h^2*p*q - (2*c
*d*f^2*g*h*p*q - c^2*f^2*h^2*p*q)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
 - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/((d*g*h^2 - c*h^3)*a^2 -
2*(d*g^2*h - c*g*h^2)*a*b + (d*g^3 - c*g^2*h)*b^2) - 2*(2*a*b*d^2*f^2*g*h*p
*q - a^2*d^2*f^2*h^2*p*q + (2*c*d*f^2*g*h*p^2 - c^2*f^2*h^2*p^2 - (p^2 + p*
q)*d^2*f^2*g^2)*b^2)*(log(b*x + a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dil
og(-(b*h*x + a*h)/(b*g - a*h)))/((d*g*h^2 - c*h^3)*a^2 - 2*(d*g^2*h - c*g*h
^2)*a*b + (d*g^3 - c*g^2*h)*b^2) + 2*(2*a*b*d^2*f^2*g*h*q^2 - a^2*d^2*f^2*h
^2*q^2 + (2*c*d*f^2*g*h*p*q - c^2*f^2*h^2*p*q - (p*q + q^2)*d^2*f^2*g^2)*b^
2)*(log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) + dilog(-(d*h*x + c*h)/
(d*g - c*h)))/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2
*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2)
- 2*(a*d^2*f^2*h*q^2 + (c*d*f^2*h*p*q - (p*q + q^2)*d^2*f^2*g)*b)*log(d*x +
 c)/((d^2*g^2*h - 2*c*d*g*h^2 + c^2*h^3)*a - (d^2*g^3 - 2*c*d*g^2*h + c^2*g
*h^2)*b) + 2*(a^2*d^2*f^2*h^2*q^2 + 2*(c*d*f^2*h^2*p*q - (p*q + q^2)*d^2*f^
2*g*h)*a*b + (c^2*f^2*h^2*p^2 + (p^2 + 2*p*q + q^2)*d^2*f^2*g^2 - 2*(p^2 +
p*q)*c*d*f^2*g*h)*b^2)*log(h*x + g)/((d^2*g^2*h^2 - 2*c*d*g*h^3 + c^2*h^4)*
a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2*g^4 - 2*c*d*g^3*
h + c^2*g^2*h^2)*b^2) + ((d^3*f^2*g^3*p^2 - 3*c*d^2*f^2*g^2*h*p^2 + 3*c^2*d
*f^2*g*h^2*p^2 - c^3*f^2*h^3*p^2)*b^2*log(b*x + a)^2 - 2*(b^2*d^2*f^2*g^2*p
*q - 2*a*b*d^2*f^2*g*h*p*q + a^2*d^2*f^2*h^2*p*q)*log(b*x + a)*log(d*x + c)
 - (b^2*d^2*f^2*g^2*q^2 - 2*a*b*d^2*f^2*g*h*q^2 + a^2*d^2*f^2*h^2*q^2)*log(
```

```
d*x + c)^2 + 2*((d^2*f^2*g*h*p*q - c*d*f^2*h^2*p*q)*a*b - (c^2*f^2*h^2*p^2
+ (p^2 + p*q)*d^2*f^2*g^2 - (2*p^2 + p*q)*c*d*f^2*g*h)*b^2)*log(b*x + a) -
2*((2*a*b*d^2*f^2*g*h*p*q - a^2*d^2*f^2*h^2*p*q + (2*c*d*f^2*g*h*p^2 - c^2*
f^2*h^2*p^2 - (p^2 + p*q)*d^2*f^2*g^2)*b^2)*log(b*x + a) + (2*a*b*d^2*f^2*g
*h*q^2 - a^2*d^2*f^2*h^2*q^2 + (2*c*d*f^2*g*h*p*q - c^2*f^2*h^2*p*q - (p*q
+ q^2)*d^2*f^2*g^2)*b^2)*log(d*x + c))*log(h*x + g))/((d^2*g^2*h^2 - 2*c*d*
g*h^3 + c^2*h^4)*a^2 - 2*(d^2*g^3*h - 2*c*d*g^2*h^2 + c^2*g*h^3)*a*b + (d^2
*g^4 - 2*c*d*g^3*h + c^2*g^2*h^2)*b^2))*r^2/(f^2*h) - 1/2*log(((b*x + a)^p*
(d*x + c)^q*f)^r*e)^2/((h*x + g)^2*h)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2\right)}{h^3x^3+3\,gh^2x^2+3\,g^2hx+g^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h^3*x^3 + 3*g*h^2*x^2 + 3*
g^2*h*x + g^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.42 $\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^4}\,dx$

Optimal. Leaf size=1957

result too large to display

```
[Out] -(b^2*p^2*r^2)/(3*h*(b*g - a*h)^2*(g + h*x)) - (2*b*d*p*q*r^2)/(3*h*(b*g -
a*h)*(d*g - c*h)*(g + h*x)) - (d^2*q^2*r^2)/(3*h*(d*g - c*h)^2*(g + h*x)) -
 (b^3*p^2*r^2*Log[a + b*x])/(3*h*(b*g - a*h)^3) - (2*b*d^2*p*q*r^2*Log[a +
b*x])/(3*h*(b*g - a*h)*(d*g - c*h)^2) - (b^2*d*p*q*r^2*Log[a + b*x])/(3*h*(
b*g - a*h)^2*(d*g - c*h)) + (b*p^2*r^2*Log[a + b*x])/(3*h*(b*g - a*h)*(g +
h*x)^2) + (d*p*q*r^2*Log[a + b*x])/(3*h*(d*g - c*h)*(g + h*x)^2) + (2*d^2*p
*q*r^2*Log[a + b*x])/(3*h*(d*g - c*h)^2*(g + h*x)) - (2*b^2*p^2*r^2*(a + b*
x)*Log[a + b*x])/(3*(b*g - a*h)^3*(g + h*x)) - (b*d^2*p*q*r^2*Log[c + d*x])
/(3*h*(b*g - a*h)*(d*g - c*h)^2) - (2*b^2*d*p*q*r^2*Log[c + d*x])/(3*h*(b*g
 - a*h)^2*(d*g - c*h)) - (d^3*q^2*r^2*Log[c + d*x])/(3*h*(d*g - c*h)^3) + (
b*p*q*r^2*Log[c + d*x])/(3*h*(b*g - a*h)*(g + h*x)^2) + (d*q^2*r^2*Log[c +
d*x])/(3*h*(d*g - c*h)*(g + h*x)^2) + (2*b^2*p*q*r^2*Log[c + d*x])/(3*h*(b*
g - a*h)^2*(g + h*x)) - (2*d^2*q^2*r^2*(c + d*x)*Log[c + d*x])/(3*(d*g - c*
h)^3*(g + h*x)) + (2*b^3*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c +
d*x])/(3*h*(b*g - a*h)^3) + (2*d^3*p*q*r^2*Log[a + b*x]*Log[(b*(c + d*x))/(
b*c - a*d)])/(3*h*(d*g - c*h)^3) - (b*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(b*g - a*h)*(g + h*x)^2)
- (d*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d
*x)^q)^r]))/(3*h*(d*g - c*h)*(g + h*x)^2) - (2*b^2*p*r*(p*r*Log[a + b*x] +
q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(b*g - a*h)^
2*(g + h*x)) - (2*d^2*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(
a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(d*g - c*h)^2*(g + h*x)) - (2*b^3*p*r*Log
[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]))/(3*h*(b*g - a*h)^3) - (2*d^3*q*r*Log[c + d*x]*(p*r*Log[a + b*x
] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(d*g - c
*h)^3) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(3*h*(g + h*x)^3) + (b^3*p^
2*r^2*Log[g + h*x])/(h*(b*g - a*h)^3) + (b*d^2*p*q*r^2*Log[g + h*x])/(h*(b*
g - a*h)*(d*g - c*h)^2) + (b^2*d*p*q*r^2*Log[g + h*x])/(h*(b*g - a*h)^2*(d*
g - c*h)) + (d^3*q^2*r^2*Log[g + h*x])/(h*(d*g - c*h)^3) + (2*b^3*p*r*(p*r*
Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log
[g + h*x])/(3*h*(b*g - a*h)^3) + (2*d^3*q*r*(p*r*Log[a + b*x] + q*r*Log[c +
 d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/(3*h*(d*g - c*h
)^3) - (2*d^3*p*q*r^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)])/(3*h*(d*
g - c*h)^3) - (2*b^3*p*q*r^2*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])/(
3*h*(b*g - a*h)^3) - (2*b^3*p^2*r^2*Log[a + b*x]*Log[1 + (b*g - a*h)/(h*(a
+ b*x))])/(3*h*(b*g - a*h)^3) - (2*d^3*q^2*r^2*Log[c + d*x]*Log[1 + (d*g -
c*h)/(h*(c + d*x))])/(3*h*(d*g - c*h)^3) + (2*b^3*p^2*r^2*PolyLog[2, -((b*g
 - a*h)/(h*(a + b*x)))])/(3*h*(b*g - a*h)^3) + (2*d^3*p*q*r^2*PolyLog[2, -(
(d*(a + b*x))/(b*c - a*d))])/(3*h*(d*g - c*h)^3) - (2*d^3*p*q*r^2*PolyLog[2
, -((h*(a + b*x))/(b*g - a*h))])/(3*h*(d*g - c*h)^3) + (2*d^3*q^2*r^2*PolyL
og[2, -((d*g - c*h)/(h*(c + d*x)))])/(3*h*(d*g - c*h)^3) + (2*b^3*p*q*r^2*P
olyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*h*(b*g - a*h)^3) - (2*b^3*p*q*r^2*
PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(3*h*(b*g - a*h)^3)
```

Rubi [A] time = 2.10192, antiderivative size = 2013, normalized size of antiderivative = 1.03, number of steps used = 61, number of rules used = 17, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.548, Rules used = {2498, 2513, 2411, 2347, 2344, 2301, 2317, 2391, 2314, 31, 2319, 44, 2418, 2394, 2393, 2395, 36}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^4,x]
```

```
[Out] -(b^2*p^2*r^2)/(3*h*(b*g - a*h)^2*(g + h*x)) - (2*b*d*p*q*r^2)/(3*h*(b*g -
a*h)*(d*g - c*h)*(g + h*x)) - (d^2*q^2*r^2)/(3*h*(d*g - c*h)^2*(g + h*x)) -
 (b^3*p^2*r^2*Log[a + b*x])/(3*h*(b*g - a*h)^3) - (2*b*d^2*p*q*r^2*Log[a +
b*x])/(3*h*(b*g - a*h)*(d*g - c*h)^2) - (b^2*d*p*q*r^2*Log[a + b*x])/(3*h*(
b*g - a*h)^2*(d*g - c*h)) + (b*p^2*r^2*Log[a + b*x])/(3*h*(b*g - a*h)*(g +
h*x)^2) + (d*p*q*r^2*Log[a + b*x])/(3*h*(d*g - c*h)*(g + h*x)^2) + (2*d^2*p
*q*r^2*Log[a + b*x])/(3*h*(d*g - c*h)^2*(g + h*x)) - (2*b^2*p^2*r^2*(a + b*
x)*Log[a + b*x])/(3*(b*g - a*h)^3*(g + h*x)) + (b^3*p^2*r^2*Log[a + b*x]^2)
/(3*h*(b*g - a*h)^3) - (b*d^2*p*q*r^2*Log[c + d*x])/(3*h*(b*g - a*h)*(d*g -
 c*h)^2) - (2*b^2*d*p*q*r^2*Log[c + d*x])/(3*h*(b*g - a*h)^2*(d*g - c*h)) -
 (d^3*q^2*r^2*Log[c + d*x])/(3*h*(d*g - c*h)^3) + (b*p*q*r^2*Log[c + d*x])/
(3*h*(b*g - a*h)*(g + h*x)^2) + (d*q^2*r^2*Log[c + d*x])/(3*h*(d*g - c*h)*(
g + h*x)^2) + (2*b^2*p*q*r^2*Log[c + d*x])/(3*h*(b*g - a*h)^2*(g + h*x)) -
(2*d^2*q^2*r^2*(c + d*x)*Log[c + d*x])/(3*(d*g - c*h)^3*(g + h*x)) + (2*b^3
*p*q*r^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*h*(b*g - a*h)^3
) + (d^3*q^2*r^2*Log[c + d*x]^2)/(3*h*(d*g - c*h)^3) + (2*d^3*p*q*r^2*Log[a
 + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(3*h*(d*g - c*h)^3) - (b*p*r*(p*r*L
og[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*
h*(b*g - a*h)*(g + h*x)^2) - (d*q*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] -
Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(d*g - c*h)*(g + h*x)^2) - (2*b
^2*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x
)^q)^r]))/(3*h*(b*g - a*h)^2*(g + h*x)) - (2*d^2*q*r*(p*r*Log[a + b*x] + q*
r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(d*g - c*h)^2*
(g + h*x)) - (2*b^3*p*r*Log[a + b*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] -
 Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]))/(3*h*(b*g - a*h)^3) - (2*d^3*q*r*Lo
g[c + d*x]*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c +
 d*x)^q)^r]))/(3*h*(d*g - c*h)^3) - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/
(3*h*(g + h*x)^3) + (b^3*p^2*r^2*Log[g + h*x])/(h*(b*g - a*h)^3) + (b*d^2*p
*q*r^2*Log[g + h*x])/(h*(b*g - a*h)*(d*g - c*h)^2) + (b^2*d*p*q*r^2*Log[g +
 h*x])/(h*(b*g - a*h)^2*(d*g - c*h)) + (d^3*q^2*r^2*Log[g + h*x])/(h*(d*g -
 c*h)^3) + (2*b^3*p*r*(p*r*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r])*Log[g + h*x])/(3*h*(b*g - a*h)^3) + (2*d^3*q*r*(p*r
*Log[a + b*x] + q*r*Log[c + d*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])*Lo
g[g + h*x])/(3*h*(d*g - c*h)^3) - (2*b^3*p^2*r^2*Log[a + b*x]*Log[(b*(g + h
*x))/(b*g - a*h)])/(3*h*(b*g - a*h)^3) - (2*d^3*p*q*r^2*Log[a + b*x]*Log[(b
*(g + h*x))/(b*g - a*h)])/(3*h*(d*g - c*h)^3) - (2*b^3*p*q*r^2*Log[c + d*x]
*Log[(d*(g + h*x))/(d*g - c*h)])/(3*h*(b*g - a*h)^3) - (2*d^3*q^2*r^2*Log[c
 + d*x]*Log[(d*(g + h*x))/(d*g - c*h)])/(3*h*(d*g - c*h)^3) + (2*d^3*p*q*r^
2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(3*h*(d*g - c*h)^3) - (2*b^3*p^
2*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))])/(3*h*(b*g - a*h)^3) - (2*d^
3*p*q*r^2*PolyLog[2, -((h*(a + b*x))/(b*g - a*h))])/(3*h*(d*g - c*h)^3) + (
2*b^3*p*q*r^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*h*(b*g - a*h)^3) -
(2*b^3*p*q*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(3*h*(b*g - a*h)^3
) - (2*d^3*q^2*r^2*PolyLog[2, -((h*(c + d*x))/(d*g - c*h))])/(3*h*(d*g - c*
h)^3)
```

<u>Rule 2498</u>

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(h*(m + 1)), x] + (-Dist[(b*p*r*
s)/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
^(s - 1))/(a + b*x), x], x] - Dist[(d*q*r*s)/(h*(m + 1)), Int[((g + h*x)^(m
 + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(c + d*x), x], x]) /; F
reeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[s, 0] && NeQ[m, -1]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(g+hx)^4}dx = -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3} + \frac{(2bpr)\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(a+bx)(g+hx)^3}dx}{3h} + \frac{(2dqr)\ldots}{\ldots}$$

$$= -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3} + \frac{\left(2bp^2r^2\right)\int \frac{\log(a+bx)}{(a+bx)(g+hx)^3}dx}{3h} + \frac{\left(2bpqr^2\right)\int \frac{\ldots}{(a\ldots}}{3\ldots}$$

$$= -\frac{\log^2\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3h(g+hx)^3} + \frac{\left(2p^2r^2\right)\text{Subst}\left(\int \frac{\log(x)}{x\left(\frac{bg-ah}{b}+\frac{hx}{b}\right)^3}dx, x, a+bx\ldots\right)}{3h}$$

$$= -\frac{bpr\left(pr\log(a+bx)+qr\log(c+dx)-\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\right)}{3h(bg-ah)(g+hx)^2} - \frac{dqr\left(\ldots\right.}{\ldots}$$

$$= \frac{bp^2r^2\log(a+bx)}{3h(bg-ah)(g+hx)^2} + \frac{dpqr^2\log(a+bx)}{3h(dg-ch)(g+hx)^2} + \frac{2d^2pqr^2\log(a+bx)}{3h(dg-ch)^2(g+hx)} + \frac{bpq\ldots}{3h(bg\ldots}$$

$$= \frac{bp^2r^2\log(a+bx)}{3h(bg-ah)(g+hx)^2} + \frac{dpqr^2\log(a+bx)}{3h(dg-ch)(g+hx)^2} + \frac{2d^2pqr^2\log(a+bx)}{3h(dg-ch)^2(g+hx)} - \frac{2b^2p^2\ldots}{3\ldots}$$

$$= -\frac{b^2p^2r^2}{3h(bg-ah)^2(g+hx)} - \frac{2bdpqr^2}{3h(bg-ah)(dg-ch)(g+hx)} - \frac{d^2q^2r^2}{3h(dg-ch)^2(g+hx)\ldots}$$

$$= -\frac{b^2p^2r^2}{3h(bg-ah)^2(g+hx)} - \frac{2bdpqr^2}{3h(bg-ah)(dg-ch)(g+hx)} - \frac{d^2q^2r^2}{3h(dg-ch)^2(g+hx)\ldots}$$

Mathematica [B] time = 6.54072, size = 47110, normalized size = 24.07

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^2/(g + h*x)^4,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.52, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)\right)^2}{\left(hx+g\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x)
```

Maxima [B] time = 5.21219, size = 6388, normalized size = 3.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x, algorithm="maxima
")
```

```
[Out] 1/3*(2*b^3*f*p*log(b*x + a)/(b^3*g^3 - 3*a*b^2*g^2*h + 3*a^2*b*g*h^2 - a^3*
h^3) + 2*d^3*f*q*log(d*x + c)/(d^3*g^3 - 3*c*d^2*g^2*h + 3*c^2*d*g*h^2 - c^
3*h^3) - 2*(3*a*b^2*d^3*f*g^2*h*q - 3*a^2*b*d^3*f*g*h^2*q + a^3*d^3*f*h^3*q
 - (d^3*f*g^3*(p + q) - 3*c*d^2*f*g^2*h*p + 3*c^2*d*f*g*h^2*p - c^3*f*h^3*p
)*b^3)*log(h*x + g)/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h
^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a
^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^
2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^3) + ((3*d^
2*f*g*h^2*q - c*d*f*h^3*q)*a^2 - (d^2*f*g^2*h*(p + 6*q) - 2*c*d*f*g*h^2*(p
+ q) + c^2*f*h^3*p)*a*b - (c*d*f*g^2*h*(6*p + q) - 3*d^2*f*g^3*(p + q) - 3*
c^2*f*g*h^2*p)*b^2 - 2*(2*a*b*d^2*f*g*h^2*q - a^2*d^2*f*h^3*q - (d^2*f*g^2*
h*(p + q) - 2*c*d*f*g*h^2*p + c^2*f*h^3*p)*b^2)*x)/((d^2*g^4*h^2 - 2*c*d*g^
3*h^3 + c^2*g^2*h^4)*a^2 - 2*(d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*a*b
+ (d^2*g^6 - 2*c*d*g^5*h + c^2*g^4*h^2)*b^2 + ((d^2*g^2*h^4 - 2*c*d*g*h^5 +
 c^2*h^6)*a^2 - 2*(d^2*g^3*h^3 - 2*c*d*g^2*h^4 + c^2*g*h^5)*a*b + (d^2*g^4*
h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*b^2)*x^2 + 2*((d^2*g^3*h^3 - 2*c*d*g^2*h
^4 + c^2*g*h^5)*a^2 - 2*(d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*a*b + (
d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*b^2)*x))*r*log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)/(f*h) + 1/3*(2*(3*a*b^2*d^3*f^2*g^2*h*p*q - 3*a^2*b*d^3*f^2*g
*h^2*p*q + a^3*d^3*f^2*h^3*p*q - (3*c*d^2*f^2*g^2*h*p*q - 3*c^2*d*f^2*g*h^2
*p*q + c^3*f^2*h^3*p*q)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) +
1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3
*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^
2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3
 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^
3*h^3)*b^3) - 2*(3*a*b^2*d^3*f^2*g^2*h*p*q - 3*a^2*b*d^3*f^2*g*h^2*p*q + a^
3*d^3*f^2*h^3*p*q + (3*c*d^2*f^2*g^2*h*p^2 - 3*c^2*d*f^2*g*h^2*p^2 + c^3*f^
2*h^3*p^2 - (p^2 + p*q)*d^3*f^2*g^3)*b^3)*(log(b*x + a)*log((b*h*x + a*h)/(
b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b*g - a*h)))/((d^3*g^3*h^3 - 3*c*d^
2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3
 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*
c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4
*h^2 - c^3*g^3*h^3)*b^3) - 2*(3*a*b^2*d^3*f^2*g^2*h*q^2 - 3*a^2*b*d^3*f^2*g
*h^2*q^2 + a^3*d^3*f^2*h^3*q^2 + (3*c*d^2*f^2*g^2*h*p*q - 3*c^2*d*f^2*g*h^2
*p*q + c^3*f^2*h^3*p*q - (p*q + q^2)*d^3*f^2*g^3)*b^3)*(log(d*x + c)*log((d
*h*x + c*h)/(d*g - c*h) + 1) + dilog(-(d*h*x + c*h)/(d*g - c*h)))/((d^3*g^3
*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*
c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2
*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h
+ 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^3) - (3*a^2*d^3*f^2*h^2*q^2 + (c*d^2*f^2
*h^2*p*q - (p*q + 6*q^2)*d^3*f^2*g*h)*a*b - (5*c*d^2*f^2*g*h*p*q - 2*c^2*d*
f^2*h^2*p*q - 3*(p*q + q^2)*d^3*f^2*g^2)*b^2)*log(d*x + c)/((d^3*g^3*h^2 -
3*c*d^2*g^2*h^3 + 3*c^2*d*g*h^4 - c^3*h^5)*a^2 - 2*(d^3*g^4*h - 3*c*d^2*g^3
*h^2 + 3*c^2*d*g^2*h^3 - c^3*g*h^4)*a*b + (d^3*g^5 - 3*c*d^2*g^4*h + 3*c^2*
d*g^3*h^2 - c^3*g^2*h^3)*b^2) + 3*(a^3*d^3*f^2*h^3*q^2 + (c*d^2*f^2*h^3*p*q
 - (p*q + 3*q^2)*d^3*f^2*g*h^2)*a^2*b - (4*c*d^2*f^2*g*h^2*p*q - c^2*d*f^2*
h^3*p*q - 3*(p*q + q^2)*d^3*f^2*g^2*h)*a*b^2 + (c^3*f^2*h^3*p^2 - (p^2 + 2*
p*q + q^2)*d^3*f^2*g^3 + 3*(p^2 + p*q)*c*d^2*f^2*g^2*h - (3*p^2 + p*q)*c^2*
d*f^2*g*h^2)*b^3)*log(h*x + g)/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*
h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c
```

```
^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^
2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^
3) - ((d^3*f^2*g*h^3*q^2 - c*d^2*f^2*h^4*q^2)*a^3 - (2*c^2*d*f^2*h^4*p*q +
(2*p*q + 3*q^2)*d^3*f^2*g^2*h^2 - (4*p*q + 3*q^2)*c*d^2*f^2*g*h^3)*a^2*b -
(c^3*f^2*h^4*p^2 - (p^2 + 4*p*q + 3*q^2)*d^3*f^2*g^3*h + (3*p^2 + 8*p*q + 3
*q^2)*c*d^2*f^2*g^2*h^2 - (3*p^2 + 4*p*q)*c^2*d*f^2*g*h^3)*a*b^2 + (c^3*f^2
*g*h^3*p^2 - (p^2 + 2*p*q + q^2)*d^3*f^2*g^4 + (3*p^2 + 4*p*q + q^2)*c*d^2*
f^2*g^3*h - (3*p^2 + 2*p*q)*c^2*d*f^2*g^2*h^2)*b^3 - ((d^3*f^2*g^3*h*p^2 -
3*c*d^2*f^2*g^2*h^2*p^2 + 3*c^2*d*f^2*g*h^3*p^2 - c^3*f^2*h^4*p^2)*b^3*x +
(d^3*f^2*g^4*p^2 - 3*c*d^2*f^2*g^3*h*p^2 + 3*c^2*d*f^2*g^2*h^2*p^2 - c^3*f^
2*g*h^3*p^2)*b^3)*log(b*x + a)^2 - 2*(b^3*d^3*f^2*g^4*p*q - 3*a*b^2*d^3*f^2
*g^3*h*p*q + 3*a^2*b*d^3*f^2*g^2*h^2*p*q - a^3*d^3*f^2*g*h^3*p*q + (b^3*d^3
*f^2*g^3*h*p*q - 3*a*b^2*d^3*f^2*g^2*h^2*p*q + 3*a^2*b*d^3*f^2*g*h^3*p*q -
a^3*d^3*f^2*h^4*p*q)*x)*log(b*x + a)*log(d*x + c) - (b^3*d^3*f^2*g^4*q^2 -
3*a*b^2*d^3*f^2*g^3*h*q^2 + 3*a^2*b*d^3*f^2*g^2*h^2*q^2 - a^3*d^3*f^2*g*h^3
*q^2 + (b^3*d^3*f^2*g^3*h*q^2 - 3*a*b^2*d^3*f^2*g^2*h^2*q^2 + 3*a^2*b*d^3*f
^2*g*h^3*q^2 - a^3*d^3*f^2*h^4*q^2)*x)*log(d*x + c)^2 - (2*(d^3*f^2*g^2*h^2
*p*q - c*d^2*f^2*g*h^3*p*q)*a^2*b - (5*d^3*f^2*g^3*h*p*q - 6*c*d^2*f^2*g^2*
h^2*p*q + c^2*d*f^2*g*h^3*p*q)*a*b^2 - (3*c^3*f^2*g*h^3*p^2 - 3*(p^2 + p*q)
*d^3*f^2*g^4 + (9*p^2 + 4*p*q)*c*d^2*f^2*g^3*h - (9*p^2 + p*q)*c^2*d*f^2*g^
2*h^2)*b^3 + (2*(d^3*f^2*g*h^3*p*q - c*d^2*f^2*h^4*p*q)*a^2*b - (5*d^3*f^2*
g^2*h^2*p*q - 6*c*d^2*f^2*g*h^3*p*q + c^2*d*f^2*h^4*p*q)*a*b^2 - (3*c^3*f^2
*h^4*p^2 - 3*(p^2 + p*q)*d^3*f^2*g^3*h + (9*p^2 + 4*p*q)*c*d^2*f^2*g^2*h^2
- (9*p^2 + p*q)*c^2*d*f^2*g*h^3)*b^3)*x)*log(b*x + a) - 2*((3*a*b^2*d^3*f^2
*g^3*h*p*q - 3*a^2*b*d^3*f^2*g^2*h^2*p*q + a^3*d^3*f^2*g*h^3*p*q + (3*c*d^2
*f^2*g^3*h*p^2 - 3*c^2*d*f^2*g^2*h^2*p^2 + c^3*f^2*g*h^3*p^2 - (p^2 + p*q)*
d^3*f^2*g^4)*b^3 + (3*a*b^2*d^3*f^2*g^2*h^2*p*q - 3*a^2*b*d^3*f^2*g*h^3*p*q
 + a^3*d^3*f^2*h^4*p*q + (3*c*d^2*f^2*g^2*h^2*p^2 - 3*c^2*d*f^2*g*h^3*p^2 +
 c^3*f^2*h^4*p^2 - (p^2 + p*q)*d^3*f^2*g^3*h)*b^3)*x)*log(b*x + a) + (3*a*b
^2*d^3*f^2*g^3*h*q^2 - 3*a^2*b*d^3*f^2*g^2*h^2*q^2 + a^3*d^3*f^2*g*h^3*q^2
+ (3*c*d^2*f^2*g^3*h*p*q - 3*c^2*d*f^2*g^2*h^2*p*q + c^3*f^2*g*h^3*p*q - (p
*q + q^2)*d^3*f^2*g^4)*b^3 + (3*a*b^2*d^3*f^2*g^2*h^2*q^2 - 3*a^2*b*d^3*f^2
*g*h^3*q^2 + a^3*d^3*f^2*h^4*q^2 + (3*c*d^2*f^2*g^2*h^2*p*q - 3*c^2*d*f^2*g
*h^3*p*q + c^3*f^2*h^4*p*q - (p*q + q^2)*d^3*f^2*g^3*h)*b^3)*x)*log(d*x + c
))*log(h*x + g))/((d^3*g^4*h^3 - 3*c*d^2*g^3*h^4 + 3*c^2*d*g^2*h^5 - c^3*g*
h^6)*a^3 - 3*(d^3*g^5*h^2 - 3*c*d^2*g^4*h^3 + 3*c^2*d*g^3*h^4 - c^3*g^2*h^5
)*a^2*b + 3*(d^3*g^6*h - 3*c*d^2*g^5*h^2 + 3*c^2*d*g^4*h^3 - c^3*g^3*h^4)*a
*b^2 - (d^3*g^7 - 3*c*d^2*g^6*h + 3*c^2*d*g^5*h^2 - c^3*g^4*h^3)*b^3 + ((d^
3*g^3*h^4 - 3*c*d^2*g^2*h^5 + 3*c^2*d*g*h^6 - c^3*h^7)*a^3 - 3*(d^3*g^4*h^3
 - 3*c*d^2*g^3*h^4 + 3*c^2*d*g^2*h^5 - c^3*g*h^6)*a^2*b + 3*(d^3*g^5*h^2 -
3*c*d^2*g^4*h^3 + 3*c^2*d*g^3*h^4 - c^3*g^2*h^5)*a*b^2 - (d^3*g^6*h - 3*c*d
^2*g^5*h^2 + 3*c^2*d*g^4*h^3 - c^3*g^3*h^4)*b^3)*x))*r^2/(f^2*h) - 1/3*log(
((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/((h*x + g)^3*h)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{h^4x^4+4gh^3x^3+6g^2h^2x^2+4g^3hx+g^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h^4*x^4 + 4*g*h^3*x^3 + 6*
g^2*h^2*x^2 + 4*g^3*h*x + g^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)**2/(h*x+g)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)^2}{\left(hx+g\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)^2/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)^2/(h*x + g)^4, x)
```

3.43 $\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=42

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^{n+1}}{bc(n+1)}$$

```
[Out] -((a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^(1 + n)/(b*c*(1 + n)))
```

Rubi [A] time = 0.0802486, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2512, 2302, 30}

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^{n+1}}{bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] -((a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^(1 + n)/(b*c*(1 + n)))
```

Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{(a+b\log(x))^n}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&= -\frac{\text{Subst}\left(\int x^n\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}\\
&= -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^{1+n}}{bc(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.0290376, size = 42, normalized size = 1.

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^{n+1}}{bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] -((a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^(1 + n)/(b*c*(1 + n)))
```

Maple [F] time = 0.485, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\ln\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algorit
hm="maxima")
```

```
[Out] -integrate((b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Fricas [A] time = 2.19345, size = 138, normalized size = 3.29

$$-\frac{\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{bcn+bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algorit
hm="fricas")
```

```
[Out] -(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)*(b*log(sqrt(-c*x + 1)/sqrt(c*x +
 1)) + a)^n/(b*c*n + b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algorit
hm="giac")
```

```
[Out] integrate(-(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

3.44 $\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=37

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4bc}$$

```
[Out] -(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(4*b*c)
```

Rubi [A] time = 0.0611698, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2512, 2302, 30}

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] -(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(4*b*c)
```

Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{(a+b\log(x))^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&= -\frac{\operatorname{Subst}\left(\int x^3\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}\\
&= -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc}
\end{aligned}$$

Mathematica [A] time = 0.0089422, size = 37, normalized size = 1.

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] -(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(4*b*c)
```

Maple [F] time = 0.411, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\ln\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

Maxima [B] time = 1.35167, size = 710, normalized size = 19.19

$$\frac{1}{2}b^3\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3+\frac{3}{2}ab^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+\frac{3}{2}a^2b\left(\frac{\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algorit
hm="maxima")
```

```
[Out] 1/2*b^3*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))
^3 + 3/2*a*b^2*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*
x + 1))^2 + 3/2*a^2*b*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/
sqrt(c*x + 1)) + 1/64*(24*(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) + l
og(c*x - 1)^2)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2/c + 8*(log(c*x + 1)^3 -
3*log(c*x + 1)^2*log(c*x - 1) + 3*log(c*x + 1)*log(c*x - 1)^2 - log(c*x - 1
)^3)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))/c + (log(c*x + 1)^4 - 4*log(c*x + 1)
^3*log(c*x - 1) + 6*log(c*x + 1)^2*log(c*x - 1)^2 - 4*log(c*x + 1)*log(c*x
- 1)^3 + log(c*x - 1)^4)/c)*b^3 + 1/8*a*b^2*(6*(log(c*x + 1)^2 - 2*log(c*x
+ 1)*log(c*x - 1) + log(c*x - 1)^2)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))/c + (
log(c*x + 1)^3 - 3*log(c*x + 1)^2*log(c*x - 1) + 3*log(c*x + 1)*log(c*x - 1
)^2 - log(c*x - 1)^3)/c) + 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 3/8*
(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) + log(c*x - 1)^2)*a^2*b/c
```

Fricas [B] time = 2.01189, size = 248, normalized size = 6.7

$$-\frac{b^3\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^4+4\,ab^2\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3+6\,a^2b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+4\,a^3\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algorit
hm="fricas")
```

```
[Out] -1/4*(b^3*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^4 + 4*a*b^2*log(sqrt(-c*x + 1)/
sqrt(c*x + 1))^3 + 6*a^2*b*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 4*a^3*log(
sqrt(-c*x + 1)/sqrt(c*x + 1)))/c
```

Sympy [A] time = 47.0202, size = 65, normalized size = 1.76

$$\begin{cases} -\frac{a^3 \operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2 \sqrt{-\frac{1}{c^2}}} & \text{for } b = 0 \\ a^3 x & \text{for } c = 0 \\ -\frac{\left(a+b \log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^4}{4bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] Piecewise((-a**3*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*
*3*x, Eq(c, 0)), (-(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**4/(4*b*c), Tr
ue))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b \log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^3}{c^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algorit
hm="giac")
```

```
[Out] integrate(-(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x)
```

3.45 $\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=37

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}$$

```
[Out] -(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(3*b*c)
```

Rubi [A] time = 0.0628152, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2512, 2302, 30}

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] -(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(3*b*c)
```

Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{(a+b\log(x))^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&= -\frac{\text{Subst}\left(\int x^2\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}\\
&= -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}
\end{aligned}$$

Mathematica [A] time = 0.0100674, size = 37, normalized size = 1.

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] -(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(3*b*c)
```

Maple [F] time = 0.342, size = 0, normalized size = 0.

$$\int\frac{1}{-c^2x^2+1}\left(a+b\ln\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

Maxima [B] time = 1.27657, size = 362, normalized size = 9.78

$$\frac{1}{2}b^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+ab\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+\frac{1}{24}b^2\left(\frac{6\left(\log(cx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algorit
hm="maxima")
```

```
[Out] 1/2*b^2*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))
^2 + a*b*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1)
) + 1/24*b^2*(6*(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) + log(c*x - 1
)^2)*log(sqrt(-c*x + 1)/sqrt(c*x + 1))/c + (log(c*x + 1)^3 - 3*log(c*x + 1)
^2*log(c*x - 1) + 3*log(c*x + 1)*log(c*x - 1)^2 - log(c*x - 1)^3)/c) + 1/2*
a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/4*(log(c*x + 1)^2 - 2*log(c*x + 1
)*log(c*x - 1) + log(c*x - 1)^2)*a*b/c
```

Fricas [B] time = 2.12897, size = 184, normalized size = 4.97

$$-\frac{b^2\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3+3\,ab\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+3\,a^2\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algorit
hm="fricas")
```

```
[Out] -1/3*(b^2*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b*log(sqrt(-c*x + 1)/sq
rt(c*x + 1))^2 + 3*a^2*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))/c
```

Sympy [A] time = 23.3409, size = 65, normalized size = 1.76

$$\begin{cases} -\frac{a^2 \operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2\sqrt{-\frac{1}{c^2}}} & \text{for } b = 0 \\ a^2 x & \text{for } c = 0 \\ -\frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^3}{3bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] Piecewise((-a**2*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*
*2*x, Eq(c, 0)), (-(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**3/(3*b*c), Tr
ue))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, algorit
hm="giac")
```

```
[Out] integrate(-(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x)
```

3.46 $\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=37

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}$$

```
[Out] -(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(2*b*c)
```

Rubi [A] time = 0.0379821, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2512, 2301}

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(2*b*c)
```

Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{a+b\log(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\ &= -\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc}\end{aligned}$$

Mathematica [A] time = 0.0063933, size = 37, normalized size = 1.

$$-\frac{\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(2*b*c)
```

Maple [F] time = 0.364, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\ln\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

Maxima [B] time = 1.20672, size = 142, normalized size = 3.84

$$\frac{1}{2}b\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+\frac{1}{2}a\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)+\frac{\left(\log(cx+1)^2-2\log(cx+1)\log\right.}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorithm
="maxima")
```

```
[Out] 1/2*b*(log(c*x + 1)/c - log(c*x - 1)/c)*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) +
 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/8*(log(c*x + 1)^2 - 2*log(c*x
+ 1)*log(c*x - 1) + log(c*x - 1)^2)*b/c
```

Fricas [A] time = 1.99544, size = 119, normalized size = 3.22

$$-\frac{b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+2\,a\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorithm
="fricas")
```

```
[Out] -1/2*(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*log(sqrt(-c*x + 1)/sqrt(c
*x + 1)))/c
```

Sympy [A] time = 30.8092, size = 61, normalized size = 1.65

$$\begin{cases} -\frac{a \operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{c^2}}}\right)}{c^2\sqrt{-\frac{1}{c^2}}} & \text{for } b = 0 \\ ax & \text{for } c = 0 \\ -\frac{\left(a+b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}{2bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] Piecewise((-a*atan(x/sqrt(-1/c**2))/(c**2*sqrt(-1/c**2)), Eq(b, 0)), (a*x,
Eq(c, 0)), (-(a + b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)))**2/(2*b*c), True))
```

Giac [B] time = 1.26335, size = 116, normalized size = 3.14

$$-\frac{b\log(cx+1)^2}{8c} + \frac{b\log(cx-1)^2}{8c} + \frac{1}{4}\left(\frac{b\log(cx+1)}{c} - \frac{b\log(cx-1)}{c}\right)\log(-cx+1) + \frac{a\log(cx+1)}{2c} - \frac{a\log(cx-1)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algorithm
="giac")
```

```
[Out] -1/8*b*log(c*x + 1)^2/c + 1/8*b*log(c*x - 1)^2/c + 1/4*(b*log(c*x + 1)/c -
b*log(c*x - 1)/c)*log(-c*x + 1) + 1/2*a*log(c*x + 1)/c - 1/2*a*log(c*x - 1)
/c
```

3.47 $\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal. Leaf size=34

$$-\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

```
[Out] -(Log[a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]]/(b*c))
```

Rubi [A] time = 0.0677073, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2512, 2302, 29}

$$-\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] -(Log[a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]]/(b*c))
```

Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x(a+b\log(x))}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}\\
&= -\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}
\end{aligned}$$

Mathematica [A] time = 0.0360779, size = 34, normalized size = 1.

$$-\frac{\log\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] -(Log[a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]]/(b*c))
```

Maple [F] time = 0.351, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\ln\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Maxima [A] time = 1.48186, size = 49, normalized size = 1.44

$$-\frac{\log\left(-\frac{b\log(cx+1)-b\log(-cx+1)-2\,a}{2\,b}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorit
hm="maxima")
```

```
[Out] -log(-1/2*(b*log(c*x + 1) - b*log(-c*x + 1) - 2*a)/b)/(b*c)
```

Fricas [A] time = 2.00456, size = 72, normalized size = 2.12

$$-\frac{\log\left(b\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorit
hm="fricas")
```

```
[Out] -log(b*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(b*c)
```

Sympy [A] time = 164.757, size = 53, normalized size = 1.56

$$\begin{cases} \frac{x}{a} & \text{for } b = 0 \wedge c = 0 \\ \frac{-\frac{\log\left(x - \frac{1}{c}\right)}{2c} + \frac{\log\left(x + \frac{1}{c}\right)}{2c}}{a} & \text{for } b = 0 \\ \frac{x}{a} & \text{for } c = 0 \\ -\frac{\log\left(\frac{a}{b} + \frac{\log(-cx+1)}{2} - \frac{\log(cx+1)}{2}\right)}{bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] Piecewise((x/a, Eq(b, 0) & Eq(c, 0)), ((-log(x - 1/c)/(2*c) + log(x + 1/c)/
(2*c))/a, Eq(b, 0)), (x/a, Eq(c, 0)), (-log(a/b + log(-c*x + 1)/2 - log(c*x
 + 1)/2)/(b*c), True))
```

Giac [A] time = 1.16082, size = 42, normalized size = 1.24

$$-\frac{\log\left(-b\log\left(cx+1\right) + b\log\left(-cx+1\right) + 2\,a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, algorit
hm="giac")
```

```
[Out] -log(-b*log(c*x + 1) + b*log(-c*x + 1) + 2*a)/(b*c)
```

3.48 $\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=34

$$\frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}$$

```
[Out] 1/(b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))
```

Rubi [A] time = 0.0654521, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2512, 2302, 30}

$$\frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] 1/(b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))
```

Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x(a+b\log(x))^2}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}\\
&= \frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}
\end{aligned}$$

Mathematica [A] time = 0.0104415, size = 34, normalized size = 1.

$$\frac{1}{bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] 1/(b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]]))
```

Maple [F] time = 0.356, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\ln\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Maxima [A] time = 1.58297, size = 46, normalized size = 1.35

$$-\frac{2}{b^2c\log(cx+1)-b^2c\log(-cx+1)-2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, algor
ithm="maxima")
```

```
[Out] -2/(b^2*c*log(c*x + 1) - b^2*c*log(-c*x + 1) - 2*a*b*c)
```

Fricas [A] time = 1.99131, size = 72, normalized size = 2.12

$$\frac{1}{b^2c\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, algor
ithm="fricas")
```

```
[Out] 1/(b^2*c*log(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a*b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25946, size = 46, normalized size = 1.35

$$-\frac{2}{b^{2}c\log\left(cx+1\right)-b^{2}c\log\left(-cx+1\right)-2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, algor
ithm="giac")
```

```
[Out] -2/(b^2*c*log(c*x + 1) - b^2*c*log(-c*x + 1) - 2*a*b*c)
```

3.49 $\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx$

Optimal. Leaf size=37

$$\frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}$$

```
[Out] 1/(2*b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
```

Rubi [A] time = 0.0651834, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2512, 2302, 30}

$$\frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3),x]
```

```
[Out] 1/(2*b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
```

Rule 2512

```
Int[((a_.) + Log[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)*(x_)]]
*(b_.))^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[g/(C*f), Subst[Int[
(a + b*Log[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, A, C, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0
]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x(a+b\log(x))^3}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{x^3}\,dx, x, a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{bc}$$
$$= \frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}$$

Mathematica [A] time = 0.0105605, size = 37, normalized size = 1.

$$\frac{1}{2bc\left(a+b\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3),x]
```

```
[Out] 1/(2*b*c*(a + b*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
```

Maple [F] time = 0.349, size = 0, normalized size = 0.

$$\int \frac{1}{-c^2x^2+1}\left(a+b\ln\left(\sqrt{-cx+1}\frac{1}{\sqrt{cx+1}}\right)\right)^{-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x)
```

Maxima [B] time = 1.60891, size = 108, normalized size = 2.92

$$\frac{2}{b^3c\log(cx+1)^2 + b^3c\log(-cx+1)^2 - 4\,ab^2c\log(cx+1) + 4\,a^2bc - 2\left(b^3c\log(cx+1) - 2\,ab^2c\right)\log(-cx+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x, algor
ithm="maxima")
```

```
[Out] 2/(b^3*c*log(c*x + 1)^2 + b^3*c*log(-c*x + 1)^2 - 4*a*b^2*c*log(c*x + 1) +
4*a^2*b*c - 2*(b^3*c*log(c*x + 1) - 2*a*b^2*c)*log(-c*x + 1))
```

Fricas [A] time = 1.84696, size = 142, normalized size = 3.84

$$\frac{1}{2\left(b^3c\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+2\,ab^2c\log\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a^2bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x, algor
ithm="fricas")
```

```
[Out] 1/2/(b^3*c*log(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b^2*c*log(sqrt(-c*x +
1)/sqrt(c*x + 1)) + a^2*b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*ln((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30372, size = 115, normalized size = 3.11

$$\frac{2}{b^3c\log\left(cx+1\right)^2-2\,b^3c\log\left(cx+1\right)\log\left(-cx+1\right)+b^3c\log\left(-cx+1\right)^2-4\,ab^2c\log\left(cx+1\right)+4\,ab^2c\log\left(-cx+1\right)+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3,x, algor
ithm="giac")
```

```
[Out] 2/(b^3*c*log(c*x + 1)^2 - 2*b^3*c*log(c*x + 1)*log(-c*x + 1) + b^3*c*log(-c
*x + 1)^2 - 4*a*b^2*c*log(c*x + 1) + 4*a*b^2*c*log(-c*x + 1) + 4*a^2*b*c)
```

3.50 $\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=30

$$-\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

```
[Out] -Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(2*a)
```

Rubi [A] time = 0.0232066, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {2505}

$$-\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(2*a)
```

Rule 2505

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{h = Simplify[u*(a + b*x)*(c + d*x)]}
, Simp[(h*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1))/(p*r*(s + 1)*(b*c -
 a*d)), x] /; FreeQ[h, x]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && N
eQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[s, -1]
```

Rubi steps

$$\int \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

Mathematica [A] time = 0.0085736, size = 30, normalized size = 1.

$$-\frac{\log^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -Log[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(2*a)
```

Maple [F] time = 0.389, size = 0, normalized size = 0.

$$\int \frac{1}{-a^{2}x^{2}+1} \ln\left(\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(ln((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Maxima [B] time = 1.9242, size = 112, normalized size = 3.73

$$\frac{1}{2}\left(\frac{\log(ax+1)}{a}-\frac{\log(ax-1)}{a}\right)\log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)+\frac{\log(ax-1)^{2}}{8\,a}+\frac{\log(ax+1)^{2}-2\,\log(ax+1)\log(ax-1)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(log(a*x + 1)/a - log(a*x - 1)/a)*log(sqrt(-a*x + 1)/sqrt(a*x + 1)) + 1
/8*log(a*x - 1)^2/a + 1/8*(log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1))/a
```

Fricas [A] time = 1.94668, size = 59, normalized size = 1.97

$$-\frac{\log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{2}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fric
as")
```

```
[Out] -1/2*log(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/a
```

Sympy [B] time = 13.5192, size = 65, normalized size = 2.17

$$-\frac{\operatorname{atan}^{2}\left(\frac{x}{\sqrt{-\frac{1}{a^{2}}}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)\operatorname{atan}\left(\frac{x}{\sqrt{-\frac{1}{a^{2}}}}\right)}{a^{2}\sqrt{-\frac{1}{a^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] -atan(x/sqrt(-1/a**2))**2/(2*a) - log(sqrt(-a*x + 1)/sqrt(a*x + 1))*atan(x/
sqrt(-1/a**2))/(a**2*sqrt(-1/a**2))
```

Giac [B] time = 1.29754, size = 78, normalized size = 2.6

$$\frac{1}{4}\left(\frac{\log(ax+1)}{a}-\frac{\log(ax-1)}{a}\right)\log(-ax+1)-\frac{\log(ax+1)^2}{8a}+\frac{\log(ax-1)^2}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="giac
")
```

```
[Out] 1/4*(log(a*x + 1)/a - log(a*x - 1)/a)*log(-a*x + 1) - 1/8*log(a*x + 1)^2/a
+ 1/8*log(a*x - 1)^2/a
```

3.51 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}{gk+hkx}\,dx$

Optimal. Leaf size=410

$$-\frac{pr\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)^2}{hk}+\frac{2nprt\text{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)}{hk}-\frac{2n^2prt^2\text{Poly}}{}$$

```
[Out] -(p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*(s + t*Log[i*(g + h*x)^n])^3)/(3*h*
k*n*t) - (q*r*Log[-((h*(c + d*x))/(d*g - c*h))]*(s + t*Log[i*(g + h*x)^n])^
3)/(3*h*k*n*t) + (Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*
x)^n])^3)/(3*h*k*n*t) - (p*r*(s + t*Log[i*(g + h*x)^n])^2*PolyLog[2, (b*(g
+ h*x))/(b*g - a*h)])/(h*k) - (q*r*(s + t*Log[i*(g + h*x)^n])^2*PolyLog[2,
(d*(g + h*x))/(d*g - c*h)])/(h*k) + (2*n*p*r*t*(s + t*Log[i*(g + h*x)^n])*P
olyLog[3, (b*(g + h*x))/(b*g - a*h)])/(h*k) + (2*n*q*r*t*(s + t*Log[i*(g +
h*x)^n])*PolyLog[3, (d*(g + h*x))/(d*g - c*h)])/(h*k) - (2*n^2*p*r*t^2*Poly
Log[4, (b*(g + h*x))/(b*g - a*h)])/(h*k) - (2*n^2*q*r*t^2*PolyLog[4, (d*(g
+ h*x))/(d*g - c*h)])/(h*k)
```

Rubi [A] time = 0.471232, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2499, 2396, 2433, 2374, 2383, 6589}

$$-\frac{pr\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)^2}{hk}+\frac{2nprt\text{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)}{hk}-\frac{2n^2prt^2\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n])^2)/(g*
k + h*k*x),x]
```


```
[Out] -(p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*(s + t*Log[i*(g + h*x)^n])^3)/(3*h*
k*n*t) - (q*r*Log[-((h*(c + d*x))/(d*g - c*h))]*(s + t*Log[i*(g + h*x)^n])^
3)/(3*h*k*n*t) + (Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*
x)^n])^3)/(3*h*k*n*t) - (p*r*(s + t*Log[i*(g + h*x)^n])^2*PolyLog[2, (b*(g
+ h*x))/(b*g - a*h)])/(h*k) - (q*r*(s + t*Log[i*(g + h*x)^n])^2*PolyLog[2,
(d*(g + h*x))/(d*g - c*h)])/(h*k) + (2*n*p*r*t*(s + t*Log[i*(g + h*x)^n])*P
olyLog[3, (b*(g + h*x))/(b*g - a*h)])/(h*k) + (2*n*q*r*t*(s + t*Log[i*(g +
h*x)^n])*PolyLog[3, (d*(g + h*x))/(d*g - c*h)])/(h*k) - (2*n^2*p*r*t^2*Poly
Log[4, (b*(g + h*x))/(b*g - a*h)])/(h*k) - (2*n^2*q*r*t^2*PolyLog[4, (d*(g
+ h*x))/(d*g - c*h)])/(h*k)
```


Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right) \left(s+t\log \left(51(g+hx)^n\right)\right)^2}{gk+hkx}\,dx = \frac{\log \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right) \left(s+t\log \left(51(g+hx)^n\right.\right.}{3hknt}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(51(g+hx)^n\right)\right)^3}{3hknt} - \frac{qr\log}{}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(51(g+hx)^n\right)\right)^3}{3hknt} - \frac{qr\log}{}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(51(g+hx)^n\right)\right)^3}{3hknt} - \frac{qr\log}{}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(51(g+hx)^n\right)\right)^3}{3hknt} - \frac{qr\log}{}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(51(g+hx)^n\right)\right)^3}{3hknt} - \frac{qr\log}{}$$

Mathematica [B] time = 7.54479, size = 22595, normalized size = 55.11

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n])^
2)/(g*k + h*k*x),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.982, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^p\left(dx+c\right)^q\right)^r\right)\left(s+t\ln\left(i\left(hx+g\right)^n\right)\right)^2}{hkx+gk}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^2/(h*k*x+g*k),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))^2/(h*k*x+g*k),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))^2/(h*k*x+
g*k),x, algorithm="maxima")
```

```
[Out] 1/3*((n^2*t^2*log(h*x + g)^3 + 3*t^2*log(h*x + g)*log((h*x + g)^n)^2 - 3*(n
*t^2*log(i) + n*s*t)*log(h*x + g)^2 + 3*(t^2*log(i)^2 + 2*s*t*log(i) + s^2)
*log(h*x + g) - 3*(n*t^2*log(h*x + g)^2 - 2*(t^2*log(i) + s*t)*log(h*x + g)
)*log((h*x + g)^n))*log(((b*x + a)^p)^r) + (n^2*t^2*log(h*x + g)^3 + 3*t^2*
log(h*x + g)*log((h*x + g)^n)^2 - 3*(n*t^2*log(i) + n*s*t)*log(h*x + g)^2 +
 3*(t^2*log(i)^2 + 2*s*t*log(i) + s^2)*log(h*x + g) - 3*(n*t^2*log(h*x + g)
^2 - 2*(t^2*log(i) + s*t)*log(h*x + g))*log((h*x + g)^n))*log(((d*x + c)^q)
^r))/(h*k) - integrate(-1/3*(3*((t^2*log(i)^2 + 2*s*t*log(i) + s^2)*h*log(e
) + (t^2*log(i)^2 + 2*s*t*log(i) + s^2)*h*log(f^r))*b*d*x^2 - ((p*r + q*r)*
b*d*h*n^2*t^2*x^2 + b*c*g*n^2*p*r*t^2 + a*d*g*n^2*q*r*t^2 + (a*d*h*n^2*q*r*
t^2 + (c*h*n^2*p*r*t^2 + (p*r + q*r)*d*g*n^2*t^2)*b)*x)*log(h*x + g)^3 + 3*
((t^2*log(i)^2 + 2*s*t*log(i) + s^2)*h*log(e) + (t^2*log(i)^2 + 2*s*t*log(i
) + s^2)*h*log(f^r))*a*c + 3*(((p*r + q*r)*n*t^2*log(i) + (p*r*s + q*r*s)*n
*t)*b*d*h*x^2 + (n*p*r*t^2*log(i) + n*p*r*s*t)*b*c*g + (n*q*r*t^2*log(i) +
n*q*r*s*t)*a*d*g + ((n*q*r*t^2*log(i) + n*q*r*s*t)*a*d*h + (((p*r + q*r)*n*
t^2*log(i) + (p*r*s + q*r*s)*n*t)*d*g + (n*p*r*t^2*log(i) + n*p*r*s*t)*c*h)
*b)*x)*log(h*x + g)^2 + 3*((h*t^2*log(e) + h*t^2*log(f^r))*b*d*x^2 + (h*t^2
*log(e) + h*t^2*log(f^r))*a*c + ((h*t^2*log(e) + h*t^2*log(f^r))*b*c + (h*t
^2*log(e) + h*t^2*log(f^r))*a*d)*x - ((p*r + q*r)*b*d*h*t^2*x^2 + b*c*g*p*r
*t^2 + a*d*g*q*r*t^2 + (a*d*h*q*r*t^2 + (c*h*p*r*t^2 + (p*r + q*r)*d*g*t^2)
*b)*x)*log(h*x + g))*log((h*x + g)^n)^2 + 3*(((t^2*log(i)^2 + 2*s*t*log(i)
+ s^2)*h*log(e) + (t^2*log(i)^2 + 2*s*t*log(i) + s^2)*h*log(f^r))*b*c + ((t
^2*log(i)^2 + 2*s*t*log(i) + s^2)*h*log(e) + (t^2*log(i)^2 + 2*s*t*log(i) +
 s^2)*h*log(f^r))*a*d)*x - 3*(((p*r + q*r)*t^2*log(i)^2 + p*r*s^2 + q*r*s^2
 + 2*(p*r*s + q*r*s)*t*log(i))*b*d*h*x^2 + (p*r*t^2*log(i)^2 + 2*p*r*s*t*lo
g(i) + p*r*s^2)*b*c*g + (q*r*t^2*log(i)^2 + 2*q*r*s*t*log(i) + q*r*s^2)*a*d
*g + ((q*r*t^2*log(i)^2 + 2*q*r*s*t*log(i) + q*r*s^2)*a*d*h + (((p*r + q*r)
*t^2*log(i)^2 + p*r*s^2 + q*r*s^2 + 2*(p*r*s + q*r*s)*t*log(i))*d*g + (p*r*
t^2*log(i)^2 + 2*p*r*s*t*log(i) + p*r*s^2)*c*h)*b)*x)*log(h*x + g) + 3*(2*(
(t^2*log(i) + s*t)*h*log(e) + (t^2*log(i) + s*t)*h*log(f^r))*b*d*x^2 + 2*((
t^2*log(i) + s*t)*h*log(e) + (t^2*log(i) + s*t)*h*log(f^r))*a*c + ((p*r + q
*r)*b*d*h*n*t^2*x^2 + b*c*g*n*p*r*t^2 + a*d*g*n*q*r*t^2 + (a*d*h*n*q*r*t^2
+ (c*h*n*p*r*t^2 + (p*r + q*r)*d*g*n*t^2)*b)*x)*log(h*x + g)^2 + 2*(((t^2*l
og(i) + s*t)*h*log(e) + (t^2*log(i) + s*t)*h*log(f^r))*b*c + ((t^2*log(i) +
 s*t)*h*log(e) + (t^2*log(i) + s*t)*h*log(f^r))*a*d)*x - 2*(((p*r + q*r)*t^
2*log(i) + (p*r*s + q*r*s)*t)*b*d*h*x^2 + (p*r*t^2*log(i) + p*r*s*t)*b*c*g
+ (q*r*t^2*log(i) + q*r*s*t)*a*d*g + ((q*r*t^2*log(i) + q*r*s*t)*a*d*h + ((
(p*r + q*r)*t^2*log(i) + (p*r*s + q*r*s)*t)*d*g + (p*r*t^2*log(i) + p*r*s*t
)*c*h)*b)*x)*log(h*x + g))*log((h*x + g)^n))/(b*d*h^2*k*x^3 + a*c*g*h*k + (
a*d*h^2*k + (d*g*h*k + c*h^2*k)*b)*x^2 + (b*c*g*h*k + (d*g*h*k + c*h^2*k)*a
)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(t^2\log\left(\left(hx+g\right)^n i\right)^2+2\,st\log\left(\left(hx+g\right)^n i\right)+s^2\right)\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hkx+gk},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))^2/(h*k*x+
g*k),x, algorithm="fricas")
```

```
[Out] integral((t^2*log((h*x + g)^n*i)^2 + 2*s*t*log((h*x + g)^n*i) + s^2)*log(((
b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)*(s+t*ln(i*(h*x+g)**n))**2/(h*k
*x+g*k),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(t\log\left(\left(hx+g\right)^{n}i\right)+s\right)^{2}\log\left(\left(\left(bx+a\right)^{p}\left(dx+c\right)^{q}f\right)^{r}e\right)}{hkx+gk}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))^2/(h*k*x+
g*k),x, algorithm="giac")
```

```
[Out] integrate((t*log((h*x + g)^n*i) + s)^2*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)
/(h*k*x + g*k), x)
```

3.52 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(i(g+hx)^n\right)\right)}{gk+hkx}\,dx$

Optimal. Leaf size=306

$$-\frac{pr\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)}{hk}+\frac{nprt\text{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)}{hk}-\frac{qr\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)\left(t\log\left(i(g+hx)^n\right)\right.}{hk}$$

```
[Out] -(p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*(s + t*Log[i*(g + h*x)^n])^2)/(2*h*
k*n*t) - (q*r*Log[-((h*(c + d*x))/(d*g - c*h))]*(s + t*Log[i*(g + h*x)^n])^
2)/(2*h*k*n*t) + (Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*
x)^n])^2)/(2*h*k*n*t) - (p*r*(s + t*Log[i*(g + h*x)^n])*PolyLog[2, (b*(g +
h*x))/(b*g - a*h)])/(h*k) - (q*r*(s + t*Log[i*(g + h*x)^n])*PolyLog[2, (d*(
g + h*x))/(d*g - c*h)])/(h*k) + (n*p*r*t*PolyLog[3, (b*(g + h*x))/(b*g - a*
h)])/(h*k) + (n*q*r*t*PolyLog[3, (d*(g + h*x))/(d*g - c*h)])/(h*k)
```

Rubi [A] time = 0.287542, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {2499, 2396, 2433, 2374, 6589}

$$-\frac{pr\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)}{hk}+\frac{nprt\text{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)}{hk}-\frac{qr\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)\left(t\log\left(i(g+hx)^n\right)\right.}{hk}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n]))/(g*k
+ h*k*x),x]
```


```
[Out] -(p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*(s + t*Log[i*(g + h*x)^n])^2)/(2*h*
k*n*t) - (q*r*Log[-((h*(c + d*x))/(d*g - c*h))]*(s + t*Log[i*(g + h*x)^n])^
2)/(2*h*k*n*t) + (Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*
x)^n])^2)/(2*h*k*n*t) - (p*r*(s + t*Log[i*(g + h*x)^n])*PolyLog[2, (b*(g +
h*x))/(b*g - a*h)])/(h*k) - (q*r*(s + t*Log[i*(g + h*x)^n])*PolyLog[2, (d*(
g + h*x))/(d*g - c*h)])/(h*k) + (n*p*r*t*PolyLog[3, (b*(g + h*x))/(b*g - a*
h)])/(h*k) + (n*q*r*t*PolyLog[3, (d*(g + h*x))/(d*g - c*h)])/(h*k)
```


Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(52(g+hx)^n\right)\right)}{gk+hkx}\,dx = \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)\left(s+t\log\left(52(g+hx)^n\right)\right)}{2hknt}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(52(g+hx)^n\right)\right)^2}{2hknt} - \frac{qr\log}{}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(52(g+hx)^n\right)\right)^2}{2hknt} - \frac{qr\log}{}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(52(g+hx)^n\right)\right)^2}{2hknt} - \frac{qr\log}{}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\left(s+t\log\left(52(g+hx)^n\right)\right)^2}{2hknt} - \frac{qr\log}{}$$

Mathematica [A] time = 2.55186, size = 436, normalized size = 1.42

$$\frac{-2pr\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)\left(t\log\left(i(g+hx)^n\right)+s\right)+2nprt\text{PolyLog}\left(3,\frac{b(g+hx)}{bg-ah}\right)-2qr\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)\left(t\log\left(i(g+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]*(s + t*Log[i*(g + h*x)^n]))
/(g*k + h*k*x),x]
```

```
[Out] (-2*p*r*s*Log[(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x] - 2*q*r*s*Log[(h*(
c + d*x))/(-(d*g) + c*h)]*Log[g + h*x] + 2*s*Log[e*(f*(a + b*x)^p*(c + d*x)
^q)^r]*Log[g + h*x] + n*p*r*t*Log[(h*(a + b*x))/(-(b*g) + a*h)]*Log[g + h*x
```

```
]^2 + n*q*r*t*Log[(h*(c + d*x))/(-(d*g) + c*h)]*Log[g + h*x]^2 - n*t*Log[e*
(f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x]^2 - 2*p*r*t*Log[(h*(a + b*x))/(
-(b*g) + a*h)]*Log[g + h*x]*Log[i*(g + h*x)^n] - 2*q*r*t*Log[(h*(c + d*x))/
(-(d*g) + c*h)]*Log[g + h*x]*Log[i*(g + h*x)^n] + 2*t*Log[e*(f*(a + b*x)^p*
(c + d*x)^q)^r]*Log[g + h*x]*Log[i*(g + h*x)^n] - 2*p*r*(s + t*Log[i*(g + h
*x)^n])*PolyLog[2, (b*(g + h*x))/(b*g - a*h)] - 2*q*r*(s + t*Log[i*(g + h*x
)^n])*PolyLog[2, (d*(g + h*x))/(d*g - c*h)] + 2*n*p*r*t*PolyLog[3, (b*(g +
h*x))/(b*g - a*h)] + 2*n*q*r*t*PolyLog[3, (d*(g + h*x))/(d*g - c*h)])/(2*h*
k)
```

Maple [F] time = 0.75, size = 0, normalized size = 0.

$$\int \frac{\ln \left(e \left(f \left(bx+a\right)^{p} \left(dx+c\right)^{q}\right)^{r}\right) \left(s+t \ln \left(i \left(hx+g\right)^{n}\right)\right)}{hkx+gk}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))/(h*k*x+g*k),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*ln(i*(h*x+g)^n))/(h*k*x+g*k),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))/(h*k*x+g*
k),x, algorithm="maxima")
```

```
[Out] -1/2*((n*t*log(h*x + g)^2 - 2*t*log(h*x + g)*log((h*x + g)^n) - 2*(t*log(i)
 + s)*log(h*x + g))*log(((b*x + a)^p)^r) + (n*t*log(h*x + g)^2 - 2*t*log(h*
x + g)*log((h*x + g)^n) - 2*(t*log(i) + s)*log(h*x + g))*log(((d*x + c)^q)^
r))/(h*k) - integrate(-1/2*(2*((t*log(i) + s)*h*log(e) + (t*log(i) + s)*h*l
og(f^r))*b*d*x^2 + 2*((t*log(i) + s)*h*log(e) + (t*log(i) + s)*h*log(f^r))*
a*c + ((p*r + q*r)*b*d*h*n*t*x^2 + b*c*g*n*p*r*t + a*d*g*n*q*r*t + (a*d*h*n
*q*r*t + (c*h*n*p*r*t + (p*r + q*r)*d*g*n*t)*b)*x)*log(h*x + g)^2 + 2*(((t*
log(i) + s)*h*log(e) + (t*log(i) + s)*h*log(f^r))*b*c + ((t*log(i) + s)*h*l
og(e) + (t*log(i) + s)*h*log(f^r))*a*d)*x - 2*((p*r*s + q*r*s + (p*r + q*r)
*t*log(i))*b*d*h*x^2 + (p*r*t*log(i) + p*r*s)*b*c*g + (q*r*t*log(i) + q*r*s
)*a*d*g + ((q*r*t*log(i) + q*r*s)*a*d*h + ((p*r*s + q*r*s + (p*r + q*r)*t*l
og(i))*d*g + (p*r*t*log(i) + p*r*s)*c*h)*b)*x)*log(h*x + g) + 2*((h*t*log(e
) + h*t*log(f^r))*b*d*x^2 + (h*t*log(e) + h*t*log(f^r))*a*c + ((h*t*log(e)
+ h*t*log(f^r))*b*c + (h*t*log(e) + h*t*log(f^r))*a*d)*x - ((p*r + q*r)*b*d
*h*t*x^2 + b*c*g*p*r*t + a*d*g*q*r*t + (a*d*h*q*r*t + (c*h*p*r*t + (p*r + q
*r)*d*g*t)*b)*x)*log(h*x + g))*log((h*x + g)^n))/(b*d*h^2*k*x^3 + a*c*g*h*k
 + (a*d*h^2*k + (d*g*h*k + c*h^2*k)*b)*x^2 + (b*c*g*h*k + (d*g*h*k + c*h^2*
k)*a)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(t\log\left(\left(hx+g\right)^{n}i\right)+s\right)\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)}{hkx+gk},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))/(h*k*x+g*
k),x, algorithm="fricas")
```

```
[Out] integral((t*log((h*x + g)^n*i) + s)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h
*k*x + g*k), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)*(s+t*ln(i*(h*x+g)**n))/(h*k*x+
g*k),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(t\log\left(\left(hx+g\right)^{n}i\right)+s\right)\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)}{hkx+gk}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)*(s+t*log(i*(h*x+g)^n))/(h*k*x+g*
k),x, algorithm="giac")
```

```
[Out] integrate((t*log((h*x + g)^n*i) + s)*log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(
h*k*x + g*k), x)
```

3.53 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{gk+hkx}\,dx$

Optimal. Leaf size=172

$$-\frac{pr\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{hk}-\frac{qr\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{hk}+\frac{\log(gk+hkx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{hk}-\frac{pr\log(gk+hkx)\log}{hk}$$

```
[Out] -((p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g*k + h*k*x])/(h*k)) - (q*r*Lo
g[-((h*(c + d*x))/(d*g - c*h))]*Log[g*k + h*k*x])/(h*k) + (Log[e*(f*(a + b*
x)^p*(c + d*x)^q)^r]*Log[g*k + h*k*x])/(h*k) - (p*r*PolyLog[2, (b*(g + h*x)
)/(b*g - a*h)])/(h*k) - (q*r*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/(h*k)
```

Rubi [A] time = 0.135941, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2494, 2394, 2393, 2391}

$$-\frac{pr\text{PolyLog}\left(2,\frac{b(g+hx)}{bg-ah}\right)}{hk}-\frac{qr\text{PolyLog}\left(2,\frac{d(g+hx)}{dg-ch}\right)}{hk}+\frac{\log(gk+hkx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{hk}-\frac{pr\log(gk+hkx)\log}{hk}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g*k + h*k*x),x]
```

```
[Out] -((p*r*Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g*k + h*k*x])/(h*k)) - (q*r*Lo
g[-((h*(c + d*x))/(d*g - c*h))]*Log[g*k + h*k*x])/(h*k) + (Log[e*(f*(a + b*
x)^p*(c + d*x)^q)^r]*Log[g*k + h*k*x])/(h*k) - (p*r*PolyLog[2, (b*(g + h*x)
)/(b*g - a*h)])/(h*k) - (q*r*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/(h*k)
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right)}{gk+hkx}\,dx = \frac{\log \left(e \left(f(a+bx)^p(c+dx)^q\right)^r\right)\log(gk+hkx)}{hk} - \frac{(bpr)\int \frac{\log(gk+hkx)}{a+bx}\,dx}{hk} - \frac{(dqr)\int}{}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(gk+hkx)}{hk} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(gk+hkx)}{hk} + \frac{\log\left(e\left(f(\right.\right.}{}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(gk+hkx)}{hk} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(gk+hkx)}{hk} + \frac{\log\left(e\left(f(\right.\right.}{}$$

$$= -\frac{pr\log\left(-\frac{h(a+bx)}{bg-ah}\right)\log(gk+hkx)}{hk} - \frac{qr\log\left(-\frac{h(c+dx)}{dg-ch}\right)\log(gk+hkx)}{hk} + \frac{\log\left(e\left(f(\right.\right.}{}$$

Mathematica [A] time = 0.0887067, size = 166, normalized size = 0.97

$$\frac{pr\text{PolyLog}\left(2,\frac{h(a+bx)}{ah-bg}\right)+qr\text{PolyLog}\left(2,\frac{h(c+dx)}{ch-dg}\right)+\log(g+hx)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)-pr\log(a+bx)\log(g+}{hk}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(g*k + h*k*x),x]
```

```
[Out] (-(p*r*Log[a + b*x]*Log[g + h*x]) - q*r*Log[c + d*x]*Log[g + h*x] + Log[e*(
f*(a + b*x)^p*(c + d*x)^q)^r]*Log[g + h*x] + p*r*Log[a + b*x]*Log[(b*(g + h
*x))/(b*g - a*h)] + q*r*Log[c + d*x]*Log[(d*(g + h*x))/(d*g - c*h)] + p*r*P
olyLog[2, (h*(a + b*x))/(-(b*g) + a*h)] + q*r*PolyLog[2, (h*(c + d*x))/(-(d
*g) + c*h)])/(h*k)
```

Maple [F] time = 0.491, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)}{hkx+gk}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x)
```

Maxima [A] time = 1.26247, size = 275, normalized size = 1.6

$$\frac{\left(\frac{\left(\log(bx+a)\log\left(\frac{bhx+ah}{bg-ah}+1\right)+\text{Li}_2\left(-\frac{bhx+ah}{bg-ah}\right)\right)fp}{hk}+\frac{\left(\log(dx+c)\log\left(\frac{dhx+ch}{dg-ch}+1\right)+\text{Li}_2\left(-\frac{dhx+ch}{dg-ch}\right)\right)fq}{hk}\right)r}{f} - \frac{\left(fp\log(bx+a)+fq\log(dx+c)\right)r\log}{fhk}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x, algorithm="maxima
")
```

```
[Out] ((log(b*x + a)*log((b*h*x + a*h)/(b*g - a*h) + 1) + dilog(-(b*h*x + a*h)/(b
*g - a*h)))*f*p/(h*k) + (log(d*x + c)*log((d*h*x + c*h)/(d*g - c*h) + 1) +
dilog(-(d*h*x + c*h)/(d*g - c*h)))*f*q/(h*k))*r/f - (f*p*log(b*x + a) + f*q
*log(d*x + c))*r*log(h*k*x + g*k)/(f*h*k) + log(h*k*x + g*k)*log(((b*x + a)
^p*(d*x + c)^q*f)^r*e)/(h*k)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hkx+gk}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x, algorithm="fricas
")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{hkx+gk}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*x + g*k), x)
```

3.54 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx$

Optimal. Leaf size=50

$$\text{Unintegrable}\left(\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(t\log\left(i(g+hx)^n\right)+s\right)},x\right)$$

```
[Out] Unintegrable[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log
[i*(g + h*x)^n])), x]
```

Rubi [A] time = 0.0532358, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*(g + h
*x)^n])),x]
```

```
[Out] Defer[Int][Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i
*(g + h*x)^n])), x]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(54(g+hx)^n\right)\right)}\,dx=\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(54(g+hx)^n\right)\right)}\,dx$$

Mathematica [A] time = 0.362222, size = 0, normalized size = 0.

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*
(g + h*x)^n])),x]
```

```
[Out] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*
(g + h*x)^n])), x]
```

Maple [A] time = 0.723, size = 0, normalized size = 0.

$$\int \frac{\ln \left(e \left(f \left(bx+a\right)^{p} \left(dx+c\right)^{q}\right)^{r}\right)}{\left(hkx+gk\right) \left(s+t \ln \left(i \left(hx+g\right)^{n}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n)),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(\left((bx+a)^{p}(dx+c)^{q} f\right)^{r} e\right)}{\left(hkx+gk\right)\left(t \log \left(\left(hx+g\right)^{n} i\right)+s\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
)),x, algorithm="maxima")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log((h*x +
 g)^n*i) + s)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log \left(\left((bx+a)^{p}(dx+c)^{q} f\right)^{r} e\right)}{hksx+gks+\left(hktx+gkt\right)\log \left(\left(hx+g\right)^{n} i\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
)),x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*s*x + g*k*s + (h*k*t*x +
 g*k*t)*log((h*x + g)^n*i)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)*
*n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{(hkx+gk)\left(t\log\left((hx+g)^n i\right)+s\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
)),x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log((h*x +
 g)^n*i) + s)), x)
```

3.55 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx$

Optimal. Leaf size=50

$$\text{Unintegrable}\left(\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(t\log\left(i(g+hx)^n\right)+s\right)^2},x\right)$$

```
[Out] Unintegrable[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log
[i*(g + h*x)^n])^2), x]
```

Rubi [A] time = 0.0525536, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*(g + h
*x)^n])^2),x]
```

```
[Out] Defer[Int][Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i
*(g + h*x)^n])^2), x]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(55(g+hx)^n\right)\right)^2}\,dx=\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(55(g+hx)^n\right)\right)^2}\,dx$$

Mathematica [A] time = 2.74292, size = 0, normalized size = 0.

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{(gk+hkx)\left(s+t\log\left(i(g+hx)^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*
(g + h*x)^n])^2),x]
```

```
[Out] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/((g*k + h*k*x)*(s + t*Log[i*
(g + h*x)^n])^2), x]
```

Maple [A] time = 0.739, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f\left(bx+a\right)^{p}\left(dx+c\right)^{q}\right)^{r}\right)}{\left(hkx+gk\right)\left(s+t\ln\left(i\left(hx+g\right)^{n}\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n))^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)^n))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(\left((bx+a)^{p}\right)^{r}\right)+\log\left(\left((dx+c)^{q}\right)^{r}\right)+\log\left(e\right)+\log\left(f^{r}\right)}{hknt^{2}\log\left(\left(hx+g\right)^{n}\right)+\left(knt^{2}\log\left(i\right)+knst\right)h}+\int \frac{}{\left(knt^{2}\log\left(i\right)+knst\right)bdhx^{2}+\left(knt^{2}\log\left(i\right)+knst\right)ach}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
))^2,x, algorithm="maxima")
```

```
[Out] -(log(((b*x + a)^p)^r) + log(((d*x + c)^q)^r) + log(e) + log(f^r))/(h*k*n*t
^2*log((h*x + g)^n) + (k*n*t^2*log(i) + k*n*s*t)*h) + integrate((b*c*p*r +
a*d*q*r + (p*r + q*r)*b*d*x)/((k*n*t^2*log(i) + k*n*s*t)*b*d*h*x^2 + (k*n*t
^2*log(i) + k*n*s*t)*a*c*h + ((k*n*t^2*log(i) + k*n*s*t)*b*c*h + (k*n*t^2*l
og(i) + k*n*s*t)*a*d*h)*x + (b*d*h*k*n*t^2*x^2 + a*c*h*k*n*t^2 + (b*c*h*k*n
*t^2 + a*d*h*k*n*t^2)*x)*log((h*x + g)^n)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)}{hks^{2}x+gks^{2}+\left(hkt^{2}x+gkt^{2}\right)\log\left(\left(hx+g\right)^{n}i\right)^{2}+2\left(hkstx+gkst\right)\log\left(\left(hx+g\right)^{n}i\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
))^2,x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(h*k*s^2*x + g*k*s^2 + (h*k*t
^2*x + g*k*t^2)*log((h*x + g)^n*i)^2 + 2*(h*k*s*t*x + g*k*s*t)*log((h*x + g
)^n*i)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/(h*k*x+g*k)/(s+t*ln(i*(h*x+g)*
*n))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{(hkx+gk)\left(t\log\left((hx+g)^n i\right)+s\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/(h*k*x+g*k)/(s+t*log(i*(h*x+g)^n
))^2,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/((h*k*x + g*k)*(t*log((h*x +
 g)^n*i) + s)^2), x)
```

3.56 $\int \frac{\log^3\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx$

Optimal. Leaf size=328

$$-6prt^2u^2\text{PolyLog}\left(4,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)-pr\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log^3\left(i\left(j(hx)^t\right)^u\right)+3prtu\text{PolyLog}\left(3,-\frac{bx}{a}\right)\log^2\left(\right.$$

```
[Out] -(p*r*Log[i*(j*(h*x)^t)^u]^4*Log[1 + (b*x)/a])/(4*t*u) + (Log[i*(j*(h*x)^t)
^u]^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*t*u) - (q*r*Log[i*(j*(h*x)^t
)^u]^4*Log[1 + (d*x)/c])/(4*t*u) - p*r*Log[i*(j*(h*x)^t)^u]^3*PolyLog[2, -(
(b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]^3*PolyLog[2, -((d*x)/c)] + 3*p*r*t*u*L
og[i*(j*(h*x)^t)^u]^2*PolyLog[3, -((b*x)/a)] + 3*q*r*t*u*Log[i*(j*(h*x)^t)^
u]^2*PolyLog[3, -((d*x)/c)] - 6*p*r*t^2*u^2*Log[i*(j*(h*x)^t)^u]*PolyLog[4,
 -((b*x)/a)] - 6*q*r*t^2*u^2*Log[i*(j*(h*x)^t)^u]*PolyLog[4, -((d*x)/c)] +
6*p*r*t^3*u^3*PolyLog[5, -((b*x)/a)] + 6*q*r*t^3*u^3*PolyLog[5, -((d*x)/c)]
```

Rubi [A] time = 1.25157, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2499, 2317, 2374, 2383, 6589, 2445}

$$-6prt^2u^2\text{PolyLog}\left(4,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)-pr\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log^3\left(i\left(j(hx)^t\right)^u\right)+3prtu\text{PolyLog}\left(3,-\frac{bx}{a}\right)\log^2\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Log[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```


```
[Out] -(p*r*Log[i*(j*(h*x)^t)^u]^4*Log[1 + (b*x)/a])/(4*t*u) + (Log[i*(j*(h*x)^t)
^u]^4*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(4*t*u) - (q*r*Log[i*(j*(h*x)^t
)^u]^4*Log[1 + (d*x)/c])/(4*t*u) - p*r*Log[i*(j*(h*x)^t)^u]^3*PolyLog[2, -(
(b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]^3*PolyLog[2, -((d*x)/c)] + 3*p*r*t*u*L
og[i*(j*(h*x)^t)^u]^2*PolyLog[3, -((b*x)/a)] + 3*q*r*t*u*Log[i*(j*(h*x)^t)^
u]^2*PolyLog[3, -((d*x)/c)] - 6*p*r*t^2*u^2*Log[i*(j*(h*x)^t)^u]*PolyLog[4,
 -((b*x)/a)] - 6*q*r*t^2*u^2*Log[i*(j*(h*x)^t)^u]*PolyLog[4, -((d*x)/c)] +
6*p*r*t^3*u^3*PolyLog[5, -((b*x)/a)] + 6*q*r*t^3*u^3*PolyLog[5, -((d*x)/c)]
```


Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int \frac{\log^3\left(56\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx = \text{Subst}\left(\int \frac{\log^3\left(56j^u(hx)^{tu}\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,d\right.$$

$$= \text{Subst}\left(\text{Subst}\left(\int \frac{\log^3\left(56h^{tu}j^ux^{tu}\right)\log\left(e\left(f(a+bx)^p(c+d\right.\right.}{x}\right.\right.$$

$$= \frac{\log^4\left(56\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{4tu} - \text{Subst}\left(\right.$$

$$= -\frac{pr\log^4\left(56\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(56\left(j(hx)^t\right)^u\right)\log}{}$$

$$= -\frac{pr\log^4\left(56\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(56\left(j(hx)^t\right)^u\right)\log}{}$$

$$= -\frac{pr\log^4\left(56\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(56\left(j(hx)^t\right)^u\right)\log}{}$$

$$= -\frac{pr\log^4\left(56\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(56\left(j(hx)^t\right)^u\right)\log}{}$$

$$= -\frac{pr\log^4\left(56\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{4tu} + \frac{\log^4\left(56\left(j(hx)^t\right)^u\right)\log}{}$$

Mathematica [B] time = 1.82702, size = 1241, normalized size = 3.78

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Log[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x
]
```

```
[Out] p*r*t^3*u^3*Log[x]*Log[h*x]^3*Log[a + b*x] - p*r*t^3*u^3*Log[h*x]^4*Log[a +
 b*x] - 3*p*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] +
 3*p*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] + 3*p*r*t*u*Log
[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[a + b*x] - 3*p*r*t*u*Log[h*x]^2*Log
[i*(j*(h*x)^t)^u]^2*Log[a + b*x] - p*r*Log[x]*Log[i*(j*(h*x)^t)^u]^3*Log[a
+ b*x] + p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[a + b*x] + (p*r*t^3*u^3*Lo
g[h*x]^4*Log[1 + (b*x)/a])/4 - p*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*
Log[1 + (b*x)/a] + (3*p*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (b*
x)/a])/2 - p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[1 + (b*x)/a] + q*r*t^3*u
^3*Log[x]*Log[h*x]^3*Log[c + d*x] - q*r*t^3*u^3*Log[h*x]^4*Log[c + d*x] - 3
*q*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] + 3*q*r*t^
2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] + 3*q*r*t*u*Log[x]*Log[h
*x]*Log[i*(j*(h*x)^t)^u]^2*Log[c + d*x] - 3*q*r*t*u*Log[h*x]^2*Log[i*(j*(h*
x)^t)^u]^2*Log[c + d*x] - q*r*Log[x]*Log[i*(j*(h*x)^t)^u]^3*Log[c + d*x] +
q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[c + d*x] - t^3*u^3*Log[x]*Log[h*x]^
3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (3*t^3*u^3*Log[h*x]^4*Log[e*(f*(a
+ b*x)^p*(c + d*x)^q)^r])/4 + 3*t^2*u^2*Log[x]*Log[h*x]^2*Log[i*(j*(h*x)^t)
^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 2*t^2*u^2*Log[h*x]^3*Log[i*(j*(h
```

```
*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - 3*t*u*Log[x]*Log[h*x]*Log[
i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (3*t*u*Log[h*x]^2
*Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/2 + Log[x]*Lo
g[i*(j*(h*x)^t)^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (q*r*t^3*u^3*Lo
g[h*x]^4*Log[1 + (d*x)/c])/4 - q*r*t^2*u^2*Log[h*x]^3*Log[i*(j*(h*x)^t)^u]*
Log[1 + (d*x)/c] + (3*q*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (d*
x)/c])/2 - q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^3*Log[1 + (d*x)/c] - p*r*Log[i
*(j*(h*x)^t)^u]^3*PolyLog[2, -((b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]^3*PolyL
og[2, -((d*x)/c)] + 3*p*r*t*u*Log[i*(j*(h*x)^t)^u]^2*PolyLog[3, -((b*x)/a)]
 + 3*q*r*t*u*Log[i*(j*(h*x)^t)^u]^2*PolyLog[3, -((d*x)/c)] - 6*p*r*t^2*u^2*
Log[i*(j*(h*x)^t)^u]*PolyLog[4, -((b*x)/a)] - 6*q*r*t^2*u^2*Log[i*(j*(h*x)^
t)^u]*PolyLog[4, -((d*x)/c)] + 6*p*r*t^3*u^3*PolyLog[5, -((b*x)/a)] + 6*q*r
*t^3*u^3*PolyLog[5, -((d*x)/c)]
```

Maple [F] time = 2.213, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(i\left(j\,(hx)^{t}\right)^{u}\right)\right)^{3}\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(i*(j*(h*x)^t)^u)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

```
[Out] int(ln(i*(j*(h*x)^t)^u)^3*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="maxima")
```

```
[Out] -1/4*(t^3*u^3*log(x)^4 - 4*(t^2*u^2*log((h^t)^u) + t^2*u^2*log(i) + t^2*u^2
*log(j^u))*log(x)^3 - 4*log(x)*log((x^t)^u)^3 + 6*(t*u*log((h^t)^u)^2 + t*u
*log(i)^2 + 2*t*u*log(i)*log(j^u) + t*u*log(j^u)^2 + 2*(t*u*log(i) + t*u*lo
g(j^u))*log((h^t)^u))*log(x)^2 + 6*(t*u*log(x)^2 - 2*(log((h^t)^u) + log(i)
 + log(j^u))*log(x))*log((x^t)^u)^2 - 4*(t^2*u^2*log(x)^3 - 3*(t*u*log((h^t
)^u) + t*u*log(i) + t*u*log(j^u))*log(x)^2 + 3*(2*(log(i) + log(j^u))*log((
h^t)^u) + log((h^t)^u)^2 + log(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log(x
))*log((x^t)^u) - 4*(3*(log(i) + log(j^u))*log((h^t)^u)^2 + log((h^t)^u)^3
+ log(i)^3 + 3*log(i)^2*log(j^u) + 3*log(i)*log(j^u)^2 + log(j^u)^3 + 3*(lo
g(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log((h^t)^u))*log(x))*log(((b*x +
a)^p)^r) - 1/4*(t^3*u^3*log(x)^4 - 4*(t^2*u^2*log((h^t)^u) + t^2*u^2*log(i)
 + t^2*u^2*log(j^u))*log(x)^3 - 4*log(x)*log((x^t)^u)^3 + 6*(t*u*log((h^t)^
u)^2 + t*u*log(i)^2 + 2*t*u*log(i)*log(j^u) + t*u*log(j^u)^2 + 2*(t*u*log(i
) + t*u*log(j^u))*log((h^t)^u))*log(x)^2 + 6*(t*u*log(x)^2 - 2*(log((h^t)^u
) + log(i) + log(j^u))*log(x))*log((x^t)^u)^2 - 4*(t^2*u^2*log(x)^3 - 3*(t*
u*log((h^t)^u) + t*u*log(i) + t*u*log(j^u))*log(x)^2 + 3*(2*(log(i) + log(j
^u))*log((h^t)^u) + log((h^t)^u)^2 + log(i)^2 + 2*log(i)*log(j^u) + log(j^u
)^2)*log(x))*log((x^t)^u) - 4*(3*(log(i) + log(j^u))*log((h^t)^u)^2 + log((
h^t)^u)^3 + log(i)^3 + 3*log(i)^2*log(j^u) + 3*log(i)*log(j^u)^2 + log(j^u)
^3 + 3*(log(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log((h^t)^u))*log(x))*lo
```

```
g(((d*x + c)^q)^r) - integrate(-1/4*(4*((3*(log(i) + log(j^u))*log((h^t)^u)
^2 + log((h^t)^u)^3 + log(i)^3 + 3*log(i)^2*log(j^u) + 3*log(i)*log(j^u)^2
+ log(j^u)^3 + 3*(log(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log((h^t)^u))*
log(e) + (3*(log(i) + log(j^u))*log((h^t)^u)^2 + log((h^t)^u)^3 + log(i)^3
+ 3*log(i)^2*log(j^u) + 3*log(i)*log(j^u)^2 + log(j^u)^3 + 3*(log(i)^2 + 2*
log(i)*log(j^u) + log(j^u)^2)*log((h^t)^u))*log(f^r))*b*d*x^2 + ((p*r*t^3*u
^3 + q*r*t^3*u^3)*b*d*x^2 + (b*c*p*r*t^3*u^3 + a*d*q*r*t^3*u^3)*x)*log(x)^4
 - 4*(((p*r*t^2*u^2 + q*r*t^2*u^2)*log((h^t)^u) + (p*r*t^2*u^2 + q*r*t^2*u^
2)*log(i) + (p*r*t^2*u^2 + q*r*t^2*u^2)*log(j^u))*b*d*x^2 + ((p*r*t^2*u^2*l
og((h^t)^u) + p*r*t^2*u^2*log(i) + p*r*t^2*u^2*log(j^u))*b*c + (q*r*t^2*u^2
*log((h^t)^u) + q*r*t^2*u^2*log(i) + q*r*t^2*u^2*log(j^u))*a*d)*x)*log(x)^3
 + 4*(b*d*x^2*(log(e) + log(f^r)) + a*c*(log(e) + log(f^r)) + (b*c*(log(e)
+ log(f^r)) + a*d*(log(e) + log(f^r)))*x - ((p*r + q*r)*b*d*x^2 + (b*c*p*r
+ a*d*q*r)*x)*log(x))*log((x^t)^u)^3 + 4*((3*(log(i) + log(j^u))*log((h^t)^
u)^2 + log((h^t)^u)^3 + log(i)^3 + 3*log(i)^2*log(j^u) + 3*log(i)*log(j^u)^
2 + log(j^u)^3 + 3*(log(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log((h^t)^u)
)*log(e) + (3*(log(i) + log(j^u))*log((h^t)^u)^2 + log((h^t)^u)^3 + log(i)^
3 + 3*log(i)^2*log(j^u) + 3*log(i)*log(j^u)^2 + log(j^u)^3 + 3*(log(i)^2 +
2*log(i)*log(j^u) + log(j^u)^2)*log((h^t)^u))*log(f^r))*a*c + 6*(((p*r*t*u
+ q*r*t*u)*log((h^t)^u)^2 + (p*r*t*u + q*r*t*u)*log(i)^2 + 2*(p*r*t*u + q*r
*t*u)*log(i)*log(j^u) + (p*r*t*u + q*r*t*u)*log(j^u)^2 + 2*((p*r*t*u + q*r*
t*u)*log(i) + (p*r*t*u + q*r*t*u)*log(j^u))*log((h^t)^u))*b*d*x^2 + ((p*r*t
*u*log((h^t)^u)^2 + p*r*t*u*log(i)^2 + 2*p*r*t*u*log(i)*log(j^u) + p*r*t*u*
log(j^u)^2 + 2*(p*r*t*u*log(i) + p*r*t*u*log(j^u))*log((h^t)^u))*b*c + (q*r
*t*u*log((h^t)^u)^2 + q*r*t*u*log(i)^2 + 2*q*r*t*u*log(i)*log(j^u) + q*r*t*
u*log(j^u)^2 + 2*(q*r*t*u*log(i) + q*r*t*u*log(j^u))*log((h^t)^u))*a*d)*x)*
log(x)^2 + 6*(2*((log((h^t)^u) + log(i) + log(j^u))*log(e) + (log((h^t)^u)
+ log(i) + log(j^u))*log(f^r))*b*d*x^2 + 2*((log((h^t)^u) + log(i) + log(j^
u))*log(e) + (log((h^t)^u) + log(i) + log(j^u))*log(f^r))*a*c + ((p*r*t*u +
 q*r*t*u)*b*d*x^2 + (b*c*p*r*t*u + a*d*q*r*t*u)*x)*log(x)^2 + 2*(((log((h^t
)^u) + log(i) + log(j^u))*log(e) + (log((h^t)^u) + log(i) + log(j^u))*log(f
^r))*b*c + ((log((h^t)^u) + log(i) + log(j^u))*log(e) + (log((h^t)^u) + log
(i) + log(j^u))*log(f^r))*a*d)*x - 2*(((p*r + q*r)*log((h^t)^u) + (p*r + q*
r)*log(i) + (p*r + q*r)*log(j^u))*b*d*x^2 + ((p*r*log((h^t)^u) + p*r*log(i)
 + p*r*log(j^u))*b*c + (q*r*log((h^t)^u) + q*r*log(i) + q*r*log(j^u))*a*d)*
x)*log(x))*log((x^t)^u)^2 + 4*(((3*(log(i) + log(j^u))*log((h^t)^u)^2 + log
((h^t)^u)^3 + log(i)^3 + 3*log(i)^2*log(j^u) + 3*log(i)*log(j^u)^2 + log(j^
u)^3 + 3*(log(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log((h^t)^u))*log(e) +
 (3*(log(i) + log(j^u))*log((h^t)^u)^2 + log((h^t)^u)^3 + log(i)^3 + 3*log(
i)^2*log(j^u) + 3*log(i)*log(j^u)^2 + log(j^u)^3 + 3*(log(i)^2 + 2*log(i)*l
og(j^u) + log(j^u)^2)*log((h^t)^u))*log(f^r))*b*c + ((3*(log(i) + log(j^u))
*log((h^t)^u)^2 + log((h^t)^u)^3 + log(i)^3 + 3*log(i)^2*log(j^u) + 3*log(i
)*log(j^u)^2 + log(j^u)^3 + 3*(log(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*l
og((h^t)^u))*log(e) + (3*(log(i) + log(j^u))*log((h^t)^u)^2 + log((h^t)^u)^
3 + log(i)^3 + 3*log(i)^2*log(j^u) + 3*log(i)*log(j^u)^2 + log(j^u)^3 + 3*(
log(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log((h^t)^u))*log(f^r))*a*d)*x -
 4*(((p*r + q*r)*log((h^t)^u)^3 + (p*r + q*r)*log(i)^3 + 3*(p*r + q*r)*log(
i)^2*log(j^u) + 3*(p*r + q*r)*log(i)*log(j^u)^2 + (p*r + q*r)*log(j^u)^3 +
3*((p*r + q*r)*log(i) + (p*r + q*r)*log(j^u))*log((h^t)^u)^2 + 3*((p*r + q*
r)*log(i)^2 + 2*(p*r + q*r)*log(i)*log(j^u) + (p*r + q*r)*log(j^u)^2)*log((
h^t)^u))*b*d*x^2 + ((p*r*log((h^t)^u)^3 + p*r*log(i)^3 + 3*p*r*log(i)^2*log
(j^u) + 3*p*r*log(i)*log(j^u)^2 + p*r*log(j^u)^3 + 3*(p*r*log(i) + p*r*log(
j^u))*log((h^t)^u)^2 + 3*(p*r*log(i)^2 + 2*p*r*log(i)*log(j^u) + p*r*log(j^
u)^2)*log((h^t)^u))*b*c + (q*r*log((h^t)^u)^3 + q*r*log(i)^3 + 3*q*r*log(i)
^2*log(j^u) + 3*q*r*log(i)*log(j^u)^2 + q*r*log(j^u)^3 + 3*(q*r*log(i) + q*
r*log(j^u))*log((h^t)^u)^2 + 3*(q*r*log(i)^2 + 2*q*r*log(i)*log(j^u) + q*r*
log(j^u)^2)*log((h^t)^u))*a*d)*x)*log(x) + 4*(3*((2*(log(i) + log(j^u))*log
((h^t)^u) + log((h^t)^u)^2 + log(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log
(e) + (2*(log(i) + log(j^u))*log((h^t)^u) + log((h^t)^u)^2 + log(i)^2 + 2*l
```

```
og(i)*log(j^u) + log(j^u)^2)*log(f^r))*b*d*x^2 - ((p*r*t^2*u^2 + q*r*t^2*u^
2)*b*d*x^2 + (b*c*p*r*t^2*u^2 + a*d*q*r*t^2*u^2)*x)*log(x)^3 + 3*((2*(log(i
) + log(j^u))*log((h^t)^u) + log((h^t)^u)^2 + log(i)^2 + 2*log(i)*log(j^u)
+ log(j^u)^2)*log(e) + (2*(log(i) + log(j^u))*log((h^t)^u) + log((h^t)^u)^2
 + log(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log(f^r))*a*c + 3*(((p*r*t*u
+ q*r*t*u)*log((h^t)^u) + (p*r*t*u + q*r*t*u)*log(i) + (p*r*t*u + q*r*t*u)*
log(j^u))*b*d*x^2 + ((p*r*t*u*log((h^t)^u) + p*r*t*u*log(i) + p*r*t*u*log(j
^u))*b*c + (q*r*t*u*log((h^t)^u) + q*r*t*u*log(i) + q*r*t*u*log(j^u))*a*d)*
x)*log(x)^2 + 3*(((2*(log(i) + log(j^u))*log((h^t)^u) + log((h^t)^u)^2 + lo
g(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log(e) + (2*(log(i) + log(j^u))*lo
g((h^t)^u) + log((h^t)^u)^2 + log(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*lo
g(f^r))*b*c + ((2*(log(i) + log(j^u))*log((h^t)^u) + log((h^t)^u)^2 + log(i
)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log(e) + (2*(log(i) + log(j^u))*log((
h^t)^u) + log((h^t)^u)^2 + log(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log(f
^r))*a*d)*x - 3*(((p*r + q*r)*log((h^t)^u)^2 + (p*r + q*r)*log(i)^2 + 2*(p*
r + q*r)*log(i)*log(j^u) + (p*r + q*r)*log(j^u)^2 + 2*((p*r + q*r)*log(i) +
 (p*r + q*r)*log(j^u))*log((h^t)^u))*b*d*x^2 + ((p*r*log((h^t)^u)^2 + p*r*l
og(i)^2 + 2*p*r*log(i)*log(j^u) + p*r*log(j^u)^2 + 2*(p*r*log(i) + p*r*log(
j^u))*log((h^t)^u))*b*c + (q*r*log((h^t)^u)^2 + q*r*log(i)^2 + 2*q*r*log(i)
*log(j^u) + q*r*log(j^u)^2 + 2*(q*r*log(i) + q*r*log(j^u))*log((h^t)^u))*a*
d)*x)*log(x))*log((x^t)^u))/(b*d*x^3 + a*c*x + (b*c + a*d)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)^3/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(i*(j*(h*x)**t)**u)**3*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^3*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)^3/x, x)
```

3.57 $\int \frac{\log^2\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx$

Optimal. Leaf size=262

$$-pr\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log^2\left(i\left(j(hx)^t\right)^u\right)+2prtu\text{PolyLog}\left(3,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)-2prt^2u^2\text{PolyLog}\left(4,-\frac{bx}{a}\right)-qr\text{Poly}$$

```
[Out] -(p*r*Log[i*(j*(h*x)^t)^u]^3*Log[1 + (b*x)/a])/(3*t*u) + (Log[i*(j*(h*x)^t)
^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*t*u) - (q*r*Log[i*(j*(h*x)^t
)^u]^3*Log[1 + (d*x)/c])/(3*t*u) - p*r*Log[i*(j*(h*x)^t)^u]^2*PolyLog[2, -(
(b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]^2*PolyLog[2, -((d*x)/c)] + 2*p*r*t*u*L
og[i*(j*(h*x)^t)^u]*PolyLog[3, -((b*x)/a)] + 2*q*r*t*u*Log[i*(j*(h*x)^t)^u]
*PolyLog[3, -((d*x)/c)] - 2*p*r*t^2*u^2*PolyLog[4, -((b*x)/a)] - 2*q*r*t^2*
u^2*PolyLog[4, -((d*x)/c)]
```

Rubi [A] time = 0.910214, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2499, 2317, 2374, 2383, 6589, 2445}

$$-pr\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log^2\left(i\left(j(hx)^t\right)^u\right)+2prtu\text{PolyLog}\left(3,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)-2prt^2u^2\text{PolyLog}\left(4,-\frac{bx}{a}\right)-qr\text{Poly}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```

```
[Out] -(p*r*Log[i*(j*(h*x)^t)^u]^3*Log[1 + (b*x)/a])/(3*t*u) + (Log[i*(j*(h*x)^t)
^u]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(3*t*u) - (q*r*Log[i*(j*(h*x)^t
)^u]^3*Log[1 + (d*x)/c])/(3*t*u) - p*r*Log[i*(j*(h*x)^t)^u]^2*PolyLog[2, -(
(b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]^2*PolyLog[2, -((d*x)/c)] + 2*p*r*t*u*L
og[i*(j*(h*x)^t)^u]*PolyLog[3, -((b*x)/a)] + 2*q*r*t*u*Log[i*(j*(h*x)^t)^u]
*PolyLog[3, -((d*x)/c)] - 2*p*r*t^2*u^2*PolyLog[4, -((b*x)/a)] - 2*q*r*t^2*
u^2*PolyLog[4, -((d*x)/c)]
```

Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 2383

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^p)/q
, x] - Dist[(b*n*p)/q, Int[(PolyLog[k + 1, e*x^q]*(a + b*Log[c*x^n])^(p - 1
))/x, x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int \frac{\log^2\left(57\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx = \text{Subst}\left(\int \frac{\log^2\left(57j^u(hx)^{tu}\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,d\right.$$

$$= \text{Subst}\left(\text{Subst}\left(\int \frac{\log^2\left(57h^{tu}j^u x^{tu}\right)\log\left(e\left(f(a+bx)^p(c+d\right.\right.}{x}\right.\right.$$

$$= \frac{\log^3\left(57\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{3tu} - \text{Subst}\left(\right.$$

$$= -\frac{pr\log^3\left(57\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu} + \frac{\log^3\left(57\left(j(hx)^t\right)^u\right)\log}{}$$

$$= -\frac{pr\log^3\left(57\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu} + \frac{\log^3\left(57\left(j(hx)^t\right)^u\right)\log}{}$$

$$= -\frac{pr\log^3\left(57\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu} + \frac{\log^3\left(57\left(j(hx)^t\right)^u\right)\log}{}$$

$$= -\frac{pr\log^3\left(57\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{3tu} + \frac{\log^3\left(57\left(j(hx)^t\right)^u\right)\log}{}$$

Mathematica [B] time = 0.913282, size = 839, normalized size = 3.2

$$prt^2u^2\log(a+bx)\log^3(hx) - \frac{1}{3}prt^2u^2\log\left(\frac{bx}{a}+1\right)\log^3(hx) + qrt^2u^2\log(c+dx)\log^3(hx) - \frac{2}{3}t^2u^2\log\left(e\left(f(a+bx)^p(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x
]
```

```
[Out] -(p*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[a + b*x]) + p*r*t^2*u^2*Log[h*x]^3*Log[
a + b*x] + 2*p*r*t*u*Log[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] - 2*
p*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] - p*r*Log[x]*Log[i*(j*
(h*x)^t)^u]^2*Log[a + b*x] + p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[a + b*
x] - (p*r*t^2*u^2*Log[h*x]^3*Log[1 + (b*x)/a])/3 + p*r*t*u*Log[h*x]^2*Log[i
*(j*(h*x)^t)^u]*Log[1 + (b*x)/a] - p*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[
1 + (b*x)/a] - q*r*t^2*u^2*Log[x]*Log[h*x]^2*Log[c + d*x] + q*r*t^2*u^2*Log
[h*x]^3*Log[c + d*x] + 2*q*r*t*u*Log[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[c
 + d*x] - 2*q*r*t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] - q*r*Log[
x]*Log[i*(j*(h*x)^t)^u]^2*Log[c + d*x] + q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^
2*Log[c + d*x] + t^2*u^2*Log[x]*Log[h*x]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q
)^r] - (2*t^2*u^2*Log[h*x]^3*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/3 - 2*t*
u*Log[x]*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
 + t*u*Log[h*x]^2*Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]
 + Log[x]*Log[i*(j*(h*x)^t)^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - (q*
r*t^2*u^2*Log[h*x]^3*Log[1 + (d*x)/c])/3 + q*r*t*u*Log[h*x]^2*Log[i*(j*(h*x
)^t)^u]*Log[1 + (d*x)/c] - q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (d*x
)/c] - p*r*Log[i*(j*(h*x)^t)^u]^2*PolyLog[2, -((b*x)/a)] - q*r*Log[i*(j*(h*
x)^t)^u]^2*PolyLog[2, -((d*x)/c)] + 2*p*r*t*u*Log[i*(j*(h*x)^t)^u]*PolyLog[
3, -((b*x)/a)] + 2*q*r*t*u*Log[i*(j*(h*x)^t)^u]*PolyLog[3, -((d*x)/c)] - 2*
p*r*t^2*u^2*PolyLog[4, -((b*x)/a)] - 2*q*r*t^2*u^2*PolyLog[4, -((d*x)/c)]
```

Maple [F] time = 1.8, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(i\left(j\,(hx)^{t}\right)^{u}\right)\right)^{2}\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(i*(j*(h*x)^t)^u)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

```
[Out] int(ln(i*(j*(h*x)^t)^u)^2*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="maxima")
```

```
[Out] 1/3*(t^2*u^2*log(x)^3 - 3*(t*u*log((h^t)^u) + t*u*log(i) + t*u*log(j^u))*lo
g(x)^2 + 3*log(x)*log((x^t)^u)^2 - 3*(t*u*log(x)^2 - 2*(log((h^t)^u) + log(
i) + log(j^u))*log(x))*log((x^t)^u) + 3*(2*(log(i) + log(j^u))*log((h^t)^u)
 + log((h^t)^u)^2 + log(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log(x))*log(
((b*x + a)^p)^r) + 1/3*(t^2*u^2*log(x)^3 - 3*(t*u*log((h^t)^u) + t*u*log(i)
 + t*u*log(j^u))*log(x)^2 + 3*log(x)*log((x^t)^u)^2 - 3*(t*u*log(x)^2 - 2*(
```

```
log((h^t)^u) + log(i) + log(j^u))*log(x))*log((x^t)^u) + 3*(2*(log(i) + log
(j^u))*log((h^t)^u) + log((h^t)^u)^2 + log(i)^2 + 2*log(i)*log(j^u) + log(j
^u)^2)*log(x))*log(((d*x + c)^q)^r) - integrate(-1/3*(3*((2*(log(i) + log(j
^u))*log((h^t)^u) + log((h^t)^u)^2 + log(i)^2 + 2*log(i)*log(j^u) + log(j^u
)^2)*log(e) + (2*(log(i) + log(j^u))*log((h^t)^u) + log((h^t)^u)^2 + log(i)
^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log(f^r))*b*d*x^2 - ((p*r*t^2*u^2 + q*
r*t^2*u^2)*b*d*x^2 + (b*c*p*r*t^2*u^2 + a*d*q*r*t^2*u^2)*x)*log(x)^3 + 3*((
2*(log(i) + log(j^u))*log((h^t)^u) + log((h^t)^u)^2 + log(i)^2 + 2*log(i)*l
og(j^u) + log(j^u)^2)*log(e) + (2*(log(i) + log(j^u))*log((h^t)^u) + log((h
^t)^u)^2 + log(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log(f^r))*a*c + 3*(((
p*r*t*u + q*r*t*u)*log((h^t)^u) + (p*r*t*u + q*r*t*u)*log(i) + (p*r*t*u + q
*r*t*u)*log(j^u))*b*d*x^2 + ((p*r*t*u*log((h^t)^u) + p*r*t*u*log(i) + p*r*t
*u*log(j^u))*b*c + (q*r*t*u*log((h^t)^u) + q*r*t*u*log(i) + q*r*t*u*log(j^u
))*a*d)*x)*log(x)^2 + 3*(b*d*x^2*(log(e) + log(f^r)) + a*c*(log(e) + log(f^
r)) + (b*c*(log(e) + log(f^r)) + a*d*(log(e) + log(f^r)))*x - ((p*r + q*r)*
b*d*x^2 + (b*c*p*r + a*d*q*r)*x)*log(x))*log((x^t)^u)^2 + 3*(((2*(log(i) +
log(j^u))*log((h^t)^u) + log((h^t)^u)^2 + log(i)^2 + 2*log(i)*log(j^u) + lo
g(j^u)^2)*log(e) + (2*(log(i) + log(j^u))*log((h^t)^u) + log((h^t)^u)^2 + l
og(i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log(f^r))*b*c + ((2*(log(i) + log
(j^u))*log((h^t)^u) + log((h^t)^u)^2 + log(i)^2 + 2*log(i)*log(j^u) + log(j
^u)^2)*log(e) + (2*(log(i) + log(j^u))*log((h^t)^u) + log((h^t)^u)^2 + log(
i)^2 + 2*log(i)*log(j^u) + log(j^u)^2)*log(f^r))*a*d)*x - 3*(((p*r + q*r)*l
og((h^t)^u)^2 + (p*r + q*r)*log(i)^2 + 2*(p*r + q*r)*log(i)*log(j^u) + (p*r
 + q*r)*log(j^u)^2 + 2*((p*r + q*r)*log(i) + (p*r + q*r)*log(j^u))*log((h^t
)^u))*b*d*x^2 + ((p*r*log((h^t)^u)^2 + p*r*log(i)^2 + 2*p*r*log(i)*log(j^u)
 + p*r*log(j^u)^2 + 2*(p*r*log(i) + p*r*log(j^u))*log((h^t)^u))*b*c + (q*r*
log((h^t)^u)^2 + q*r*log(i)^2 + 2*q*r*log(i)*log(j^u) + q*r*log(j^u)^2 + 2*
(q*r*log(i) + q*r*log(j^u))*log((h^t)^u))*a*d)*x)*log(x) + 3*(2*((log((h^t)
^u) + log(i) + log(j^u))*log(e) + (log((h^t)^u) + log(i) + log(j^u))*log(f^
r))*b*d*x^2 + 2*((log((h^t)^u) + log(i) + log(j^u))*log(e) + (log((h^t)^u)
+ log(i) + log(j^u))*log(f^r))*a*c + ((p*r*t*u + q*r*t*u)*b*d*x^2 + (b*c*p*
r*t*u + a*d*q*r*t*u)*x)*log(x)^2 + 2*(((log((h^t)^u) + log(i) + log(j^u))*l
og(e) + (log((h^t)^u) + log(i) + log(j^u))*log(f^r))*b*c + ((log((h^t)^u) +
 log(i) + log(j^u))*log(e) + (log((h^t)^u) + log(i) + log(j^u))*log(f^r))*a
*d)*x - 2*(((p*r + q*r)*log((h^t)^u) + (p*r + q*r)*log(i) + (p*r + q*r)*log
(j^u))*b*d*x^2 + ((p*r*log((h^t)^u) + p*r*log(i) + p*r*log(j^u))*b*c + (q*r
*log((h^t)^u) + q*r*log(i) + q*r*log(j^u))*a*d)*x)*log(x))*log((x^t)^u))/(b
*d*x^3 + a*c*x + (b*c + a*d)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)^2/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(i*(j*(h*x)**t)**u)**2*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)\log\left(\left((hx)^{t}j\right)^{u}i\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)^2*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algo
rithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)^2/x, x)
```

3.58 $\int \frac{\log\left(i\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx$

Optimal. Leaf size=194

$$-pr\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)+prtu\text{PolyLog}\left(3,-\frac{bx}{a}\right)-qr\text{PolyLog}\left(2,-\frac{dx}{c}\right)\log\left(i\left(j(hx)^t\right)^u\right)+qrtu\text{PolyL}$$

```
[Out] -(p*r*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (b*x)/a])/(2*t*u) + (Log[i*(j*(h*x)^t)
^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*t*u) - (q*r*Log[i*(j*(h*x)^t
)^u]^2*Log[1 + (d*x)/c])/(2*t*u) - p*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((b
*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((d*x)/c)] + p*r*t*u*PolyLog
[3, -((b*x)/a)] + q*r*t*u*PolyLog[3, -((d*x)/c)]
```

Rubi [A] time = 0.603361, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2499, 2317, 2374, 6589, 2445}

$$-pr\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)+prtu\text{PolyLog}\left(3,-\frac{bx}{a}\right)-qr\text{PolyLog}\left(2,-\frac{dx}{c}\right)\log\left(i\left(j(hx)^t\right)^u\right)+qrtu\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```



```
[Out] -(p*r*Log[i*(j*(h*x)^t)^u]^2*Log[1 + (b*x)/a])/(2*t*u) + (Log[i*(j*(h*x)^t)
^u]^2*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(2*t*u) - (q*r*Log[i*(j*(h*x)^t
)^u]^2*Log[1 + (d*x)/c])/(2*t*u) - p*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((b
*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((d*x)/c)] + p*r*t*u*PolyLog
[3, -((b*x)/a)] + q*r*t*u*PolyLog[3, -((d*x)/c)]
```


Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_)), x_Symbol] :> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
```

```
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_.)*((e_.) + (f_.)*(x_))^(m_.))^(n_)]*(b_.))^(p_.
)*(u_.), x_Symbol] :> Subst[Int[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x],
 c*d^n*(e + f*x)^(m*n), c*(d*(e + f*x)^m)^n] /; FreeQ[{a, b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[n] &&  !(EqQ[d, 1] && EqQ[m, 1]) && IntegralFreeQ[
IntHide[u*(a + b*Log[c*d^n*(e + f*x)^(m*n)])^p, x]]
```

Rubi steps

$$\int \frac{\log\left(58\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx = \text{Subst}\left(\int \frac{\log\left(58j^u(hx)^{tu}\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx, 58j\right.$$

$$= \text{Subst}\left(\text{Subst}\left(\int \frac{\log\left(58h^{tu}j^u x^{tu}\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\right.\right.$$

$$= \frac{\log^2\left(58\left(j(hx)^t\right)^u\right)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{2tu} - \text{Subst}\left(\text{Subs}\right.$$

$$= -\frac{pr\log^2\left(58\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{2tu} + \frac{\log^2\left(58\left(j(hx)^t\right)^u\right)\log\left(e\right.}{2tu}$$

$$= -\frac{pr\log^2\left(58\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{2tu} + \frac{\log^2\left(58\left(j(hx)^t\right)^u\right)\log\left(e\right.}{2tu}$$

$$= -\frac{pr\log^2\left(58\left(j(hx)^t\right)^u\right)\log\left(1+\frac{bx}{a}\right)}{2tu} + \frac{\log^2\left(58\left(j(hx)^t\right)^u\right)\log\left(e\right.}{2tu}$$

Mathematica [B] time = 0.448249, size = 451, normalized size = 2.32

$$-pr\text{PolyLog}\left(2,-\frac{bx}{a}\right)\log\left(i\left(j(hx)^t\right)^u\right)+prtu\text{PolyLog}\left(3,-\frac{bx}{a}\right)-qr\text{PolyLog}\left(2,-\frac{dx}{c}\right)\log\left(i\left(j(hx)^t\right)^u\right)+qrtu\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/x,x]
```

```
[Out] p*r*t*u*Log[x]*Log[h*x]*Log[a + b*x] - p*r*t*u*Log[h*x]^2*Log[a + b*x] - p*
r*Log[x]*Log[i*(j*(h*x)^t)^u]*Log[a + b*x] + p*r*Log[h*x]*Log[i*(j*(h*x)^t)
^u]*Log[a + b*x] + (p*r*t*u*Log[h*x]^2*Log[1 + (b*x)/a])/2 - p*r*Log[h*x]*L
og[i*(j*(h*x)^t)^u]*Log[1 + (b*x)/a] + q*r*t*u*Log[x]*Log[h*x]*Log[c + d*x]
 - q*r*t*u*Log[h*x]^2*Log[c + d*x] - q*r*Log[x]*Log[i*(j*(h*x)^t)^u]*Log[c
+ d*x] + q*r*Log[h*x]*Log[i*(j*(h*x)^t)^u]*Log[c + d*x] - t*u*Log[x]*Log[h*
x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] + (t*u*Log[h*x]^2*Log[e*(f*(a + b*x
```

```
)^p*(c + d*x)^q)^r])/2 + Log[x]*Log[i*(j*(h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(
c + d*x)^q)^r] + (q*r*t*u*Log[h*x]^2*Log[1 + (d*x)/c])/2 - q*r*Log[h*x]*Log
[i*(j*(h*x)^t)^u]*Log[1 + (d*x)/c] - p*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -(
(b*x)/a)] - q*r*Log[i*(j*(h*x)^t)^u]*PolyLog[2, -((d*x)/c)] + p*r*t*u*PolyL
og[3, -((b*x)/a)] + q*r*t*u*PolyLog[3, -((d*x)/c)]
```

Maple [F] time = 1.784, size = 0, normalized size = 0.

$$\int \frac{\ln\left(i\left(j\,(hx)^{t}\right)^{u}\right)\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(i*(j*(h*x)^t)^u)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

```
[Out] int(ln(i*(j*(h*x)^t)^u)*ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algori
thm="maxima")
```

```
[Out] -1/2*(t*u*log(x)^2 - 2*(log((h^t)^u) + log(i) + log(j^u))*log(x) - 2*log(x)
*log((x^t)^u))*log(((b*x + a)^p)^r) - 1/2*(t*u*log(x)^2 - 2*(log((h^t)^u) +
 log(i) + log(j^u))*log(x) - 2*log(x)*log((x^t)^u))*log(((d*x + c)^q)^r) -
integrate(-1/2*(2*((log((h^t)^u) + log(i) + log(j^u))*log(e) + (log((h^t)^u
) + log(i) + log(j^u))*log(f^r))*b*d*x^2 + 2*((log((h^t)^u) + log(i) + log(
j^u))*log(e) + (log((h^t)^u) + log(i) + log(j^u))*log(f^r))*a*c + ((p*r*t*u
 + q*r*t*u)*b*d*x^2 + (b*c*p*r*t*u + a*d*q*r*t*u)*x)*log(x)^2 + 2*(((log((h
^t)^u) + log(i) + log(j^u))*log(e) + (log((h^t)^u) + log(i) + log(j^u))*log
(f^r))*b*c + ((log((h^t)^u) + log(i) + log(j^u))*log(e) + (log((h^t)^u) + l
og(i) + log(j^u))*log(f^r))*a*d)*x - 2*(((p*r + q*r)*log((h^t)^u) + (p*r +
q*r)*log(i) + (p*r + q*r)*log(j^u))*b*d*x^2 + ((p*r*log((h^t)^u) + p*r*log(
i) + p*r*log(j^u))*b*c + (q*r*log((h^t)^u) + q*r*log(i) + q*r*log(j^u))*a*d
)*x)*log(x) + 2*(b*d*x^2*(log(e) + log(f^r)) + a*c*(log(e) + log(f^r)) + (b
*c*(log(e) + log(f^r)) + a*d*(log(e) + log(f^r)))*x - ((p*r + q*r)*b*d*x^2
+ (b*c*p*r + a*d*q*r)*x)*log(x))*log((x^t)^u))/(b*d*x^3 + a*c*x + (b*c + a*
d)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)\log\left(\left((hx)^{t}j\right)^{u}i\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algori
thm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(i*(j*(h*x)**t)**u)*ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\log\left(\left((hx)^t j\right)^u i\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(i*(j*(h*x)^t)^u)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algori
thm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)*log(((h*x)^t*j)^u*i)/x, x)
```

3.59 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx$

Optimal. Leaf size=81

$$-pr\text{PolyLog}\left(2,-\frac{bx}{a}\right)-qr\text{PolyLog}\left(2,-\frac{dx}{c}\right)+\log(x)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)-pr\log(x)\log\left(\frac{bx}{a}+1\right)-qr\log$$

```
[Out] -(p*r*Log[x]*Log[1 + (b*x)/a]) + Log[x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
] - q*r*Log[x]*Log[1 + (d*x)/c] - p*r*PolyLog[2, -((b*x)/a)] - q*r*PolyLog[
2, -((d*x)/c)]
```

Rubi [A] time = 0.0657538, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2494, 2317, 2391}

$$-pr\text{PolyLog}\left(2,-\frac{bx}{a}\right)-qr\text{PolyLog}\left(2,-\frac{dx}{c}\right)+\log(x)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)-pr\log(x)\log\left(\frac{bx}{a}+1\right)-qr\log$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/x,x]
```

```
[Out] -(p*r*Log[x]*Log[1 + (b*x)/a]) + Log[x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r
] - q*r*Log[x]*Log[1 + (d*x)/c] - p*r*PolyLog[2, -((b*x)/a)] - q*r*PolyLog[
2, -((d*x)/c)]
```

Rule 2494

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(Log[g + h*x]*Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*
x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{
a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x}\,dx = \log(x)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - (bpr)\int \frac{\log(x)}{a+bx}\,dx - (dqr)\int \frac{\log(x)}{c+dx}\,dx$$

$$= -pr\log(x)\log\left(1+\frac{bx}{a}\right) + \log(x)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - qr\log(x)\log\left(1+\right.$$

$$= -pr\log(x)\log\left(1+\frac{bx}{a}\right) + \log(x)\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - qr\log(x)\log\left(1+\right.$$

Mathematica [A] time = 0.0630908, size = 78, normalized size = 0.96

$$-pr\text{PolyLog}\left(2,-\frac{bx}{a}\right) - qr\text{PolyLog}\left(2,-\frac{dx}{c}\right) + \log(x)\left(\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right) - pr\log\left(\frac{bx}{a}+1\right) - qr\log\left(\frac{dx}{c}+1\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/x,x]
```

```
[Out] Log[x]*(-(p*r*Log[1 + (b*x)/a]) + Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - q*
r*Log[1 + (d*x)/c]) - p*r*PolyLog[2, -((b*x)/a)] - q*r*PolyLog[2, -((d*x)/c
)]
```

Maple [F] time = 0.123, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f\,(bx+a)^p\,(dx+c)^q\right)^r\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x)
```

Maxima [A] time = 1.20753, size = 170, normalized size = 2.1

$$-\frac{\left(fp\log(bx+a) + fq\log(dx+c)\right)r\log(x)}{f} + \log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)\log(x) + \frac{\left(\left(\log(bx+a)\log\left(-\frac{bx+a}{a}+1\right) + \text{L}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algorithm="maxima")
```

```
[Out] -(f*p*log(b*x + a) + f*q*log(d*x + c))*r*log(x)/f + log(((b*x + a)^p*(d*x +
 c)^q*f)^r*e)*log(x) + ((log(b*x + a)*log(-(b*x + a)/a + 1) + dilog((b*x +
a)/a))*f*p + (log(d*x + c)*log(-(d*x + c)/c + 1) + dilog((d*x + c)/c))*f*q)
*r/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algorithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x,x, algorithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/x, x)
```

3.60 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)}\,dx$

Optimal. Leaf size=41

$$\text{CannotIntegrate}\left(\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)},x\right)$$

```
[Out] CannotIntegrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u
]), x]
```

Rubi [A] time = 0.544652, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]),x]
```

```
[Out] Defer[Int][Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]), x
]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log\left(60\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log\left(60\left(j(hx)^t\right)^u\right)}\,dx$$

Mathematica [A] time = 0.444816, size = 0, normalized size = 0.

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]),x]
```

```
[Out] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]), x]
```

Maple [A] time = 1.772, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f(bx+a)^p(dx+c)^q\right)^r\right)}{x\ln\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u),x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x\log\left(\left((hx)^t j\right)^u i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u),x, algori
thm="maxima")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x\log\left(\left((hx)^t j\right)^u i\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u),x, algori
thm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x/ln(i*(j*(h*x)**t)**u),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p(dx+c)^q f\right)^r e\right)}{x\log\left(\left((hx)^t j\right)^u i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u),x, algori
thm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)), x)
```

3.61 $\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx$

Optimal. Leaf size=41

$$\text{CannotIntegrate}\left(\frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)},x\right)$$

```
[Out] CannotIntegrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u
]^2), x]
```

Rubi [A] time = 0.42531, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),x]
```

```
[Out] Defer[Int][Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),
 x]
```

Rubi steps

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log^2\left(61\left(j(hx)^t\right)^u\right)}\,dx=\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log^2\left(61\left(j(hx)^t\right)^u\right)}\,dx$$

Mathematica [A] time = 2.46198, size = 0, normalized size = 0.

$$\int \frac{\log\left(e\left(f(a+bx)^p(c+dx)^q\right)^r\right)}{x\log^2\left(i\left(j(hx)^t\right)^u\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),x
]
```

```
[Out] Integrate[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(x*Log[i*(j*(h*x)^t)^u]^2),
x]
```

Maple [A] time = 1.78, size = 0, normalized size = 0.

$$\int \frac{\ln\left(e\left(f\,(bx+a)^{p}\,(dx+c)^{q}\right)^{r}\right)}{x\left(\ln\left(i\left(j\,(hx)^{t}\right)^{u}\right)\right)^{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u)^2,x)
```

```
[Out] int(ln(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/ln(i*(j*(h*x)^t)^u)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(\left((bx+a)^{p}\right)^{r}\right)+\log\left(\left((dx+c)^{q}\right)^{r}\right)+\log\left(e\right)+\log\left(f^{r}\right)}{tu\log\left((h^{t})^{u}\right)+tu\log\left(i\right)+tu\log\left(j^{u}\right)+tu\log\left((x^{t})^{u}\right)}+\int\frac{}{\left(tu\log\left((h^{t})^{u}\right)+tu\log\left(i\right)+tu\log\left(j^{u}\right)\right)bdx^{2}+\left(tu\log\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u)^2,x, algo
rithm="maxima")
```

```
[Out] -(log(((b*x + a)^p)^r) + log(((d*x + c)^q)^r) + log(e) + log(f^r))/(t*u*log
((h^t)^u) + t*u*log(i) + t*u*log(j^u) + t*u*log((x^t)^u)) + integrate((b*c*
p*r + a*d*q*r + (p*r + q*r)*b*d*x)/((t*u*log((h^t)^u) + t*u*log(i) + t*u*lo
g(j^u))*b*d*x^2 + (t*u*log((h^t)^u) + t*u*log(i) + t*u*log(j^u))*a*c + ((t*
u*log((h^t)^u) + t*u*log(i) + t*u*log(j^u))*b*c + (t*u*log((h^t)^u) + t*u*l
og(i) + t*u*log(j^u))*a*d)*x + (b*d*t*u*x^2 + a*c*t*u + (b*c*t*u + a*d*t*u)
*x)*log((x^t)^u)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left((bx+a)^{p}(dx+c)^{q}f\right)^{r}e\right)}{x\log\left(\left((hx)^{t}j\right)^{u}i\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u)^2,x, algo
rithm="fricas")
```

```
[Out] integral(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)^2), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(f*(b*x+a)**p*(d*x+c)**q)**r)/x/ln(i*(j*(h*x)**t)**u)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left((bx+a)^p (dx+c)^q f\right)^r e\right)}{x \log\left(\left((hx)^t j\right)^u i\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(f*(b*x+a)^p*(d*x+c)^q)^r)/x/log(i*(j*(h*x)^t)^u)^2,x, algo
rithm="giac")
```

```
[Out] integrate(log(((b*x + a)^p*(d*x + c)^q*f)^r*e)/(x*log(((h*x)^t*j)^u*i)^2),
x)
```

3.62 $\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\log(x)\log^3\left(\frac{a+bx}{x(bc-ad)}\right)}{x},x\right)$$

```
[Out] Unintegrable[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x, x]
```

Rubi [A] time = 0.021354, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x,x]

[Out] Defer[Int][(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x, x]
```

Rubi steps

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx=\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Mathematica [A] time = 5.09395, size = 0, normalized size = 0.

$$\int \frac{\log(x)\log^3\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x,x]

[Out] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^3)/x, x]
```

Maple [A] time = 0.385, size = 0, normalized size = 0.

$$\int \frac{\ln(x)}{x}\left(\ln\left(\frac{bx+a}{(-ad+bc)\,x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^3/x,x)
```

```
[Out] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^3/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\log(bx+a)^3\log(x)^2 - \int \frac{2(bx+a)\log(x)^4 + 6(bx\log(bc-ad) + a\log(bc-ad))\log(x)^3 + 3\left((3bx+2a)\log(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^3/x,x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x + a)^3*log(x)^2 - integrate(1/2*(2*(b*x + a)*log(x)^4 + 6*(b*x*
log(b*c - a*d) + a*log(b*c - a*d))*log(x)^3 + 3*((3*b*x + 2*a)*log(x)^2 + 2
*(b*x*log(b*c - a*d) + a*log(b*c - a*d))*log(x))*log(b*x + a)^2 + 6*(b*x*lo
g(b*c - a*d)^2 + a*log(b*c - a*d)^2)*log(x)^2 - 6*((b*x + a)*log(x)^3 + 2*(
b*x*log(b*c - a*d) + a*log(b*c - a*d))*log(x)^2 + (b*x*log(b*c - a*d)^2 + a
*log(b*c - a*d)^2)*log(x))*log(b*x + a) + 2*(b*x*log(b*c - a*d)^3 + a*log(b
*c - a*d)^3)*log(x))/(b*x^2 + a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^3/x,x, algorithm="fricas")
```

```
[Out] integral(log(x)*log((b*x + a)/((b*c - a*d)*x))^3/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{3a\int \frac{\log(x)^2\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)^2}{ax+bx^2}\,dx}{2} + \frac{\log(x)^2\log\left(\frac{a+bx}{x(-ad+bc)}\right)^3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)**3/x,x)
```

```
[Out] 3*a*Integral(log(x)**2*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))**2/(a
*x + b*x**2), x)/2 + log(x)**2*log((a + b*x)/(x*(-a*d + b*c)))**3/2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^3/x,x, algorithm="giac")
```

```
[Out] integrate(log(x)*log((b*x + a)/((b*c - a*d)*x))^3/x, x)
```

3.63 $\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\log(x)\log^2\left(\frac{a+bx}{x(bc-ad)}\right)}{x},x\right)$$

```
[Out] Unintegrable[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x, x]
```

Rubi [A] time = 0.0213293, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x,x]
```

```
[Out] Defer[Int][(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x, x]
```

Rubi steps

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Mathematica [A] time = 5.08442, size = 0, normalized size = 0.

$$\int \frac{\log(x)\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x,x]
```

```
[Out] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)]^2)/x, x]
```

Maple [A] time = 0.209, size = 0, normalized size = 0.

$$\int \frac{\ln(x)}{x}\left(\ln\left(\frac{bx+a}{(-ad+bc)\,x}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^2/x,x)
```

```
[Out] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)^2/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\log(bx+a)^2\log(x)^2 - \int -\frac{(bx+a)\log(x)^3 + 2(bx\log(bc-ad) + a\log(bc-ad))\log(x)^2 - \left((3bx+2a)\log(x)^2 + 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x + a)^2*log(x)^2 - integrate(-((b*x + a)*log(x)^3 + 2*(b*x*log(b
*c - a*d) + a*log(b*c - a*d))*log(x)^2 - ((3*b*x + 2*a)*log(x)^2 + 2*(b*x*l
og(b*c - a*d) + a*log(b*c - a*d))*log(x))*log(b*x + a) + (b*x*log(b*c - a*d
)^2 + a*log(b*c - a*d)^2)*log(x))/(b*x^2 + a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(log(x)*log((b*x + a)/((b*c - a*d)*x))^2/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$a\int\frac{\log(x)^2\log\left(\frac{a}{-adx+bcx} + \frac{bx}{-adx+bcx}\right)}{ax+bx^2}\,dx + \frac{\log(x)^2\log\left(\frac{a+bx}{x(-ad+bc)}\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)**2/x,x)
```

```
[Out] a*Integral(log(x)**2*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))/(a*x +
b*x**2), x) + log(x)**2*log((a + b*x)/(x*(-a*d + b*c)))**2/2
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(log(x)*log((b*x + a)/((b*c - a*d)*x))^2/x, x)
```

3.64 $\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx$

Optimal. Leaf size=82

$$\text{PolyLog}\left(3,-\frac{a}{bx}\right)+\log(x)\text{PolyLog}\left(2,-\frac{a}{bx}\right)+\frac{1}{2}\log^2(x)\log\left(\frac{a}{x(bc-ad)}+\frac{b}{bc-ad}\right)-\frac{1}{2}\log^2(x)\log\left(\frac{a}{bx}+1\right)$$

```
[Out] -(Log[1 + a/(b*x)]*Log[x]^2)/2 + (Log[b/(b*c - a*d) + a/((b*c - a*d)*x)]*Lo
g[x]^2)/2 + Log[x]*PolyLog[2, -(a/(b*x))] + PolyLog[3, -(a/(b*x))]
```

Rubi [A] time = 0.176511, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2380, 2375, 2337, 2374, 6589}

$$\text{PolyLog}\left(3,-\frac{a}{bx}\right)+\log(x)\text{PolyLog}\left(2,-\frac{a}{bx}\right)+\frac{1}{2}\log^2(x)\log\left(\frac{a}{x(bc-ad)}+\frac{b}{bc-ad}\right)-\frac{1}{2}\log^2(x)\log\left(\frac{a}{bx}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)])/x,x]
```

```
[Out] -(Log[1 + a/(b*x)]*Log[x]^2)/2 + (Log[b/(b*c - a*d) + a/((b*c - a*d)*x)]*Lo
g[x]^2)/2 + Log[x]*PolyLog[2, -(a/(b*x))] + PolyLog[3, -(a/(b*x))]
```

Rule 2380

```
Int[Log[(d_.)*(u_)^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((g_.
)*(x_))^(q_.), x_Symbol] :> Int[(g*x)^q*Log[d*ExpandToSum[u, x]^r]*(a + b*L
og[c*x^n])^p, x] /; FreeQ[{a, b, c, d, g, r, n, p, q}, x] && BinomialQ[u, x
] &&  !BinomialMatchQ[u, x]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2337

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[(f^m*Log[1 + (e*x^r)/d]*(a + b*Log[c*
x^n])^p)/(e*r), x] - Dist[(b*f^m*n*p)/(e*r), Int[(Log[1 + (e*x^r)/d]*(a + b
*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\log(x)\log\left(\frac{a+bx}{(bc-ad)x}\right)}{x}\,dx = \int \frac{\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log(x)}{x}\,dx$$

$$= \frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x) + \frac{a\int \frac{\log^2(x)}{\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)x^2}\,dx}{2(bc-ad)}$$

$$= -\frac{1}{2}\log\left(1+\frac{a}{bx}\right)\log^2(x) + \frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x) + \int \frac{\log\left(1+\frac{a}{bx}\right)\log(x)}{x}\,dx$$

$$= -\frac{1}{2}\log\left(1+\frac{a}{bx}\right)\log^2(x) + \frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x) + \log(x)\text{Li}_2\left(-\frac{a}{bx}\right) - \int \frac{\text{Li}$$

$$= -\frac{1}{2}\log\left(1+\frac{a}{bx}\right)\log^2(x) + \frac{1}{2}\log\left(\frac{b}{bc-ad}+\frac{a}{(bc-ad)x}\right)\log^2(x) + \log(x)\text{Li}_2\left(-\frac{a}{bx}\right) + \text{Li}_3\left(-\right.$$

Mathematica [A] time = 5.02716, size = 66, normalized size = 0.8

$$\text{PolyLog}\left(3,-\frac{bx}{a}\right) - \log(x)\text{PolyLog}\left(2,-\frac{bx}{a}\right) + \frac{1}{6}\log^2(x)\left(3\log\left(\frac{a+bx}{bcx-adx}\right) - 3\log\left(\frac{bx}{a}+1\right) + \log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[x]*Log[(a + b*x)/((b*c - a*d)*x)])/x,x]
```

```
[Out] (Log[x]^2*(Log[x] - 3*Log[1 + (b*x)/a] + 3*Log[(a + b*x)/(b*c*x - a*d*x)]))
/6 - Log[x]*PolyLog[2, -((b*x)/a)] + PolyLog[3, -((b*x)/a)]
```

Maple [C] time = 0.121, size = 450, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)/x,x)
```

```
[Out] 1/2*ln(x)^2*ln(b*x+a)-1/3*ln(x)^3-1/4*I*ln(x)^2*Pi*csgn(I/x)*csgn(I*(b*x+a)
/(a*d-b*c))*csgn(I/x*(b*x+a)/(a*d-b*c))+1/2*I*ln(x)^2*Pi+1/4*I*ln(x)^2*Pi*c
sgn(I*(b*x+a))*csgn(I*(b*x+a)/(a*d-b*c))^2-1/4*I*ln(x)^2*Pi*csgn(I*(b*x+a)/
(a*d-b*c))^3+1/4*I*ln(x)^2*Pi*csgn(I*(b*x+a)/(a*d-b*c))*csgn(I/x*(b*x+a)/(a
*d-b*c))^2+1/4*I*ln(x)^2*Pi*csgn(I/x*(b*x+a)/(a*d-b*c))^3-1/2*I*ln(x)^2*Pi*
csgn(I/x*(b*x+a)/(a*d-b*c))^2-1/4*I*ln(x)^2*Pi*csgn(I*(b*x+a))*csgn(I/(a*d-
b*c))*csgn(I*(b*x+a)/(a*d-b*c))+1/4*I*ln(x)^2*Pi*csgn(I/x)*csgn(I/x*(b*x+a)
/(a*d-b*c))^2+1/4*I*ln(x)^2*Pi*csgn(I/(a*d-b*c))*csgn(I*(b*x+a)/(a*d-b*c))^
2-1/2*ln(x)^2*ln(a*d-b*c)-1/2*ln(x)^2*ln(1+x*b/a)-ln(x)*polylog(2,-x*b/a)+p
olylog(3,-x*b/a)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: TypeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)/x,x, algorithm="fricas")
```

```
[Out] integral(log(x)*log((b*x + a)/((b*c - a*d)*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\int\frac{\log(x)^2}{ax+bx^2}\,dx}{2} + \frac{\log(x)^2\log\left(\frac{a+bx}{x(-ad+bc)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*ln((b*x+a)/(-a*d+b*c)/x)/x,x)
```

```
[Out] a*Integral(log(x)**2/(a*x + b*x**2), x)/2 + log(x)**2*log((a + b*x)/(x*(-a*
d + b*c)))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log(x)\log\left(\frac{bx+a}{(bc-ad)x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log((b*x+a)/(-a*d+b*c)/x)/x,x, algorithm="giac")
```

```
[Out] integrate(log(x)*log((b*x + a)/((b*c - a*d)*x))/x, x)
```

3.65 $\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\log(x)}{x\log\left(\frac{a+bx}{x(bc-ad)}\right)},x\right)$$

```
[Out] Unintegrable[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]), x]
```

Rubi [A] time = 0.0216231, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]),x]
```

```
[Out] Defer[Int][Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]), x]
```

Rubi steps

$$\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx=\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx$$

Mathematica [A] time = 5.0975, size = 0, normalized size = 0.

$$\int \frac{\log(x)}{x\log\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]),x]
```

```
[Out] Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]), x]
```

Maple [A] time = 0.089, size = 0, normalized size = 0.

$$\int {\frac{\ln(x)}{x}}\left(\ln\left({\frac{bx+a}{(-ad+bc)x}}\right)\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
```

```
[Out] int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x)}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x),x, algorithm="maxima")
```

```
[Out] integrate(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log(x)}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x),x, algorithm="fricas")
```

```
[Out] integral(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x)}{x\log\left(\frac{a}{-adx+bcx} + \frac{bx}{-adx+bcx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x),x)
```

```
[Out] Integral(log(x)/(x*log(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x)}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x),x, algorithm="giac")
```

```
[Out] integrate(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))), x)
```

3.66 $\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\log(x)}{x\log^2\left(\frac{a+bx}{x(bc-ad)}\right)},x\right)$$

```
[Out] Unintegrable[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2), x]
```

Rubi [A] time = 0.0219153, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2),x]
```

```
[Out] Defer[Int][Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2), x]
```

Rubi steps

$$\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx=\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx$$

Mathematica [A] time = 33.1028, size = 0, normalized size = 0.

$$\int \frac{\log(x)}{x\log^2\left(\frac{a+bx}{(bc-ad)x}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2),x]
```

```
[Out] Integrate[Log[x]/(x*Log[(a + b*x)/((b*c - a*d)*x)]^2), x]
```

Maple [A] time = 0.327, size = 0, normalized size = 0.

$$\int {\frac{\ln(x)}{x}}\left(\ln\left({\frac{bx+a}{(-ad+bc)x}}\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)^2,x)
```

```
[Out] int(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{(bx+a)\log(x)}{a\log(bc-ad)-a\log(bx+a)+a\log(x)}-\int -\frac{bx\log(x)+bx+a}{ax\log(bc-ad)-ax\log(bx+a)+ax\log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x)^2,x, algorithm="maxima")
```

```
[Out] -(b*x + a)*log(x)/(a*log(b*c - a*d) - a*log(b*x + a) + a*log(x)) - integrat
e(-(b*x*log(x) + b*x + a)/(a*x*log(b*c - a*d) - a*x*log(b*x + a) + a*x*log(
x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log(x)}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x)^2,x, algorithm="fricas")
```

```
[Out] integral(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{a\log(x)+bx\log(x)}{a\log\left(\frac{a+bx}{x(-ad+bc)}\right)}-\frac{\int\frac{b}{\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}\,dx+\int\frac{a}{x\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}\,dx+\int\frac{b\log(x)}{\log\left(\frac{a}{-adx+bcx}+\frac{bx}{-adx+bcx}\right)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x/ln((b*x+a)/(-a*d+b*c)/x)**2,x)
```

```
[Out] (a*log(x) + b*x*log(x))/(a*log((a + b*x)/(x*(-a*d + b*c)))) - (Integral(b/l
og(a/(-a*d*x + b*c*x) + b*x/(-a*d*x + b*c*x)), x) + Integral(a/(x*log(a/(-a
*d*x + b*c*x) + b*x/(-a*d*x + b*c*x))), x) + Integral(b*log(x)/log(a/(-a*d*
x + b*c*x) + b*x/(-a*d*x + b*c*x)), x))/a
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log(x)}{x\log\left(\frac{bx+a}{(bc-ad)x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/log((b*x+a)/(-a*d+b*c)/x)^2,x, algorithm="giac")
```

```
[Out] integrate(log(x)/(x*log((b*x + a)/((b*c - a*d)*x))^2), x)
```

3.67 $\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=620

$$-\frac{6mn^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(4,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad}-\frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{3mn\log^2\left(e\left(\frac{a+bx}{c+dx}\right.\right.}{}$$

```
[Out] (m*Log[e*((a + b*x)/(c + d*x))^n]^4*Log[(b*c - a*d)/(b*(c + d*x))])/(4*(b*c
 - a*d)*n) + (Log[e*((a + b*x)/(c + d*x))^n]^4*Log[h*(f + g*x)^m])/(4*(b*c
- a*d)*n) - (m*Log[e*((a + b*x)/(c + d*x))^n]^4*Log[1 - ((d*f - c*g)*(a + b
*x))/((b*f - a*g)*(c + d*x))])/(4*(b*c - a*d)*n) + (m*Log[e*((a + b*x)/(c +
 d*x))^n]^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) - (m*Log[e
*((a + b*x)/(c + d*x))^n]^3*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)
*(c + d*x))])/(b*c - a*d) - (3*m*n*Log[e*((a + b*x)/(c + d*x))^n]^2*PolyLog
[3, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) + (3*m*n*Log[e*((a + b*x)/(c
+ d*x))^n]^2*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(
b*c - a*d) + (6*m*n^2*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[4, (d*(a + b*x
))/(b*(c + d*x))])/(b*c - a*d) - (6*m*n^2*Log[e*((a + b*x)/(c + d*x))^n]*Po
lyLog[4, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(b*c - a*d) - (6
*m*n^3*PolyLog[5, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) + (6*m*n^3*Poly
Log[5, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(b*c - a*d)
```

Rubi [A] time = 1.13263, antiderivative size = 649, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2507, 2489, 2488, 2506, 2508, 6610, 2503}

$$-\frac{6mn^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(4,1-\frac{(f+gx)(bc-ad)}{(c+dx)(bf-ag)}\right)}{bc-ad}-\frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(2,1-\frac{(f+gx)(bc-ad)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{3mn\log^2\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*((a + b*x)/(c + d*x))^n]^3*Log[h*(f + g*x)^m])/((a + b*x)*(c + d
*x)),x]
```

```
[Out] (m*Log[e*((a + b*x)/(c + d*x))^n]^4*Log[(b*c - a*d)/(b*(c + d*x))])/(4*(b*c
 - a*d)*n) - (m*Log[e*((a + b*x)/(c + d*x))^n]^4*Log[((b*c - a*d)*(f + g*x)
)/((b*f - a*g)*(c + d*x))])/(4*(b*c - a*d)*n) + (Log[e*((a + b*x)/(c + d*x)
)^n]^4*Log[h*(f + g*x)^m])/(4*(b*c - a*d)*n) + (m*Log[e*((a + b*x)/(c + d*x
))^n]^3*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d) - (m*Log[e*(
(a + b*x)/(c + d*x))^n]^3*PolyLog[2, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*
g)*(c + d*x))])/(b*c - a*d) - (3*m*n*Log[e*((a + b*x)/(c + d*x))^n]^2*PolyL
og[3, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d) + (3*m*n*Log[e*((a + b*x)
/(c + d*x))^n]^2*PolyLog[3, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d
*x))])/(b*c - a*d) + (6*m*n^2*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[4, 1 -
 (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d) - (6*m*n^2*Log[e*((a + b*x)/(c + d
*x))^n]*PolyLog[4, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))])/(b
*c - a*d) - (6*m*n^3*PolyLog[5, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d)
 + (6*m*n^3*PolyLog[5, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))]
)/(b*c - a*d)
```

Rule 2507

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[(k*Log[i*(j*(g +
 h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1))/(p*r*(s + 1)*(b*c
 - a*d)), x] - Dist[(k*h*t*u)/(p*r*(s + 1)*(b*c - a*d)), Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[{
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[p + q, 0] && NeQ[s, -1]
```

Rule 2489

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[d/h, Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^s/(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{a
, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0
] && NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 1]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2508

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[(v*(c +
 d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(h*PolyLog[n
+ 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] - Dist[h*p*
r*s, Int[(PolyLog[n + 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(
(a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Coeff[Simplify[1/(u*(a + b*x))],
 x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
```

```
)))])/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))])/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
 x]
```

Rubi steps

$$\int \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n} - \frac{(gm)\int\frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}\,dx}{4(bc-ad)n}$$

$$= \frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{4(bc-ad)n} - \frac{(dm)\int\frac{\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{4(bc-ad)n} + \frac{((df-cg)m \cdots}{4(b \cdots}$$

$$= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n} + \cdots$$

$$= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n} + \cdots$$

$$= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n} + \cdots$$

$$= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n} + \cdots$$

$$= \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{4(bc-ad)n} - \frac{m\log^4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{4(bc-ad)n} + \cdots$$

Mathematica [B] time = 21.5443, size = 18164, normalized size = 29.3

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^3*Log[h*(f + g*x)^m])/((a + b*x)*
(c + d*x)),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 7.167, size = 0, normalized size = 0.

$$\int \frac{\ln\left(h\left(gx+f\right)^m\right)}{(bx+a)\,(dx+c)}\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)^3*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)^3*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^3*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="maxima")
```

```
[Out] -1/4*(n^3*log(b*x + a)^4 + n^3*log(d*x + c)^4 - 4*n^2*log(b*x + a)^3*log(e)
 + 6*n*log(b*x + a)^2*log(e)^2 - 4*(n^3*log(b*x + a) - n^2*log(e))*log(d*x
+ c)^3 - 4*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n)^3 + 4*(log(b*x +
a) - log(d*x + c))*log((d*x + c)^n)^3 - 4*log(b*x + a)*log(e)^3 + 6*(n^3*lo
g(b*x + a)^2 - 2*n^2*log(b*x + a)*log(e) + n*log(e)^2)*log(d*x + c)^2 + 6*(
n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))*log(d*x +
 c) - 2*log(b*x + a)*log(e))*log((b*x + a)^n)^2 + 6*(n*log(b*x + a)^2 + n*l
og(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))*log(d*x + c) - 2*(log(b*x + a)
- log(d*x + c))*log((b*x + a)^n) - 2*log(b*x + a)*log(e))*log((d*x + c)^n)^
2 - 4*(n^3*log(b*x + a)^3 - 3*n^2*log(b*x + a)^2*log(e) + 3*n*log(b*x + a)*
log(e)^2 - log(e)^3)*log(d*x + c) - 4*(n^2*log(b*x + a)^3 - n^2*log(d*x + c
)^3 - 3*n*log(b*x + a)^2*log(e) + 3*(n^2*log(b*x + a) - n*log(e))*log(d*x +
 c)^2 + 3*log(b*x + a)*log(e)^2 - 3*(n^2*log(b*x + a)^2 - 2*n*log(b*x + a)*
log(e) + log(e)^2)*log(d*x + c))*log((b*x + a)^n) + 4*(n^2*log(b*x + a)^3 -
 n^2*log(d*x + c)^3 - 3*n*log(b*x + a)^2*log(e) + 3*(n^2*log(b*x + a) - n*l
og(e))*log(d*x + c)^2 + 3*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n)^2
+ 3*log(b*x + a)*log(e)^2 - 3*(n^2*log(b*x + a)^2 - 2*n*log(b*x + a)*log(e)
 + log(e)^2)*log(d*x + c) - 3*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*l
og(b*x + a) - log(e))*log(d*x + c) - 2*log(b*x + a)*log(e))*log((b*x + a)^n
))*log((d*x + c)^n))*log((g*x + f)^m)/(b*c - a*d) + integrate(1/4*(4*b*c*f*
log(e)^3*log(h) - 4*a*d*f*log(e)^3*log(h) + (b*d*g*m*n^3*x^2 + a*c*g*m*n^3
+ (b*c*g*m*n^3 + a*d*g*m*n^3)*x)*log(b*x + a)^4 + (b*d*g*m*n^3*x^2 + a*c*g*
m*n^3 + (b*c*g*m*n^3 + a*d*g*m*n^3)*x)*log(d*x + c)^4 - 4*(b*d*g*m*n^2*x^2*
log(e) + a*c*g*m*n^2*log(e) + (b*c*g*m*n^2*log(e) + a*d*g*m*n^2*log(e))*x)*
log(b*x + a)^3 + 4*(b*d*g*m*n^2*x^2*log(e) + a*c*g*m*n^2*log(e) + (b*c*g*m*
n^2*log(e) + a*d*g*m*n^2*log(e))*x - (b*d*g*m*n^3*x^2 + a*c*g*m*n^3 + (b*c*
g*m*n^3 + a*d*g*m*n^3)*x)*log(b*x + a))*log(d*x + c)^3 + 4*(b*c*f*log(h) -
a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (
b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*
d*g*m)*x)*log(d*x + c))*log((b*x + a)^n)^3 - 4*(b*c*f*log(h) - a*d*f*log(h)
 + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*
d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*lo
g(d*x + c))*log((d*x + c)^n)^3 + 6*(b*d*g*m*n*x^2*log(e)^2 + a*c*g*m*n*log(
e)^2 + (b*c*g*m*n*log(e)^2 + a*d*g*m*n*log(e)^2)*x)*log(b*x + a)^2 + 6*(b*d
*g*m*n*x^2*log(e)^2 + a*c*g*m*n*log(e)^2 + (b*d*g*m*n^3*x^2 + a*c*g*m*n^3 +
 (b*c*g*m*n^3 + a*d*g*m*n^3)*x)*log(b*x + a)^2 + (b*c*g*m*n*log(e)^2 + a*d*
g*m*n*log(e)^2)*x - 2*(b*d*g*m*n^2*x^2*log(e) + a*c*g*m*n^2*log(e) + (b*c*g
*m*n^2*log(e) + a*d*g*m*n^2*log(e))*x)*log(b*x + a))*log(d*x + c)^2 + 6*(2*
b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b*d*g*m*n*x^2 + a*c*g*m*n +
(b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d*g*m*n*x^2 + a*c*g*m*n + (b
*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c*g*log(e)*log(h) - a*d*g*lo
g(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) +
 a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e) +
 (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*
m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c))*log((b*x + a)^n)^2 + 6*(2*b
*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b*d*g*m*n*x^2 + a*c*g*m*n + (
```

```
b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*
c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c*g*log(e)*log(h) - a*d*g*log
(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) +
a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e) +
(b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m
*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c) + 2*(b*c*f*log(h) - a*d*f*log
(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m +
 a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)
*log(d*x + c))*log((b*x + a)^n))*log((d*x + c)^n)^2 + 4*(b*c*g*log(e)^3*log
(h) - a*d*g*log(e)^3*log(h))*x - 4*(b*d*g*m*x^2*log(e)^3 + a*c*g*m*log(e)^3
 + (b*c*g*m*log(e)^3 + a*d*g*m*log(e)^3)*x)*log(b*x + a) + 4*(b*d*g*m*x^2*l
og(e)^3 + a*c*g*m*log(e)^3 - (b*d*g*m*n^3*x^2 + a*c*g*m*n^3 + (b*c*g*m*n^3
+ a*d*g*m*n^3)*x)*log(b*x + a)^3 + 3*(b*d*g*m*n^2*x^2*log(e) + a*c*g*m*n^2*
log(e) + (b*c*g*m*n^2*log(e) + a*d*g*m*n^2*log(e))*x)*log(b*x + a)^2 + (b*c
*g*m*log(e)^3 + a*d*g*m*log(e)^3)*x - 3*(b*d*g*m*n*x^2*log(e)^2 + a*c*g*m*n
*log(e)^2 + (b*c*g*m*n*log(e)^2 + a*d*g*m*n*log(e)^2)*x)*log(b*x + a))*log(
d*x + c) + 4*(3*b*c*f*log(e)^2*log(h) - 3*a*d*f*log(e)^2*log(h) - (b*d*g*m*
n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^3 + (b*
d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(d*x + c)^3
 + 3*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m
*n*log(e))*x)*log(b*x + a)^2 + 3*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) +
 (b*c*g*m*n*log(e) + a*d*g*m*n*log(e))*x - (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 +
 (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a))*log(d*x + c)^2 + 3*(b*c*g*log
(e)^2*log(h) - a*d*g*log(e)^2*log(h))*x - 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m
*log(e)^2 + (b*c*g*m*log(e)^2 + a*d*g*m*log(e)^2)*x)*log(b*x + a) + 3*(b*d*
g*m*x^2*log(e)^2 + a*c*g*m*log(e)^2 + (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c
*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^2 + (b*c*g*m*log(e)^2 + a*d*g*m*log
(e)^2)*x - 2*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) +
 a*d*g*m*n*log(e))*x)*log(b*x + a))*log(d*x + c))*log((b*x + a)^n) - 4*(3*b
*c*f*log(e)^2*log(h) - 3*a*d*f*log(e)^2*log(h) - (b*d*g*m*n^2*x^2 + a*c*g*m
*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^3 + (b*d*g*m*n^2*x^2 + a
*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(d*x + c)^3 + 3*(b*d*g*m*n*x
^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m*n*log(e))*x)*log
(b*x + a)^2 + 3*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e
) + a*d*g*m*n*log(e))*x - (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a
*d*g*m*n^2)*x)*log(b*x + a))*log(d*x + c)^2 + 3*(b*c*f*log(h) - a*d*f*log(h
) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a
*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*l
og(d*x + c))*log((b*x + a)^n)^2 + 3*(b*c*g*log(e)^2*log(h) - a*d*g*log(e)^2
*log(h))*x - 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m*log(e)^2 + (b*c*g*m*log(e)^2
 + a*d*g*m*log(e)^2)*x)*log(b*x + a) + 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m*lo
g(e)^2 + (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*lo
g(b*x + a)^2 + (b*c*g*m*log(e)^2 + a*d*g*m*log(e)^2)*x - 2*(b*d*g*m*n*x^2*l
og(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m*n*log(e))*x)*log(b*x
 + a))*log(d*x + c) + 3*(2*b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b
*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d
*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c
*g*log(e)*log(h) - a*d*g*log(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m
*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^
2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m*
n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c))*
log((b*x + a)^n))*log((d*x + c)^n))/(a*b*c^2*f - a^2*c*d*f + (b^2*c*d*g - a
*b*d^2*g)*x^3 - (a*b*d^2*f + a^2*d^2*g - (c*d*f + c^2*g)*b^2)*x^2 + (b^2*c^
2*f + a*b*c^2*g - (d^2*f + c*d*g)*a^2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(gx+f\right)^{m}h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{3}}{bdx^{2}+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^3*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^3/(b*d*x^2 + a*c
 + (b*c + a*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)**3*ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(\left(gx+f\right)^{m}h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{3}}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^3*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^3/((b*x + a)*(d
*x + c)), x)
```

3.68 $\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=496

$$-\frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{2mn\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(3,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{m\operatorname{PolyLog}\left(2,\frac{d(a+}{b(c+}\right.}{bc}$$

```
[Out] (m*Log[e*((a + b*x)/(c + d*x))^n]^3*Log[(b*c - a*d)/(b*(c + d*x))])/(3*(b*c
 - a*d)*n) + (Log[e*((a + b*x)/(c + d*x))^n]^3*Log[h*(f + g*x)^m])/(3*(b*c
- a*d)*n) - (m*Log[e*((a + b*x)/(c + d*x))^n]^3*Log[1 - ((d*f - c*g)*(a + b
*x))/((b*f - a*g)*(c + d*x))])/(3*(b*c - a*d)*n) + (m*Log[e*((a + b*x)/(c +
 d*x))^n]^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) - (m*Log[e
*((a + b*x)/(c + d*x))^n]^2*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)
*(c + d*x))])/(b*c - a*d) - (2*m*n*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[3
, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) + (2*m*n*Log[e*((a + b*x)/(c +
d*x))^n]*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(b*c
- a*d) + (2*m*n^2*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) - (2
*m*n^2*PolyLog[4, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(b*c -
a*d)
```

Rubi [A] time = 0.818883, antiderivative size = 517, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 7, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2507, 2489, 2488, 2506, 2508, 6610, 2503}

$$-\frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(2,1-\frac{(f+gx)(bc-ad)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{2mn\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\operatorname{PolyLog}\left(3,1-\frac{(f+gx)(bc-ad)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{m\operatorname{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[h*(f + g*x)^m])/((a + b*x)*(c + d
*x)),x]
```

```
[Out] (m*Log[e*((a + b*x)/(c + d*x))^n]^3*Log[(b*c - a*d)/(b*(c + d*x))])/(3*(b*c
 - a*d)*n) - (m*Log[e*((a + b*x)/(c + d*x))^n]^3*Log[((b*c - a*d)*(f + g*x)
)/((b*f - a*g)*(c + d*x))])/(3*(b*c - a*d)*n) + (Log[e*((a + b*x)/(c + d*x)
)^n]^3*Log[h*(f + g*x)^m])/(3*(b*c - a*d)*n) + (m*Log[e*((a + b*x)/(c + d*x
))^n]^2*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d) - (m*Log[e*(
(a + b*x)/(c + d*x))^n]^2*PolyLog[2, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*
g)*(c + d*x))])/(b*c - a*d) - (2*m*n*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog
[3, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d) + (2*m*n*Log[e*((a + b*x)/(
c + d*x))^n]*PolyLog[3, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))
])/(b*c - a*d) + (2*m*n^2*PolyLog[4, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c -
 a*d) - (2*m*n^2*PolyLog[4, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d
*x))])/(b*c - a*d)
```

Rule 2507

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[(k*Log[i*(j*(g +
 h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1))/(p*r*(s + 1)*(b*c
 - a*d)), x] - Dist[(k*h*t*u)/(p*r*(s + 1)*(b*c - a*d)), Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[{
```

```
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[p + q, 0] && NeQ[s, -1]
```

Rule 2489

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[d/h, Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^s/(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{a
, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0
] && NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 1]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2508

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[(v*(c +
 d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(h*PolyLog[n
+ 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] - Dist[h*p*
r*s, Int[(PolyLog[n + 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(
(a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Coeff[Simplify[1/(u*(a + b*x))],
 x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
)))])/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))])/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
 x]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{3(bc-ad)n} - \frac{(gm)\int\frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}dx}{3(bc-ad)n}$$

$$= \frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{3(bc-ad)n} - \frac{(dm)\int\frac{\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}dx}{3(bc-ad)n} + \frac{((df-cg)m}{3(b}$$

$$= \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{3(bc-ad)n} + -$$

$$= \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{3(bc-ad)n} + -$$

$$= \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{3(bc-ad)n} + -$$

$$= \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3(bc-ad)n} - \frac{m\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{3(bc-ad)n} + -$$

Mathematica [B] time = 10.071, size = 9211, normalized size = 18.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[h*(f + g*x)^m])/((a + b*x)*
(c + d*x)),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 5.039, size = 0, normalized size = 0.

$$\int \frac{\ln\left(h\left(gx+f\right)^m\right)}{(bx+a)(dx+c)}\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="maxima")
```

```
[Out] 1/3*(n^2*log(b*x + a)^3 - n^2*log(d*x + c)^3 - 3*n*log(b*x + a)^2*log(e) +
3*(n^2*log(b*x + a) - n*log(e))*log(d*x + c)^2 + 3*(log(b*x + a) - log(d*x
+ c))*log((b*x + a)^n)^2 + 3*(log(b*x + a) - log(d*x + c))*log((d*x + c)^n)
^2 + 3*log(b*x + a)*log(e)^2 - 3*(n^2*log(b*x + a)^2 - 2*n*log(b*x + a)*log
(e) + log(e)^2)*log(d*x + c) - 3*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(
n*log(b*x + a) - log(e))*log(d*x + c) - 2*log(b*x + a)*log(e))*log((b*x + a
)^n) + 3*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))
*log(d*x + c) - 2*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n) - 2*log(b*
x + a)*log(e))*log((d*x + c)^n))*log((g*x + f)^m)/(b*c - a*d) - integrate(-
1/3*(3*b*c*f*log(e)^2*log(h) - 3*a*d*f*log(e)^2*log(h) - (b*d*g*m*n^2*x^2 +
 a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*x + a)^3 + (b*d*g*m*n^2
*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(d*x + c)^3 + 3*(b*d
*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m*n*log(e)
)*x)*log(b*x + a)^2 + 3*(b*d*g*m*n*x^2*log(e) + a*c*g*m*n*log(e) + (b*c*g*m
*n*log(e) + a*d*g*m*n*log(e))*x - (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m
*n^2 + a*d*g*m*n^2)*x)*log(b*x + a))*log(d*x + c)^2 + 3*(b*c*f*log(h) - a*d
*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c
*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g
*m)*x)*log(d*x + c))*log((b*x + a)^n)^2 + 3*(b*c*f*log(h) - a*d*f*log(h) +
(b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g
*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(d
*x + c))*log((d*x + c)^n)^2 + 3*(b*c*g*log(e)^2*log(h) - a*d*g*log(e)^2*log
(h))*x - 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m*log(e)^2 + (b*c*g*m*log(e)^2 + a
*d*g*m*log(e)^2)*x)*log(b*x + a) + 3*(b*d*g*m*x^2*log(e)^2 + a*c*g*m*log(e)
^2 + (b*d*g*m*n^2*x^2 + a*c*g*m*n^2 + (b*c*g*m*n^2 + a*d*g*m*n^2)*x)*log(b*
x + a)^2 + (b*c*g*m*log(e)^2 + a*d*g*m*log(e)^2)*x - 2*(b*d*g*m*n*x^2*log(e
) + a*c*g*m*n*log(e) + (b*c*g*m*n*log(e) + a*d*g*m*n*log(e))*x)*log(b*x + a
))*log(d*x + c) + 3*(2*b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b*d*g
*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d*g*m
*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c*g*l
og(e)*log(h) - a*d*g*log(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log
(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^2*lo
g(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m*n*x^
2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c))*log(
(b*x + a)^n) - 3*(2*b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log(h) + (b*d*g*m*
n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^2 + (b*d*g*m*n*
x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2 + 2*(b*c*g*log(
e)*log(h) - a*d*g*log(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) + a*c*g*m*log(e)
 + (b*c*g*m*log(e) + a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*d*g*m*x^2*log(e
) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (b*d*g*m*n*x^2 +
 a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*x + c) + 2*(b*c
*f*log(h) - a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2 +
 a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m + (
b*c*g*m + a*d*g*m)*x)*log(d*x + c))*log((b*x + a)^n))*log((d*x + c)^n))/(a*
b*c^2*f - a^2*c*d*f + (b^2*c*d*g - a*b*d^2*g)*x^3 - (a*b*d^2*f + a^2*d^2*g
- (c*d*f + c^2*g)*b^2)*x^2 + (b^2*c^2*f + a*b*c^2*g - (d^2*f + c*d*g)*a^2)*
x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(gx+f\right)^{m}h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}}{bdx^{2}+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^2/(b*d*x^2 + a*c
 + (b*c + a*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)**2*ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x,
algorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)^2/((b*x + a)*(d
*x + c)), x)
```

3.69 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=371

$$-\frac{m\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(2,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{m\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad}+\frac{mn\text{PolyLog}\left(3,\frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{bc-ad}-$$

```
[Out] (m*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*(c + d*x))])/(2*(b*c
 - a*d)*n) + (Log[e*((a + b*x)/(c + d*x))^n]^2*Log[h*(f + g*x)^m])/(2*(b*c
- a*d)*n) - (m*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[1 - ((d*f - c*g)*(a + b
*x))/((b*f - a*g)*(c + d*x))])/(2*(b*c - a*d)*n) + (m*Log[e*((a + b*x)/(c +
 d*x))^n]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*c - a*d) - (m*Log[e*(
(a + b*x)/(c + d*x))^n]*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c
+ d*x))])/(b*c - a*d) - (m*n*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(b*c
- a*d) + (m*n*PolyLog[3, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/
(b*c - a*d)
```

Rubi [A] time = 0.557658, antiderivative size = 384, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.14, Rules used = {2507, 2489, 2488, 2506, 6610, 2503}

$$-\frac{m\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{PolyLog}\left(2,1-\frac{(f+gx)(bc-ad)}{(c+dx)(bf-ag)}\right)}{bc-ad}+\frac{m\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bc-ad}+\frac{mn\text{PolyLog}\left(3,1-\frac{(f+g}{(c+d}\right.}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(f + g*x)^m])/((a + b*x)*(c + d*x
)),x]
```

```
[Out] (m*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*(c + d*x))])/(2*(b*c
 - a*d)*n) - (m*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[((b*c - a*d)*(f + g*x)
)/((b*f - a*g)*(c + d*x))])/(2*(b*c - a*d)*n) + (Log[e*((a + b*x)/(c + d*x)
)^n]^2*Log[h*(f + g*x)^m])/(2*(b*c - a*d)*n) + (m*Log[e*((a + b*x)/(c + d*x
))^n]*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d) - (m*Log[e*((a
 + b*x)/(c + d*x))^n]*PolyLog[2, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(
c + d*x))])/(b*c - a*d) - (m*n*PolyLog[3, 1 - (b*c - a*d)/(b*(c + d*x))])/(
b*c - a*d) + (m*n*PolyLog[3, 1 - ((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c +
d*x))])/(b*c - a*d)
```

Rule 2507

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[(k*Log[i*(j*(g +
 h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1))/(p*r*(s + 1)*(b*c
 - a*d)), x] - Dist[(k*h*t*u)/(p*r*(s + 1)*(b*c - a*d)), Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[{
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[p + q, 0] && NeQ[s, -1]
```

Rule 2489

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[d/h, Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^s/(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{a
, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0
] && NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 1]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Coeff[Simplify[1/(u*(a + b*x))],
 x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
)))])/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))])/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
 x]
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)}\,dx = \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{2(bc-ad)n} - \frac{(gm)\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx}dx}{2(bc-ad)n}$$

$$= \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(h(f+gx)^m\right)}{2(bc-ad)n} - \frac{(dm)\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}dx}{2(bc-ad)n} + \frac{((df-cg)m)\int -}{2(bc-}$$

$$= \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n} + \frac{\log^2(}{}$$

$$= \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n} + \frac{\log^2(}{}$$

$$= \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2(bc-ad)n} - \frac{m\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{(bc-ad)(f+gx)}{(bf-ag)(c+dx)}\right)}{2(bc-ad)n} + \frac{\log^2(}{}$$

Mathematica [B] time = 6.02764, size = 1408, normalized size = 3.8

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(f + g*x)^m])/((a + b*x)*(c
 + d*x)),x]
```

```
[Out] (m*n*Log[(a + b*x)/(c + d*x)]^2*Log[(b*c - a*d)/(b*c + b*d*x)] - m*n*Log[a/
b + x]^2*Log[f + g*x] - m*n*Log[c/d + x]^2*Log[f + g*x] + 2*m*n*Log[a/b + x
]*Log[a + b*x]*Log[f + g*x] - 2*m*n*Log[c/d + x]*Log[a + b*x]*Log[f + g*x]
+ 2*m*n*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[f + g*x] + 2*m*L
og[a/b + x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] - 2*m*Log[c/d + x]*
Log[e*((a + b*x)/(c + d*x))^n]*Log[f + g*x] - 2*m*Log[a + b*x]*Log[e*((a +
b*x)/(c + d*x))^n]*Log[f + g*x] - 2*m*n*Log[a/b + x]*Log[(a + b*x)/(c + d*x
)]*Log[f + g*x] + 2*m*n*Log[c/d + x]*Log[(a + b*x)/(c + d*x)]*Log[f + g*x]
+ m*n*Log[(a + b*x)/(c + d*x)]^2*Log[f + g*x] - 2*m*n*Log[a/b + x]*Log[c +
d*x]*Log[f + g*x] + 2*m*n*Log[c/d + x]*Log[c + d*x]*Log[f + g*x] + 2*m*Log[
e*((a + b*x)/(c + d*x))^n]*Log[c + d*x]*Log[f + g*x] + 2*m*n*Log[a/b + x]*L
og[(b*(c + d*x))/(b*c - a*d)]*Log[f + g*x] - 2*m*Log[a/b + x]*Log[e*((a + b
*x)/(c + d*x))^n]*Log[(b*(f + g*x))/(b*f - a*g)] + 2*m*n*Log[a/b + x]*Log[(
a + b*x)/(c + d*x)]*Log[(b*(f + g*x))/(b*f - a*g)] + 2*m*Log[c/d + x]*Log[e
*((a + b*x)/(c + d*x))^n]*Log[(d*(f + g*x))/(d*f - c*g)] - 2*m*n*Log[c/d +
x]*Log[(a + b*x)/(c + d*x)]*Log[(d*(f + g*x))/(d*f - c*g)] - m*n*Log[(a + b
*x)/(c + d*x)]^2*Log[((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))] + n*L
og[a/b + x]^2*Log[h*(f + g*x)^m] + n*Log[c/d + x]^2*Log[h*(f + g*x)^m] - 2*
n*Log[a/b + x]*Log[a + b*x]*Log[h*(f + g*x)^m] + 2*n*Log[c/d + x]*Log[a + b
*x]*Log[h*(f + g*x)^m] - 2*n*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]
*Log[h*(f + g*x)^m] + 2*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n]*Log[h*(
f + g*x)^m] + 2*n*Log[a/b + x]*Log[c + d*x]*Log[h*(f + g*x)^m] - 2*n*Log[c/
d + x]*Log[c + d*x]*Log[h*(f + g*x)^m] - 2*Log[e*((a + b*x)/(c + d*x))^n]*L
og[c + d*x]*Log[h*(f + g*x)^m] - 2*n*Log[a/b + x]*Log[(b*(c + d*x))/(b*c -
a*d)]*Log[h*(f + g*x)^m] + 2*n*(m*Log[f + g*x] - Log[h*(f + g*x)^m])*PolyLo
g[2, (d*(a + b*x))/(-(b*c) + a*d)] - 2*m*(Log[e*((a + b*x)/(c + d*x))^n] -
n*Log[(a + b*x)/(c + d*x)])*PolyLog[2, (g*(a + b*x))/(-(b*f) + a*g)] + 2*m*
n*Log[(a + b*x)/(c + d*x)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*m*n*
Log[(a + b*x)/(c + d*x)]*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c
```

```
 + d*x))] + 2*m*n*Log[f + g*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*n*
Log[h*(f + g*x)^m]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 2*m*Log[e*((a +
b*x)/(c + d*x))^n]*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)] - 2*m*n*Log[(a
+ b*x)/(c + d*x)]*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)] - 2*m*n*PolyLog[
3, (d*(a + b*x))/(b*(c + d*x))] + 2*m*n*PolyLog[3, ((d*f - c*g)*(a + b*x))/
((b*f - a*g)*(c + d*x))])/(2*b*c - 2*a*d)
```

Maple [F] time = 4.115, size = 0, normalized size = 0.

$$\int \frac{\ln\left(h\left(gx+f\right)^{m}\right)}{(bx+a)\,(dx+c)}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)*ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x, al
gorithm="maxima")
```

```
[Out] -1/2*(n*log(b*x + a)^2 + n*log(d*x + c)^2 - 2*(n*log(b*x + a) - log(e))*log
(d*x + c) - 2*(log(b*x + a) - log(d*x + c))*log((b*x + a)^n) + 2*(log(b*x +
 a) - log(d*x + c))*log((d*x + c)^n) - 2*log(b*x + a)*log(e))*log((g*x + f)
^m)/(b*c - a*d) + integrate(1/2*(2*b*c*f*log(e)*log(h) - 2*a*d*f*log(e)*log
(h) + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a)^
2 + (b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(d*x + c)^2
+ 2*(b*c*g*log(e)*log(h) - a*d*g*log(e)*log(h))*x - 2*(b*d*g*m*x^2*log(e) +
 a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x)*log(b*x + a) + 2*(b*
d*g*m*x^2*log(e) + a*c*g*m*log(e) + (b*c*g*m*log(e) + a*d*g*m*log(e))*x - (
b*d*g*m*n*x^2 + a*c*g*m*n + (b*c*g*m*n + a*d*g*m*n)*x)*log(b*x + a))*log(d*
x + c) + 2*(b*c*f*log(h) - a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x -
 (b*d*g*m*x^2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^
2 + a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(d*x + c))*log((b*x + a)^n) - 2*(b*
c*f*log(h) - a*d*f*log(h) + (b*c*g*log(h) - a*d*g*log(h))*x - (b*d*g*m*x^2
+ a*c*g*m + (b*c*g*m + a*d*g*m)*x)*log(b*x + a) + (b*d*g*m*x^2 + a*c*g*m +
(b*c*g*m + a*d*g*m)*x)*log(d*x + c))*log((d*x + c)^n))/(a*b*c^2*f - a^2*c*d
*f + (b^2*c*d*g - a*b*d^2*g)*x^3 - (a*b*d^2*f + a^2*d^2*g - (c*d*f + c^2*g)
*b^2)*x^2 + (b^2*c^2*f + a*b*c^2*g - (d^2*f + c*d*g)*a^2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(gx+f\right)^{m}h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{bdx^{2}+ac+(bc+ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x, al
gorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)/(b*d*x^2 + a*c +
 (b*c + a*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)*ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)*log(h*(g*x+f)^m)/(b*x+a)/(d*x+c),x, al
gorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)*log(e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*x
 + c)), x)
```

3.70 $\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal. Leaf size=106

$$\frac{b\text{Unintegrable}\left(\frac{\log\left(h(f+gx)^m\right)}{(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)}{bc-ad}-\frac{d\text{Unintegrable}\left(\frac{\log\left(h(f+gx)^m\right)}{(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)}{bc-ad}$$

```
[Out] (b*Unintegrable[Log[h*(f + g*x)^m]/((a + b*x)*Log[e*((a + b*x)/(c + d*x))^n
]), x])/(b*c - a*d) - (d*Unintegrable[Log[h*(f + g*x)^m]/((c + d*x)*Log[e*(
(a + b*x)/(c + d*x))^n]), x])/(b*c - a*d)
```

Rubi [A] time = 0.500247, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n])
,x]
```

```
[Out] (b*Defer[Int][Log[h*(f + g*x)^m]/((a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])
, x])/(b*c - a*d) - (d*Defer[Int][Log[h*(f + g*x)^m]/((c + d*x)*Log[e*((a +
 b*x)/(c + d*x))^n]), x])/(b*c - a*d)
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx &= \int \left(\frac{b\log\left(h(f+gx)^m\right)}{(bc-ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}-\frac{d\log\left(h(f+gx)^m\right)}{(bc-ad)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\right)dx\\ &= \frac{b\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx}{bc-ad}-\frac{d\int \frac{\log\left(h(f+gx)^m\right)}{(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx}{bc-ad}\end{aligned}$$

Mathematica [A] time = 1.90844, size = 0, normalized size = 0.

$$\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x
))^n]),x]
```

```
[Out] Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x
))^n]), x]
```

Maple [A] time = 38.288, size = 0, normalized size = 0.

$$\int \frac{\ln \left(h \left(gx+f\right)^{m}\right)}{\left(bx+a\right)\left(dx+c\right)}\left(\ln \left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n),x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)}{(bx+a)(dx+c)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, al
gorithm="maxima")
```

```
[Out] integrate(log((g*x + f)^m*h)/((b*x + a)*(d*x + c)*log(e*((b*x + a)/(d*x + c
))^n)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(gx+f\right)^{m}h\right)}{\left(bdx^{2}+ac+(bc+ad)x\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, al
gorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)/((b*d*x^2 + a*c + (b*c + a*d)*x)*log(e*((b*x +
a)/(d*x + c))^n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))**n),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)}{(bx+a)(dx+c)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, al
gorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/((b*x + a)*(d*x + c)*log(e*((b*x + a)/(d*x + c
))^n)), x)
```

3.71 $\int \frac{\log \left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal. Leaf size=92

$$\frac{gm\text{Unintegrable}\left(\frac{1}{(f+gx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)},x\right)}{n(bc-ad)}-\frac{\log\left(h(f+gx)^m\right)}{n(bc-ad)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}$$

```
[Out] -(Log[h*(f + g*x)^m]/((b*c - a*d)*n*Log[e*((a + b*x)/(c + d*x))^n])) + (g*m
*Unintegrable[1/((f + g*x)*Log[e*((a + b*x)/(c + d*x))^n]), x])/((b*c - a*d
)*n)
```

Rubi [A] time = 0.128186, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n]^
2),x]
```

```
[Out] -(Log[h*(f + g*x)^m]/((b*c - a*d)*n*Log[e*((a + b*x)/(c + d*x))^n])) + (g*m
*Defer[Int][1/((f + g*x)*Log[e*((a + b*x)/(c + d*x))^n]), x])/((b*c - a*d)*
n)
```

Rubi steps

$$\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=-\frac{\log\left(h(f+gx)^m\right)}{(bc-ad)n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}+\frac{(gm)\int \frac{1}{(f+gx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx}{(bc-ad)n}$$

Mathematica [A] time = 1.28668, size = 0, normalized size = 0.

$$\int \frac{\log\left(h(f+gx)^m\right)}{(a+bx)(c+dx)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x
))^n]^2),x]
```

```
[Out] Integrate[Log[h*(f + g*x)^m]/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x
))^n]^2), x]
```

Maple [A] time = 48.165, size = 0, normalized size = 0.

$$\int \frac{\ln\left(h\left(gx+f\right)^{m}\right)}{\left(bx+a\right)\left(dx+c\right)}\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n)^2,x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$gm\int \frac{1}{bcfn\log(e) - adfn\log(e) + \left(bcgn\log(e) - adgn\log(e)\right)x + \left(bcfn - adfn + \left(bcgn - adgn\right)x\right)\log\left((bx+a)^{n}\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n)^2,x,
algorithm="maxima")
```

```
[Out] g*m*integrate(1/(b*c*f*n*log(e) - a*d*f*n*log(e) + (b*c*g*n*log(e) - a*d*g*
n*log(e))*x + (b*c*f*n - a*d*f*n + (b*c*g*n - a*d*g*n)*x)*log((b*x + a)^n)
- (b*c*f*n - a*d*f*n + (b*c*g*n - a*d*g*n)*x)*log((d*x + c)^n)), x) - (log(
(g*x + f)^m) + log(h))/(b*c*n*log(e) - a*d*n*log(e) + (b*c*n - a*d*n)*log((
b*x + a)^n) - (b*c*n - a*d*n)*log((d*x + c)^n))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(gx+f\right)^{m}h\right)}{\left(bdx^{2}+ac+(bc+ad)x\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n)^2,x,
algorithm="fricas")
```

```
[Out] integral(log((g*x + f)^m*h)/((b*d*x^2 + a*c + (b*c + a*d)*x)*log(e*((b*x +
a)/(d*x + c))^n)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))**n)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(gx+f\right)^{m}h\right)}{(bx+a)(dx+c)\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n)^2,x,
algorithm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/((b*x + a)*(d*x + c)*log(e*((b*x + a)/(d*x + c
))^n)^2), x)
```

3.72 $\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$

Optimal. Leaf size=112

$$\frac{b\text{CannotIntegrate}\left(\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}-\frac{d\text{CannotIntegrate}\left(\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}$$

```
[Out] (b*CannotIntegrate[Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*Log[(a + b*x)/(c
 + d*x)]^2), x])/(b*c - a*d) - (d*CannotIntegrate[Log[1 - (a + b*x)/(c + d*
x)]/((c + d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi [A] time = 0.519287, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*
x)]^2),x]
```

```
[Out] (b*Defer[Int][Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*Log[(a + b*x)/(c + d*
x)]^2), x])/(b*c - a*d) - (d*Defer[Int][Log[1 - (a + b*x)/(c + d*x)]/((c +
d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx&=\int \left(\frac{b\log\left(1-\frac{a+bx}{c+dx}\right)}{(bc-ad)(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}-\frac{d\log\left(1-\frac{a+bx}{c+dx}\right)}{(bc-ad)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx\\&=\frac{b\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad}-\frac{d\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad}\end{aligned}$$

Mathematica [A] time = 0.631728, size = 0, normalized size = 0.

$$\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(
c + d*x)]^2),x]
```

```
[Out] Integrate[Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(
c + d*x)]^2), x]
```

Maple [A] time = 1.07, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)}\ln\left(1+\frac{-bx-a}{dx+c}\right)\left(\ln\left(\frac{bx+a}{dx+c}\right)\right)^{-2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

```
[Out] int(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\log(-(b-d)x-a+c)-\log(bx+a)}{(bc-ad)\log(bx+a)-(bc-ad)\log(dx+c)}-\int -\frac{1}{\left(\left(bd-d^2\right)x^2+ac-c^2+(bc+ad-2\,cd)x\right)\log(bx+a)-\left(\left(bd-d^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="maxima")
```

```
[Out] -(log(-(b - d)*x - a + c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c
- a*d)*log(d*x + c)) - integrate(-1/(((b*d - d^2)*x^2 + a*c - c^2 + (b*c +
a*d - 2*c*d)*x)*log(b*x + a) - ((b*d - d^2)*x^2 + a*c - c^2 + (b*c + a*d -
2*c*d)*x)*log(d*x + c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(-\frac{(b-d)x+a-c}{dx+c}\right)}{\left(bdx^2+ac+(bc+ad)x\right)\log\left(\frac{bx+a}{dx+c}\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="fricas")
```

```
[Out] integral(log(-((b - d)*x + a - c)/(d*x + c))/((b*d*x^2 + a*c + (b*c + a*d)*
x)*log((b*x + a)/(d*x + c))^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(-\frac{bx+a}{dx+c}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-b*x-a)/(d*x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="giac")
```

```
[Out] integrate(log(-(b*x + a)/(d*x + c) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/
(d*x + c))^2), x)
```

3.73 $\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$

Optimal. Leaf size=112

$$\frac{b\text{CannotIntegrate}\left(\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}-\frac{d\text{CannotIntegrate}\left(\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)},x\right)}{bc-ad}$$

```
[Out] (b*CannotIntegrate[Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*Log[(a + b*x)/(c
 + d*x)]^2), x])/(b*c - a*d) - (d*CannotIntegrate[Log[1 - (c + d*x)/(a + b*
x)]/((c + d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi [A] time = 0.483652, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*
x)]^2),x]
```

```
[Out] (b*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*Log[(a + b*x)/(c + d*
x)]^2), x])/(b*c - a*d) - (d*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((c +
d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx &= \int \left(\frac{b\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}-\frac{d\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx\\ &= \frac{b\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad}-\frac{d\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad}\end{aligned}$$

Mathematica [A] time = 0.574993, size = 0, normalized size = 0.

$$\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(
c + d*x)]^2),x]
```

```
[Out] Integrate[Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(
c + d*x)]^2), x]
```

Maple [A] time = 1.008, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)} \ln\left(1+\frac{-dx-c}{bx+a}\right)\left(\ln\left(\frac{bx+a}{dx+c}\right)\right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

```
[Out] int(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\log((b-d)x+a-c)-\log(bx+a)}{(bc-ad)\log(bx+a)-(bc-ad)\log(dx+c)} - \int -\frac{1}{\left(\left(b^2-bd\right)x^2+a^2-ac+(a(2b-d)-bc)x\right)\log(bx+a)-\left(\left(b^2-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="maxima")
```

```
[Out] -(log((b - d)*x + a - c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c -
 a*d)*log(d*x + c)) - integrate(-1/(((b^2 - b*d)*x^2 + a^2 - a*c + (a*(2*b
- d) - b*c)*x)*log(b*x + a) - ((b^2 - b*d)*x^2 + a^2 - a*c + (a*(2*b - d) -
 b*c)*x)*log(d*x + c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\frac{(b-d)x+a-c}{bx+a}\right)}{\left(bdx^2+ac+(bc+ad)x\right)\log\left(\frac{bx+a}{dx+c}\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="fricas")
```

```
[Out] integral(log(((b - d)*x + a - c)/(b*x + a))/((b*d*x^2 + a*c + (b*c + a*d)*x
)*log((b*x + a)/(d*x + c))^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(-\frac{dx+c}{bx+a}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+(-d*x-c)/(b*x+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x,
 algorithm="giac")
```

```
[Out] integrate(log(-(d*x + c)/(b*x + a) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/
(d*x + c))^2), x)
```

3.74 $\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$

Optimal. Leaf size=45

$$-\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(bc-ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

```
[Out] -(Log[1 - (a + b*x)/(c + d*x)]/((b*c - a*d)*Log[(a + b*x)/(c + d*x)]))
```

Rubi [F] time = 0.515643, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(-a + c + (-b + d)*x)*Log[(a + b*x)/(c + d*x)]) + Log[1 -
(a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(-a + c + (-b + d)*x)*Log[(a + b*x)/(c + d*x)]), x]
 + (b*Defer[Int][Log[1 - (a + b*x)/(c + d*x)]/((a + b*x)*Log[(a + b*x)/(c +
 d*x)]^2), x])/(b*c - a*d) - (d*Defer[Int][Log[1 - (a + b*x)/(c + d*x)]/((c
 + d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi steps

$$\int \left(\frac{1}{(c+dx)(-a+c+(-b+d)x)\log\left(\frac{a+bx}{c+dx}\right)}+\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)}\right)dx=\int \frac{1}{(c+dx)(-a+c+(-b+d)x)\log$$

$$=\int \frac{1}{(c+dx)(-a+c+(-b+d)x)\log$$

$$=\frac{b\int \frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}dx}{bc-ad}-\frac{d\int \frac{\log(1-}{(c+dx)\log}}{bc-}$$

Mathematica [A] time = 0.311511, size = 44, normalized size = 0.98

$$\frac{\log\left(1-\frac{a+bx}{c+dx}\right)}{(ad-bc)\log\left(\frac{a+bx}{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(-a + c + (-b + d)*x)*Log[(a + b*x)/(c + d*x)]) + Lo
g[1 - (a + b*x)/(c + d*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2)
,x]
```

```
[Out] Log[1 - (a + b*x)/(c + d*x)]/((-(b*c) + a*d)*Log[(a + b*x)/(c + d*x)])
```

Maple [C] time = 1.322, size = 662, normalized size = 14.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(-a+c+(-b+d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-b*x-a)/(d*x+c))/(b
*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

```
[Out] 2*I/(a*d-b*c)/(csgn(I*(b*x+a)/(d*x+c))*csgn(I*(b*x+a))*csgn(I/(d*x+c))*Pi-c
sgn(I*(b*x+a)/(d*x+c))^2*csgn(I*(b*x+a))*Pi-csgn(I*(b*x+a)/(d*x+c))^2*csgn(
I/(d*x+c))*Pi+csgn(I*(b*x+a)/(d*x+c))^3*Pi+2*I*ln(b*x+a)-2*I*ln(d*x+c))*ln(
b*x-d*x+a-c)-(-I*csgn(I*(b*x+a)/(d*x+c))*csgn(I*(b*x+a))*csgn(I/(d*x+c))*Pi
+I*csgn(I*(b*x+a)/(d*x+c))^2*csgn(I*(b*x+a))*Pi+I*csgn(I*(b*x+a)/(d*x+c))^2
*csgn(I/(d*x+c))*Pi+I*Pi*csgn(I/(d*x+c))*csgn(I*(b*x-d*x+a-c))*csgn(I/(d*x+
c)*(b*x-d*x+a-c))-I*Pi*csgn(I/(d*x+c))*csgn(I/(d*x+c)*(b*x-d*x+a-c))^2-I*cs
gn(I*(b*x+a)/(d*x+c))^3*Pi+2*I*Pi*csgn(I/(d*x+c)*(b*x-d*x+a-c))^2-I*Pi*csgn
(I*(b*x-d*x+a-c))*csgn(I/(d*x+c)*(b*x-d*x+a-c))^2-I*Pi*csgn(I/(d*x+c)*(b*x-
d*x+a-c))^3-2*I*Pi+2*ln(b*x+a))/(a*d-b*c)/(-I*csgn(I*(b*x+a)/(d*x+c))^3*Pi+
I*csgn(I*(b*x+a)/(d*x+c))^2*csgn(I*(b*x+a))*Pi+I*csgn(I*(b*x+a)/(d*x+c))^2*
csgn(I/(d*x+c))*Pi-I*csgn(I*(b*x+a)/(d*x+c))*csgn(I*(b*x+a))*csgn(I/(d*x+c)
)*Pi+2*ln(b*x+a)-2*ln(d*x+c))
```

Maxima [A] time = 1.49692, size = 80, normalized size = 1.78

$$-\frac{\log\left(-(b-d)x-a+c\right)-\log\left(bx+a\right)}{(bc-ad)\log\left(bx+a\right)-(bc-ad)\log\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/log((b*x+a)/(d*x+c))+log(1+(-b*x-a)/(d*
x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(log(-(b - d)*x - a + c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c
- a*d)*log(d*x + c))
```

Fricas [A] time = 2.06832, size = 104, normalized size = 2.31

$$-\frac{\log\left(-\frac{(b-d)x+a-c}{dx+c}\right)}{(bc-ad)\log\left(\frac{bx+a}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/log((b*x+a)/(d*x+c))+log(1+(-b*x-a)/(d*
x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -log(-((b - d)*x + a - c)/(d*x + c))/((b*c - a*d)*log((b*x + a)/(d*x + c)))
```

Sympy [A] time = 3.05976, size = 44, normalized size = 0.98

$$\frac{\log\left(\frac{-a-bx}{c+dx}+1\right)}{ad\log\left(\frac{a+bx}{c+dx}\right)-bc\log\left(\frac{a+bx}{c+dx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-b*x-a)/(d*x+
c))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

```
[Out] log((-a - b*x)/(c + d*x) + 1)/(a*d*log((a + b*x)/(c + d*x)) - b*c*log((a +
b*x)/(c + d*x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{((b-d)x+a-c)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)}+\frac{\log\left(-\frac{bx+a}{dx+c}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(-a+c+(-b+d)*x)/log((b*x+a)/(d*x+c))+log(1+(-b*x-a)/(d*
x+c))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(-1/(((b - d)*x + a - c)*(d*x + c)*log((b*x + a)/(d*x + c))) + log
(-(b*x + a)/(d*x + c) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/(d*x + c))^2)
, x)
```

3.75 $\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)} \right) dx$

Optimal. Leaf size=45

$$-\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

```
[Out] -(Log[1 - (c + d*x)/(a + b*x)]/((b*c - a*d)*Log[(a + b*x)/(c + d*x)]))
```

Rubi [F] time = 0.512104, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)} \right) dx$$

Verification is Not applicable to the result.

```
[In] Int[-(1/((a + b*x)*(a - c + (b - d)*x)*Log[(a + b*x)/(c + d*x)])) + Log[1 -
 (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2),x]
```

```
[Out] -Defer[Int][1/((a + b*x)*(a - c + (b - d)*x)*Log[(a + b*x)/(c + d*x)]), x]
+ (b*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((a + b*x)*Log[(a + b*x)/(c +
d*x)]^2), x])/(b*c - a*d) - (d*Defer[Int][Log[1 - (c + d*x)/(a + b*x)]/((c
+ d*x)*Log[(a + b*x)/(c + d*x)]^2), x])/(b*c - a*d)
```

Rubi steps

$$\begin{aligned}
\int \left(-\frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)} + \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)(c+dx)\log^2\left(\frac{a+bx}{c+dx}\right)} \right) dx &= -\int \frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)} \\
&= -\int \frac{1}{(a+bx)(a-c+(b-d)x)\log\left(\frac{a+bx}{c+dx}\right)} \\
&= \frac{b\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(a+bx)\log^2\left(\frac{a+bx}{c+dx}\right)}\,dx}{bc-ad} - \frac{d\int \frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(c+dx)\log^2\left(\frac{a+b}{c+d}\right.}}{bc-ad}
\end{aligned}$$

Mathematica [A] time = 0.0851547, size = 45, normalized size = 1.

$$-\frac{\log\left(1-\frac{c+dx}{a+bx}\right)}{(bc-ad)\log\left(\frac{a+bx}{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[-(1/((a + b*x)*(a - c + (b - d)*x)*Log[(a + b*x)/(c + d*x)])) + L
og[1 - (c + d*x)/(a + b*x)]/((a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^2
),x]
```

```
[Out] -(Log[1 - (c + d*x)/(a + b*x)]/((b*c - a*d)*Log[(a + b*x)/(c + d*x)]))
```

Maple [C] time = 1.214, size = 503, normalized size = 11.2

$$\frac{2i\ln(bx-dx+a-c)}{ad-bc}\left(\operatorname{csgn}\left(\frac{i(bx+a)}{dx+c}\right)\operatorname{csgn}(i(bx+a))\operatorname{csgn}\left(\frac{i}{dx+c}\right)\pi-\left(\operatorname{csgn}\left(\frac{i(bx+a)}{dx+c}\right)\right)^2\operatorname{csgn}(i(bx+a))\pi -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(b*x+a)/(a-c+(b-d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-d*x-c)/(b*x+a))/(b*
x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))^2,x)
```

```
[Out] 2*I/(a*d-b*c)/(csgn(I*(b*x+a)/(d*x+c))*csgn(I*(b*x+a))*csgn(I/(d*x+c))*Pi-c
sgn(I*(b*x+a)/(d*x+c))^2*csgn(I*(b*x+a))*Pi-csgn(I*(b*x+a)/(d*x+c))^2*csgn(
I/(d*x+c))*Pi+csgn(I*(b*x+a)/(d*x+c))^3*Pi+2*I*ln(b*x+a)-2*I*ln(d*x+c))*ln(
b*x-d*x+a-c)-(I*Pi*csgn(I*(b*x-d*x+a-c))*csgn(I/(b*x+a))*csgn(I/(b*x+a)*(b*
x-d*x+a-c))-I*Pi*csgn(I*(b*x-d*x+a-c))*csgn(I/(b*x+a)*(b*x-d*x+a-c))^2-I*Pi
*csgn(I/(b*x+a))*csgn(I/(b*x+a)*(b*x-d*x+a-c))^2+I*Pi*csgn(I/(b*x+a)*(b*x-d
*x+a-c))^3+2*ln(b*x+a))/(a*d-b*c)/(-I*csgn(I*(b*x+a)/(d*x+c))^3*Pi+I*csgn(I
*(b*x+a)/(d*x+c))^2*csgn(I*(b*x+a))*Pi+I*csgn(I*(b*x+a)/(d*x+c))^2*csgn(I/(
d*x+c))*Pi-I*csgn(I*(b*x+a)/(d*x+c))*csgn(I*(b*x+a))*csgn(I/(d*x+c))*Pi+2*l
n(b*x+a)-2*ln(d*x+c))
```

Maxima [A] time = 1.67998, size = 78, normalized size = 1.73

$$-\frac{\log\left((b-d)x+a-c\right)-\log\left(bx+a\right)}{(bc-ad)\log\left(bx+a\right)-(bc-ad)\log\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b*x+a)/(a-c+(b-d)*x)/log((b*x+a)/(d*x+c))+log(1+(-d*x-c)/(b*x
+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(log((b - d)*x + a - c) - log(b*x + a))/((b*c - a*d)*log(b*x + a) - (b*c -
 a*d)*log(d*x + c))
```

Fricas [A] time = 1.99166, size = 103, normalized size = 2.29

$$-\frac{\log\left(\frac{(b-d)x+a-c}{bx+a}\right)}{(bc-ad)\log\left(\frac{bx+a}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b*x+a)/(a-c+(b-d)*x)/log((b*x+a)/(d*x+c))+log(1+(-d*x-c)/(b*x
+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -log(((b - d)*x + a - c)/(b*x + a))/((b*c - a*d)*log((b*x + a)/(d*x + c)))
```

Sympy [A] time = 3.32851, size = 44, normalized size = 0.98

$$\frac{\log\left(1+\frac{-c-dx}{a+bx}\right)}{ad\log\left(\frac{a+bx}{c+dx}\right)-bc\log\left(\frac{a+bx}{c+dx}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b*x+a)/(a-c+(b-d)*x)/ln((b*x+a)/(d*x+c))+ln(1+(-d*x-c)/(b*x+a
))/(b*x+a)/(d*x+c)/ln((b*x+a)/(d*x+c))**2,x)
```

```
[Out] log(1 + (-c - d*x)/(a + b*x))/(a*d*log((a + b*x)/(c + d*x)) - b*c*log((a +
b*x)/(c + d*x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{((b-d)x+a-c)(bx+a)\log\left(\frac{bx+a}{dx+c}\right)}+\frac{\log\left(-\frac{dx+c}{bx+a}+1\right)}{(bx+a)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b*x+a)/(a-c+(b-d)*x)/log((b*x+a)/(d*x+c))+log(1+(-d*x-c)/(b*x
+a))/(b*x+a)/(d*x+c)/log((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(-1/(((b - d)*x + a - c)*(b*x + a)*log((b*x + a)/(d*x + c))) + log
(-(d*x + c)/(b*x + a) + 1)/((b*x + a)*(d*x + c)*log((b*x + a)/(d*x + c))^2)
, x)
```

3.76 $\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$

Optimal. Leaf size=560

$$-\frac{fn\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^2}-\frac{fn\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^2}+\frac{fn\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^2}+\frac{fn\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g^2}+$$

```
[Out] -(a*n*x)/(2*b*g) + (c*n*x)/(2*d*g) + (a^2*n*Log[a + b*x])/(2*b^2*g) - (n*x^
2*Log[a + b*x])/(2*g) - (c^2*n*Log[c + d*x])/(2*d^2*g) + (n*x^2*Log[c + d*x
])/(2*g) + (x^2*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c
+ d*x]))/(2*g) - (f*n*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f]
 + a*Sqrt[g])])/(2*g^2) + (f*n*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(
d*Sqrt[f] + c*Sqrt[g])])/(2*g^2) - (f*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt
[g]*x))/(b*Sqrt[f] - a*Sqrt[g])])/(2*g^2) + (f*n*Log[c + d*x]*Log[(d*(Sqrt[
f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])])/(2*g^2) + (f*(n*Log[a + b*x] - L
og[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f - g*x^2])/(2*g^2) - (
f*n*PolyLog[2, -((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*g^2) - (
f*n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^2) + (f*n
*PolyLog[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*g^2) + (f*n
*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^2)
```

Rubi [A] time = 0.734765, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2513, 266, 43, 2416, 2395, 260, 2394, 2393, 2391}

$$-\frac{fn\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^2}-\frac{fn\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^2}+\frac{fn\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^2}+\frac{fn\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```


```
[Out] -(a*n*x)/(2*b*g) + (c*n*x)/(2*d*g) + (a^2*n*Log[a + b*x])/(2*b^2*g) - (n*x^
2*Log[a + b*x])/(2*g) - (c^2*n*Log[c + d*x])/(2*d^2*g) + (n*x^2*Log[c + d*x
])/(2*g) + (x^2*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c
+ d*x]))/(2*g) - (f*n*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f]
 + a*Sqrt[g])])/(2*g^2) + (f*n*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(
d*Sqrt[f] + c*Sqrt[g])])/(2*g^2) - (f*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt
[g]*x))/(b*Sqrt[f] - a*Sqrt[g])])/(2*g^2) + (f*n*Log[c + d*x]*Log[(d*(Sqrt[
f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])])/(2*g^2) + (f*(n*Log[a + b*x] - L
og[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f - g*x^2])/(2*g^2) - (
f*n*PolyLog[2, -((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*g^2) - (
f*n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^2) + (f*n
*PolyLog[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*g^2) + (f*n
*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^2)
```


Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
```

```
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = n\int \frac{x^3\log(a+bx)}{f-gx^2}\,dx - n\int\frac{x^3\log(c+dx)}{f-gx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\,\mathrm{lo}\right.$$

$$= n\int\left(-\frac{x\log(a+bx)}{g} + \frac{fx\log(a+bx)}{g\left(f-gx^2\right)}\right)dx - n\int\left(-\frac{x\log(c+dx)}{g} + \frac{fx\log(c+dx)}{g\left(f-gx^2\right)}\right)dx -$$

$$= -\frac{n\int x\log(a+bx)\,dx}{g} + \frac{n\int x\log(c+dx)\,dx}{g} + \frac{(fn)\int\frac{x\log(a+bx)}{f-gx^2}\,dx}{g} - \frac{(fn)\int\frac{x\log(c+dx)}{f-gx^2}\,dx}{g}$$

$$= -\frac{nx^2\log(a+bx)}{2g} + \frac{nx^2\log(c+dx)}{2g} + \frac{x^2\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{2g}$$

$$= -\frac{nx^2\log(a+bx)}{2g} + \frac{nx^2\log(c+dx)}{2g} + \frac{x^2\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{2g}$$

$$= -\frac{anx}{2bg} + \frac{cnx}{2dg} + \frac{a^2n\log(a+bx)}{2b^2g} - \frac{nx^2\log(a+bx)}{2g} - \frac{c^2n\log(c+dx)}{2d^2g} + \frac{nx^2\log(c+dx)}{2g} + \cdots$$

$$= -\frac{anx}{2bg} + \frac{cnx}{2dg} + \frac{a^2n\log(a+bx)}{2b^2g} - \frac{nx^2\log(a+bx)}{2g} - \frac{c^2n\log(c+dx)}{2d^2g} + \frac{nx^2\log(c+dx)}{2g} + \cdots$$

$$= -\frac{anx}{2bg} + \frac{cnx}{2dg} + \frac{a^2n\log(a+bx)}{2b^2g} - \frac{nx^2\log(a+bx)}{2g} - \frac{c^2n\log(c+dx)}{2d^2g} + \frac{nx^2\log(c+dx)}{2g} + \cdots$$

Mathematica [A] time = 0.387355, size = 461, normalized size = 0.82

$$fn\left(\mathrm{PolyLog}\left(2,\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{a\sqrt{g}+b\sqrt{f}}\right) - \mathrm{PolyLog}\left(2,\frac{d\left(\sqrt{f}-\sqrt{g}x\right)}{c\sqrt{g}+d\sqrt{f}}\right) + \log\left(\sqrt{f}-\sqrt{g}x\right)\left(\log\left(\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right) - \log\left(\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)\right)\right) + fn\left(\mathrm{Po}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] (-(g*x^2*Log[e*((a + b*x)/(c + d*x))^n]) + (g*n*(a^2*d^2*Log[a + b*x] - b*(
d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])))/(b^2*d^2) - f*Log[e*((a + b*x)/(
c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] - f*Log[e*((a + b*x)/(c + d*x))^n]*Lo
g[Sqrt[f] + Sqrt[g]*x] + f*n*((Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[
g])] - Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])*Log[Sqrt[f] - Sqrt
[g]*x] + PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] - Po
lyLog[2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])]) + f*n*((Log[-(
(Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))] - Log[-((Sqrt[g]*(c + d*x))/(
d*Sqrt[f] - c*Sqrt[g]))])*Log[Sqrt[f] + Sqrt[g]*x] + PolyLog[2, (b*(Sqrt[f]
 + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] - PolyLog[2, (d*(Sqrt[f] + Sqrt[g]*
x))/(d*Sqrt[f] - c*Sqrt[g])]))/(2*g^2)
```

Maple [F] time = 0.493, size = 0, normalized size = 0.

$$\int \frac{x^3}{-gx^2+f}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

```
[Out] int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] -integrate(x^3*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(-x^3*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")

[Out] integrate(-x^3*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

3.77 $\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$

Optimal. Leaf size=550

$$\frac{\sqrt{f}n\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}}-\frac{\sqrt{f}n\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^{3/2}}-\frac{\sqrt{f}n\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}}+\frac{\sqrt{f}n\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g^{3/2}}$$

```
[Out] -((n*(a + b*x)*Log[a + b*x])/(b*g)) + (n*(c + d*x)*Log[c + d*x])/(d*g) + (x
*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/g - (S
qrt[f]*ArcTanh[(Sqrt[g]*x)/Sqrt[f]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c +
 d*x))^n] - n*Log[c + d*x]))/g^(3/2) - (Sqrt[f]*n*Log[a + b*x]*Log[(b*(Sqrt
[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^(3/2)) + (Sqrt[f]*n*Log[c
+ d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^(3/2))
+ (Sqrt[f]*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt
[g])])/(2*g^(3/2)) - (Sqrt[f]*n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/
(d*Sqrt[f] - c*Sqrt[g])])/(2*g^(3/2)) + (Sqrt[f]*n*PolyLog[2, -((Sqrt[g]*(a
 + b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*g^(3/2)) - (Sqrt[f]*n*PolyLog[2, (Sq
rt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^(3/2)) - (Sqrt[f]*n*PolyLog
[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*g^(3/2)) + (Sqrt[f]
*n*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^(3/2))
```

Rubi [A] time = 0.565428, antiderivative size = 550, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2513, 321, 208, 2416, 2389, 2295, 2409, 2394, 2393, 2391}

$$\frac{\sqrt{f}n\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g^{3/2}}-\frac{\sqrt{f}n\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g^{3/2}}-\frac{\sqrt{f}n\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g^{3/2}}+\frac{\sqrt{f}n\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```


```
[Out] -((n*(a + b*x)*Log[a + b*x])/(b*g)) + (n*(c + d*x)*Log[c + d*x])/(d*g) + (x
*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/g - (S
qrt[f]*ArcTanh[(Sqrt[g]*x)/Sqrt[f]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c +
 d*x))^n] - n*Log[c + d*x]))/g^(3/2) - (Sqrt[f]*n*Log[a + b*x]*Log[(b*(Sqrt
[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^(3/2)) + (Sqrt[f]*n*Log[c
+ d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^(3/2))
+ (Sqrt[f]*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt
[g])])/(2*g^(3/2)) - (Sqrt[f]*n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/
(d*Sqrt[f] - c*Sqrt[g])])/(2*g^(3/2)) + (Sqrt[f]*n*PolyLog[2, -((Sqrt[g]*(a
 + b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*g^(3/2)) - (Sqrt[f]*n*PolyLog[2, (Sq
rt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*g^(3/2)) - (Sqrt[f]*n*PolyLog
[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*g^(3/2)) + (Sqrt[f]
*n*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*g^(3/2))
```


Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
```

```
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = n\int \frac{x^2\log(a+bx)}{f-gx^2}\,dx - n\int \frac{x^2\log(c+dx)}{f-gx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c$$

$$= \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{g} + n\int\left(-\frac{\log(a+bx)}{g} + \frac{f\log(a+bx)}{g\left(f-gx^2\right)}\right)dx$$

$$= \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{g} - \frac{\sqrt{f}\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\right.\right.}{g^{3/2}}$$

$$= \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{g} - \frac{\sqrt{f}\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\right.\right.}{g^{3/2}}$$

$$= -\frac{n(a+bx)\log(a+bx)}{bg} + \frac{n(c+dx)\log(c+dx)}{dg} + \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c\right.}{g}$$

$$= -\frac{n(a+bx)\log(a+bx)}{bg} + \frac{n(c+dx)\log(c+dx)}{dg} + \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c\right.}{g}$$

$$= -\frac{n(a+bx)\log(a+bx)}{bg} + \frac{n(c+dx)\log(c+dx)}{dg} + \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c\right.}{g}$$

$$= -\frac{n(a+bx)\log(a+bx)}{bg} + \frac{n(c+dx)\log(c+dx)}{dg} + \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c\right.}{g}$$

Mathematica [A] time = 0.24023, size = 467, normalized size = 0.85

$$\sqrt{f}n\left(\text{PolyLog}\left(2,\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{a\sqrt{g}+b\sqrt{f}}\right) - \text{PolyLog}\left(2,\frac{d\left(\sqrt{f}-\sqrt{g}x\right)}{c\sqrt{g}+d\sqrt{f}}\right) + \log\left(\sqrt{f}-\sqrt{g}x\right)\left(\log\left(\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right) - \log\left(\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)\right)\right) - \sqrt{f}n\left(\text{Po}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] ((-2*Sqrt[g]*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/b + (2*(b*c - a*d)*S
qrt[g]*n*Log[c + d*x])/(b*d) - Sqrt[f]*Log[e*((a + b*x)/(c + d*x))^n]*Log[S
qrt[f] - Sqrt[g]*x] + Sqrt[f]*Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] +
Sqrt[g]*x] + Sqrt[f]*n*((Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])] -
 Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])*Log[Sqrt[f] - Sqrt[g]*x]
 + PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] - PolyLog[
2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])]) - Sqrt[f]*n*((Log[-(
(Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))] - Log[-((Sqrt[g]*(c + d*x))/(
d*Sqrt[f] - c*Sqrt[g]))])*Log[Sqrt[f] + Sqrt[g]*x] + PolyLog[2, (b*(Sqrt[f]
 + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] - PolyLog[2, (d*(Sqrt[f] + Sqrt[g]*
x))/(d*Sqrt[f] - c*Sqrt[g])]))/(2*g^(3/2))
```

Maple [F] time = 0.418, size = 0, normalized size = 0.

$$\int \frac{x^2}{-g x^2+f} \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

```
[Out] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{g x^2-f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(-x^2*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{g x^2-f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(-x^2*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

3.78 $\int \frac{x \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$

Optimal. Leaf size=403

$$-\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g}-\frac{n\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g}+\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g}+\frac{n\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g}+\frac{\log(\ldots}{}$$

```
[Out] -(n*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2
*g) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])
])/(2*g) - (n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqr
t[g])])/(2*g) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f] -
c*Sqrt[g])])/(2*g) + ((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*
Log[c + d*x])*Log[f - g*x^2])/(2*g) - (n*PolyLog[2, -((Sqrt[g]*(a + b*x))/(
b*Sqrt[f] - a*Sqrt[g]))])/(2*g) - (n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt
[f] + a*Sqrt[g])])/(2*g) + (n*PolyLog[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] -
 c*Sqrt[g]))])/(2*g) + (n*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqr
t[g])])/(2*g)
```

Rubi [A] time = 0.352342, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2513, 260, 2416, 2394, 2393, 2391}

$$-\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g}-\frac{n\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2g}+\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2g}+\frac{n\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2g}+\frac{\log(\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```


```
[Out] -(n*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2
*g) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])
])/(2*g) - (n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqr
t[g])])/(2*g) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f] -
c*Sqrt[g])])/(2*g) + ((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*
Log[c + d*x])*Log[f - g*x^2])/(2*g) - (n*PolyLog[2, -((Sqrt[g]*(a + b*x))/(
b*Sqrt[f] - a*Sqrt[g]))])/(2*g) - (n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*Sqrt
[f] + a*Sqrt[g])])/(2*g) + (n*PolyLog[2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] -
 c*Sqrt[g]))])/(2*g) + (n*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqr
t[g])])/(2*g)
```


Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = n\int \frac{x\log(a+bx)}{f-gx^2}\,dx - n\int \frac{x\log(c+dx)}{f-gx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+\right.$$

$$= \frac{\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f-gx^2\right)}{2g} + n\int\left(\frac{\log(a+bx)}{2\sqrt{g}\left(\sqrt{f}-\sqrt{g}x\right)} - \frac{}{2\sqrt{}}\right.$$

$$= \frac{\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f-gx^2\right)}{2g} + \frac{n\int\frac{\log(a+bx)}{\sqrt{f}-\sqrt{g}x}\,dx}{2\sqrt{g}} - \frac{n\int\frac{\log(a+b}{\sqrt{f}+\sqrt{g}}}{2\sqrt{g}}$$

$$= -\frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g} + \frac{n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}-\sqrt{g}x\right)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g} - \frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}+\sqrt{g}x\right)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g}$$

$$= -\frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g} + \frac{n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}-\sqrt{g}x\right)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g} - \frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}+\sqrt{g}x\right)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g}$$

$$= -\frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{b\sqrt{f}+a\sqrt{g}}\right)}{2g} + \frac{n\log(c+dx)\log\left(\frac{d\left(\sqrt{f}-\sqrt{g}x\right)}{d\sqrt{f}+c\sqrt{g}}\right)}{2g} - \frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}+\sqrt{g}x\right)}{b\sqrt{f}-a\sqrt{g}}\right)}{2g}$$

Mathematica [A] time = 0.121141, size = 413, normalized size = 1.02

$$-\frac{-n\text{PolyLog}\left(2,\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{a\sqrt{g}+b\sqrt{f}}\right)-n\text{PolyLog}\left(2,\frac{b\left(\sqrt{f}+\sqrt{g}x\right)}{b\sqrt{f}-a\sqrt{g}}\right)+n\text{PolyLog}\left(2,\frac{d\left(\sqrt{f}-\sqrt{g}x\right)}{c\sqrt{g}+d\sqrt{f}}\right)+n\text{PolyLog}\left(2,\frac{d\left(\sqrt{f}+\sqrt{g}x\right)}{d\sqrt{f}-c\sqrt{g}}\right)+\text{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f - g*x^2),x]
```

```
[Out] -(-(n*Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])]*Log[Sqrt[f] - Sqrt[g
]*x]) + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] + n*Log[(Sq
rt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])]*Log[Sqrt[f] - Sqrt[g]*x] - n*Log[
-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x] +
Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] + Sqrt[g]*x] + n*Log[-((Sqrt[g]*
(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x] - n*PolyLog[2
, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] + n*PolyLog[2, (d*(Sqr
t[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])] - n*PolyLog[2, (b*(Sqrt[f] + Sq
rt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] + n*PolyLog[2, (d*(Sqrt[f] + Sqrt[g]*x))
/(d*Sqrt[f] - c*Sqrt[g])])/(2*g)
```

Maple [F] time = 0.44, size = 0, normalized size = 0.

$$\int \frac{x}{-gx^2+f}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

```
[Out] int(x*ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] -integrate(x*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(-x*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x \log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{gx^{2}-f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(-x*log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

3.79 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx$

Optimal. Leaf size=291

$$\frac{n\text{PolyLog}\left(2,\frac{(a+bx)\left(d\sqrt{f}-c\sqrt{g}\right)}{(c+dx)\left(b\sqrt{f}-a\sqrt{g}\right)}\right)}{2\sqrt{f}\sqrt{g}}-\frac{n\text{PolyLog}\left(2,\frac{(a+bx)\left(c\sqrt{g}+d\sqrt{f}\right)}{(c+dx)\left(a\sqrt{g}+b\sqrt{f}\right)}\right)}{2\sqrt{f}\sqrt{g}}+\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(1-\frac{(a+bx)\left(d\sqrt{f}-c\sqrt{g}\right)}{(c+dx)\left(b\sqrt{f}-a\sqrt{g}\right)}\right)}{2\sqrt{f}\sqrt{g}}-\ldots$$

```
[Out] (Log[e*((a + b*x)/(c + d*x))^n]*Log[1 - ((d*Sqrt[f] - c*Sqrt[g])*(a + b*x))
/((b*Sqrt[f] - a*Sqrt[g])*(c + d*x))])/(2*Sqrt[f]*Sqrt[g]) - (Log[e*((a + b
*x)/(c + d*x))^n]*Log[1 - ((d*Sqrt[f] + c*Sqrt[g])*(a + b*x))/((b*Sqrt[f] +
 a*Sqrt[g])*(c + d*x))])/(2*Sqrt[f]*Sqrt[g]) + (n*PolyLog[2, ((d*Sqrt[f] -
c*Sqrt[g])*(a + b*x))/((b*Sqrt[f] - a*Sqrt[g])*(c + d*x))])/(2*Sqrt[f]*Sqrt
[g]) - (n*PolyLog[2, ((d*Sqrt[f] + c*Sqrt[g])*(a + b*x))/((b*Sqrt[f] + a*Sq
rt[g])*(c + d*x))])/(2*Sqrt[f]*Sqrt[g])
```

Rubi [A] time = 0.317435, antiderivative size = 468, normalized size of antiderivative = 1.61, number of steps used = 18, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2513, 2409, 2394, 2393, 2391, 208}

$$\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2\sqrt{f}\sqrt{g}}-\frac{n\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}}-\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2\sqrt{f}\sqrt{g}}+\frac{n\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(f - g*x^2),x]
```

```
[Out] -((ArcTanh[(Sqrt[g]*x)/Sqrt[f]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x
))^n] - n*Log[c + d*x]))/(Sqrt[f]*Sqrt[g])) - (n*Log[a + b*x]*Log[(b*(Sqrt[
f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*Sqrt[f]*Sqrt[g]) + (n*Log[c +
 d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*Sqrt[f]*Sq
rt[g]) + (n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[
g])])/(2*Sqrt[f]*Sqrt[g]) - (n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(
d*Sqrt[f] - c*Sqrt[g])])/(2*Sqrt[f]*Sqrt[g]) + (n*PolyLog[2, -((Sqrt[g]*(a
+ b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*Sqrt[f]*Sqrt[g]) - (n*PolyLog[2, (Sqr
t[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*Sqrt[f]*Sqrt[g]) - (n*PolyLog[
2, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*Sqrt[f]*Sqrt[g]) + (
n*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*Sqrt[f]*Sqrt[
g])
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
```

```
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f-gx^2}\,dx = n\int \frac{\log(a+bx)}{f-gx^2}\,dx - n\int \frac{\log(c+dx)}{f-gx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\ldots$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{f}\sqrt{g}} + n\int\left(\frac{\log(a+bx)}{2\sqrt{f}\left(\sqrt{f}-\sqrt{g}x\right)} + \frac{\ldots}{2\sqrt{\ldots}}\right.$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{f}\sqrt{g}} + \frac{n\int \frac{\log(a+bx)}{\sqrt{f}-\sqrt{g}x}\,dx}{2\sqrt{f}} + \frac{n\int \frac{\log(a+b\ldots}{\sqrt{f}+\sqrt{g}\ldots}}{2\sqrt{f}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{f}\sqrt{g}} - \frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{b\sqrt{f}+a\sqrt{g}}\right)\ldots}{2\sqrt{f}\sqrt{g}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{f}\sqrt{g}} - \frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{b\sqrt{f}+a\sqrt{g}}\right)\ldots}{2\sqrt{f}\sqrt{g}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{f}\sqrt{g}} - \frac{n\log(a+bx)\log\left(\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{b\sqrt{f}+a\sqrt{g}}\right)\ldots}{2\sqrt{f}\sqrt{g}}$$

Mathematica [A] time = 0.101591, size = 421, normalized size = 1.45

$$n\text{PolyLog}\left(2,\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{a\sqrt{g}+b\sqrt{f}}\right) - n\text{PolyLog}\left(2,\frac{b\left(\sqrt{f}+\sqrt{g}x\right)}{b\sqrt{f}-a\sqrt{g}}\right) - n\text{PolyLog}\left(2,\frac{d\left(\sqrt{f}-\sqrt{g}x\right)}{c\sqrt{g}+d\sqrt{f}}\right) + n\text{PolyLog}\left(2,\frac{d\left(\sqrt{f}+\sqrt{g}x\right)}{d\sqrt{f}-c\sqrt{g}}\right) - \log\left(\sqrt{f}\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(f - g*x^2),x]
```

```
[Out] (n*Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])]*Log[Sqrt[f] - Sqrt[g]*x
] - Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] - n*Log[(Sqrt[g
]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])]*Log[Sqrt[f] - Sqrt[g]*x] - n*Log[-((S
qrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x] + Log[
e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] + Sqrt[g]*x] + n*Log[-((Sqrt[g]*(c +
 d*x))/(d*Sqrt[f] - c*Sqrt[g]))]*Log[Sqrt[f] + Sqrt[g]*x] + n*PolyLog[2, (b
*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] - n*PolyLog[2, (d*(Sqrt[f]
 - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])] - n*PolyLog[2, (b*(Sqrt[f] + Sqrt[g
]*x))/(b*Sqrt[f] - a*Sqrt[g])] + n*PolyLog[2, (d*(Sqrt[f] + Sqrt[g]*x))/(d*
Sqrt[f] - c*Sqrt[g])])/(2*Sqrt[f]*Sqrt[g])
```

Maple [F] time = 0.474, size = 0, normalized size = 0.

$$\int \frac{1}{-gx^2+f}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/(-g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^2-f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(-g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^2 - f), x)
```

3.80 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(f-gx^2)}\,dx$

Optimal. Leaf size=518

$$-\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f}-\frac{n\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f}+\frac{n\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{f}+\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f}+\frac{n\text{PolyI}}{}$$

```
[Out] (n*Log[-((b*x)/a)]*Log[a + b*x])/f - (n*Log[-((d*x)/c)]*Log[c + d*x])/f - (
Log[x]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/
f - (n*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])
/(2*f) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[
g])])/(2*f) - (n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*
Sqrt[g])])/(2*f) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f]
 - c*Sqrt[g])])/(2*f) + ((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] -
 n*Log[c + d*x])*Log[f - g*x^2])/(2*f) - (n*PolyLog[2, -((Sqrt[g]*(a + b*x)
)/(b*Sqrt[f] - a*Sqrt[g]))])/(2*f) - (n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*S
qrt[f] + a*Sqrt[g])])/(2*f) + (n*PolyLog[2, 1 + (b*x)/a])/f + (n*PolyLog[2,
 -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*f) + (n*PolyLog[2, (Sq
rt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f) - (n*PolyLog[2, 1 + (d*x)/
c])/f
```

Rubi [A] time = 0.602265, antiderivative size = 518, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 11, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2513, 266, 36, 29, 31, 2416, 2394, 2315, 260, 2393, 2391}

$$-\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f}-\frac{n\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f}+\frac{n\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{f}+\frac{n\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f}+\frac{n\text{PolyI}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f - g*x^2)),x]
```


```
[Out] (n*Log[-((b*x)/a)]*Log[a + b*x])/f - (n*Log[-((d*x)/c)]*Log[c + d*x])/f - (
Log[x]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/
f - (n*Log[a + b*x]*Log[(b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])
/(2*f) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[
g])])/(2*f) - (n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*
Sqrt[g])])/(2*f) + (n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f]
 - c*Sqrt[g])])/(2*f) + ((n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] -
 n*Log[c + d*x])*Log[f - g*x^2])/(2*f) - (n*PolyLog[2, -((Sqrt[g]*(a + b*x)
)/(b*Sqrt[f] - a*Sqrt[g]))])/(2*f) - (n*PolyLog[2, (Sqrt[g]*(a + b*x))/(b*S
qrt[f] + a*Sqrt[g])])/(2*f) + (n*PolyLog[2, 1 + (b*x)/a])/f + (n*PolyLog[2,
 -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*f) + (n*PolyLog[2, (Sq
rt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f) - (n*PolyLog[2, 1 + (d*x)/
c])/f
```


Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
```

```
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f-gx^2\right)}\,dx = n\int \frac{\log(a+bx)}{x\left(f-gx^2\right)}\,dx - n\int \frac{\log(c+dx)}{x\left(f-gx^2\right)}\,dx - \left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx\right.$$

$$= n\int\left(\frac{\log(a+bx)}{fx}-\frac{gx\log(a+bx)}{f\left(-f+gx^2\right)}\right)dx - n\int\left(\frac{\log(c+dx)}{fx}-\frac{gx\log(c+dx)}{f\left(-f+gx^2\right)}\right)dx - \frac{1}{2}\left(n\log(\right.$$

$$= \frac{n\int\frac{\log(a+bx)}{x}\,dx}{f} - \frac{n\int\frac{\log(c+dx)}{x}\,dx}{f} - \frac{(gn)\int\frac{x\log(a+bx)}{-f+gx^2}\,dx}{f} + \frac{(gn)\int\frac{x\log(c+dx)}{-f+gx^2}\,dx}{f} - \frac{\left(n\log(a+b\right.}{}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-\right.}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-\right.}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-\right.}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-\right.}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-\right.}{f}$$

Mathematica [A] time = 0.186835, size = 487, normalized size = 0.94

$$-\frac{-n\text{PolyLog}\left(2,\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{a\sqrt{g}+b\sqrt{f}}\right)-n\text{PolyLog}\left(2,\frac{b\left(\sqrt{f}+\sqrt{g}x\right)}{b\sqrt{f}-a\sqrt{g}}\right)+2n\text{PolyLog}\left(2,-\frac{bx}{a}\right)+n\text{PolyLog}\left(2,\frac{d\left(\sqrt{f}-\sqrt{g}x\right)}{c\sqrt{g}+d\sqrt{f}}\right)+n\text{PolyI}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f - g*x^2)),x]
```

```
[Out] -(2*n*Log[x]*Log[1 + (b*x)/a] - 2*Log[x]*Log[e*((a + b*x)/(c + d*x))^n] - 2
*n*Log[x]*Log[1 + (d*x)/c] - n*Log[(Sqrt[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[
g])]*Log[Sqrt[f] - Sqrt[g]*x] + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f]
- Sqrt[g]*x] + n*Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])]*Log[Sqrt[
f] - Sqrt[g]*x] - n*Log[-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))]*Log
[Sqrt[f] + Sqrt[g]*x] + Log[e*((a + b*x)/(c + d*x))^n]*Log[Sqrt[f] + Sqrt[g
]*x] + n*Log[-((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))]*Log[Sqrt[f] +
Sqrt[g]*x] + 2*n*PolyLog[2, -((b*x)/a)] - 2*n*PolyLog[2, -((d*x)/c)] - n*Po
lyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])] + n*PolyLog[2,
(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])] - n*PolyLog[2, (b*(Sqrt[
f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] + n*PolyLog[2, (d*(Sqrt[f] + Sqrt
[g]*x))/(d*Sqrt[f] - c*Sqrt[g])])/(2*f)
```

Maple [F] time = 0.457, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-g x^{2}+f\right)}\ln \left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{\left(gx^{2}-f\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] -integrate(log(e*((b*x + a)/(d*x + c))^n)/((g*x^2 - f)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{gx^{3}-fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^3 - f*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/x/(-g*x**2+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{\left(gx^{2}-f\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(-g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(-log(e*((b*x + a)/(d*x + c))^n)/((g*x^2 - f)*x), x)
```

3.81 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f-gx^2)}\,dx$

Optimal. Leaf size=596

$$\frac{\sqrt{g}n\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}}-\frac{\sqrt{g}n\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f^{3/2}}-\frac{\sqrt{g}n\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}}+\frac{\sqrt{g}n\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2f^{3/2}}$$

```
[Out] (b*n*Log[x])/(a*f) - (d*n*Log[x])/(c*f) - (b*n*Log[a + b*x])/(a*f) - (n*Log
[a + b*x])/(f*x) + (d*n*Log[c + d*x])/(c*f) + (n*Log[c + d*x])/(f*x) + (n*L
og[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])/(f*x) - (Sqr
t[g]*ArcTanh[(Sqrt[g]*x)/Sqrt[f]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d
*x))^n] - n*Log[c + d*x]))/f^(3/2) - (Sqrt[g]*n*Log[a + b*x]*Log[(b*(Sqrt[f
] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*f^(3/2)) + (Sqrt[g]*n*Log[c +
d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f^(3/2)) +
(Sqrt[g]*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g
])])/(2*f^(3/2)) - (Sqrt[g]*n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d
*Sqrt[f] - c*Sqrt[g])])/(2*f^(3/2)) + (Sqrt[g]*n*PolyLog[2, -((Sqrt[g]*(a +
 b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*f^(3/2)) - (Sqrt[g]*n*PolyLog[2, (Sqrt
[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*f^(3/2)) - (Sqrt[g]*n*PolyLog[2
, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*f^(3/2)) + (Sqrt[g]*n
*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f^(3/2))
```

Rubi [A] time = 0.583434, antiderivative size = 596, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 12, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2513, 325, 208, 2416, 2395, 36, 29, 31, 2409, 2394, 2393, 2391}

$$\frac{\sqrt{g}n\text{PolyLog}\left(2,-\frac{\sqrt{g}(a+bx)}{b\sqrt{f}-a\sqrt{g}}\right)}{2f^{3/2}}-\frac{\sqrt{g}n\text{PolyLog}\left(2,\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)}{2f^{3/2}}-\frac{\sqrt{g}n\text{PolyLog}\left(2,-\frac{\sqrt{g}(c+dx)}{d\sqrt{f}-c\sqrt{g}}\right)}{2f^{3/2}}+\frac{\sqrt{g}n\text{PolyLog}\left(2,\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)}{2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f - g*x^2)),x]
```

```
[Out] (b*n*Log[x])/(a*f) - (d*n*Log[x])/(c*f) - (b*n*Log[a + b*x])/(a*f) - (n*Log
[a + b*x])/(f*x) + (d*n*Log[c + d*x])/(c*f) + (n*Log[c + d*x])/(f*x) + (n*L
og[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])/(f*x) - (Sqr
t[g]*ArcTanh[(Sqrt[g]*x)/Sqrt[f]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d
*x))^n] - n*Log[c + d*x]))/f^(3/2) - (Sqrt[g]*n*Log[a + b*x]*Log[(b*(Sqrt[f
] - Sqrt[g]*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*f^(3/2)) + (Sqrt[g]*n*Log[c +
d*x]*Log[(d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f^(3/2)) +
(Sqrt[g]*n*Log[a + b*x]*Log[(b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g
])])/(2*f^(3/2)) - (Sqrt[g]*n*Log[c + d*x]*Log[(d*(Sqrt[f] + Sqrt[g]*x))/(d
*Sqrt[f] - c*Sqrt[g])])/(2*f^(3/2)) + (Sqrt[g]*n*PolyLog[2, -((Sqrt[g]*(a +
 b*x))/(b*Sqrt[f] - a*Sqrt[g]))])/(2*f^(3/2)) - (Sqrt[g]*n*PolyLog[2, (Sqrt
[g]*(a + b*x))/(b*Sqrt[f] + a*Sqrt[g])])/(2*f^(3/2)) - (Sqrt[g]*n*PolyLog[2
, -((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])/(2*f^(3/2)) + (Sqrt[g]*n
*PolyLog[2, (Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])/(2*f^(3/2))
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
```

```
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2\left(f-gx^2\right)}\,dx = n\int \frac{\log(a+bx)}{x^2\left(f-gx^2\right)}\,dx - n\int \frac{\log(c+dx)}{x^2\left(f-gx^2\right)}\,dx - \left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right.$$

$$= \frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx} + n\int\left(\frac{\log(a+bx)}{fx^2}+\frac{g\log(a+bx)}{f\left(f-gx^2\right)}\right)dx - n\int\Big($$

$$= \frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx} - \frac{\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{f^{3/2}}$$

$$= -\frac{n\log(a+bx)}{fx}+\frac{n\log(c+dx)}{fx}+\frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx} - \frac{\sqrt{g}\tanh^{-1}\Big(}{}$$

$$= -\frac{n\log(a+bx)}{fx}+\frac{n\log(c+dx)}{fx}+\frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx} - \frac{\sqrt{g}\tanh^{-1}\Big(}{}$$

$$= \frac{bn\log(x)}{af}-\frac{dn\log(x)}{cf}-\frac{bn\log(a+bx)}{af}-\frac{n\log(a+bx)}{fx}+\frac{dn\log(c+dx)}{cf}+\frac{n\log(c+dx)}{fx}+\frac{n\,\mathrm{lo}}{}$$

$$= \frac{bn\log(x)}{af}-\frac{dn\log(x)}{cf}-\frac{bn\log(a+bx)}{af}-\frac{n\log(a+bx)}{fx}+\frac{dn\log(c+dx)}{cf}+\frac{n\log(c+dx)}{fx}+\frac{n\,\mathrm{lo}}{}$$

$$= \frac{bn\log(x)}{af}-\frac{dn\log(x)}{cf}-\frac{bn\log(a+bx)}{af}-\frac{n\log(a+bx)}{fx}+\frac{dn\log(c+dx)}{cf}+\frac{n\log(c+dx)}{fx}+\frac{n\,\mathrm{lo}}{}$$

Mathematica [A] time = 0.30863, size = 479, normalized size = 0.8

$$\frac{\sqrt{g}n\left(\mathrm{PolyLog}\left(2,\frac{b\left(\sqrt{f}-\sqrt{g}x\right)}{a\sqrt{g}+b\sqrt{f}}\right)-\mathrm{PolyLog}\left(2,\frac{d\left(\sqrt{f}-\sqrt{g}x\right)}{c\sqrt{g}+d\sqrt{f}}\right)+\log\left(\sqrt{f}-\sqrt{g}x\right)\left(\log\left(\frac{\sqrt{g}(a+bx)}{a\sqrt{g}+b\sqrt{f}}\right)-\log\left(\frac{\sqrt{g}(c+dx)}{c\sqrt{g}+d\sqrt{f}}\right)\right)\right)-\sqrt{g}n\Big(\mathrm{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f - g*x^2)),x]
```

```
[Out] ((-2*Sqrt[f]*Log[e*((a + b*x)/(c + d*x))^n])/x + (2*Sqrt[f]*n*((b*c - a*d)*
Log[x] - b*c*Log[a + b*x] + a*d*Log[c + d*x]))/(a*c) - Sqrt[g]*Log[e*((a +
b*x)/(c + d*x))^n]*Log[Sqrt[f] - Sqrt[g]*x] + Sqrt[g]*Log[e*((a + b*x)/(c +
 d*x))^n]*Log[Sqrt[f] + Sqrt[g]*x] + Sqrt[g]*n*((Log[(Sqrt[g]*(a + b*x))/(b
*Sqrt[f] + a*Sqrt[g])] - Log[(Sqrt[g]*(c + d*x))/(d*Sqrt[f] + c*Sqrt[g])])*
Log[Sqrt[f] - Sqrt[g]*x] + PolyLog[2, (b*(Sqrt[f] - Sqrt[g]*x))/(b*Sqrt[f]
+ a*Sqrt[g])] - PolyLog[2, (d*(Sqrt[f] - Sqrt[g]*x))/(d*Sqrt[f] + c*Sqrt[g]
)]) - Sqrt[g]*n*((Log[-((Sqrt[g]*(a + b*x))/(b*Sqrt[f] - a*Sqrt[g]))] - Log
[-((Sqrt[g]*(c + d*x))/(d*Sqrt[f] - c*Sqrt[g]))])*Log[Sqrt[f] + Sqrt[g]*x]
+ PolyLog[2, (b*(Sqrt[f] + Sqrt[g]*x))/(b*Sqrt[f] - a*Sqrt[g])] - PolyLog[2
, (d*(Sqrt[f] + Sqrt[g]*x))/(d*Sqrt[f] - c*Sqrt[g])]))/(2*f^(3/2))
```

Maple [F] time = 0.455, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(-gx^2+f\right)}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{gx^4-fx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(-log(e*((b*x + a)/(d*x + c))^n)/(g*x^4 - f*x^2), x)
```

<u>**Sympy**</u> **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/x**2/(-g*x**2+f),x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{\left(gx^{2}-f\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(-g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(-log(e*((b*x + a)/(d*x + c))^n)/((g*x^2 - f)*x^2), x)
```

3.82 $\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx$

Optimal. Leaf size=1046

result too large to display

```
[Out] (a*n*x)/(2*b*h) - (c*n*x)/(2*d*h) - (a^2*n*Log[a + b*x])/(2*b^2*h) + (n*x^2
*Log[a + b*x])/(2*h) - (g*n*(a + b*x)*Log[a + b*x])/(b*h^2) + (c^2*n*Log[c
+ d*x])/(2*d^2*h) - (n*x^2*Log[c + d*x])/(2*h) + (g*n*(c + d*x)*Log[c + d*x
])/(d*h^2) + (g*x*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[
c + d*x]))/h^2 - (x^2*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*
Log[c + d*x]))/(2*h) - (g*(g^2 - 3*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*
h]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h^
3*Sqrt[g^2 - 4*f*h]) + ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n
*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqr
t[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*
f*h])*n*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(
g - Sqrt[g^2 - 4*f*h])))])/(2*h^3) + ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[g
^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*
h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h + (g*(g^2 - 3*f*h))
/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x)
)/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h)*(n*Log[a +
b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2
])/(2*h^3) + ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*PolyLog[2
, (2*h*(a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^3) + ((g^2 - f
*h + (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a
*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*h^3) - ((g^2 - f*h - (g*(g^2 - 3*f*h))
/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^2 -
 4*f*h]))])/(2*h^3) - ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*
PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*h^3)
```

Rubi [A] time = 1.71701, antiderivative size = 1046, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 14, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2513, 2418, 2389, 2295, 2395, 43, 2394, 2393, 2391, 701, 634, 618, 206, 628}

$$-\frac{n\log(a+bx)a^2}{2b^2h}+\frac{nxa}{2bh}-\frac{cnx}{2dh}+\frac{nx^2\log(a+bx)}{2h}-\frac{gn(a+bx)\log(a+bx)}{bh^2}-\frac{nx^2\log(c+dx)}{2h}+\frac{c^2n\log(c+dx)}{2d^2h}+\frac{gn}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] (a*n*x)/(2*b*h) - (c*n*x)/(2*d*h) - (a^2*n*Log[a + b*x])/(2*b^2*h) + (n*x^2
*Log[a + b*x])/(2*h) - (g*n*(a + b*x)*Log[a + b*x])/(b*h^2) + (c^2*n*Log[c
+ d*x])/(2*d^2*h) - (n*x^2*Log[c + d*x])/(2*h) + (g*n*(c + d*x)*Log[c + d*x
])/(d*h^2) + (g*x*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[
c + d*x]))/h^2 - (x^2*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*
Log[c + d*x]))/(2*h) - (g*(g^2 - 3*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*
h]]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h^
3*Sqrt[g^2 - 4*f*h]) + ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n
*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqr
t[g^2 - 4*f*h])))])/(2*h^3) - ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*
```

```
f*h])*n*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(
g - Sqrt[g^2 - 4*f*h]))))])/(2*h^3) + ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[g
^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*
h - b*(g + Sqrt[g^2 - 4*f*h]))))])/(2*h^3) - ((g^2 - f*h + (g*(g^2 - 3*f*h))
/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x)
)/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))))])/(2*h^3) - ((g^2 - f*h)*(n*Log[a +
b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2
])/(2*h^3) + ((g^2 - f*h - (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*PolyLog[2
, (2*h*(a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^3) + ((g^2 - f
*h + (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a
*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*h^3) - ((g^2 - f*h - (g*(g^2 - 3*f*h))
/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^2 -
 4*f*h]))])/(2*h^3) - ((g^2 - f*h + (g*(g^2 - 3*f*h))/Sqrt[g^2 - 4*f*h])*n*
PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*h^3)
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = n\int \frac{x^3\log(a+bx)}{f+gx+hx^2}\,dx - n\int \frac{x^3\log(c+dx)}{f+gx+hx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c\right.$$

$$= n\int\left(-\frac{g\log(a+bx)}{h^2} + \frac{x\log(a+bx)}{h} + \frac{\left(fg+\left(g^2-fh\right)x\right)\log(a+bx)}{h^2\left(f+gx+hx^2\right)}\right)dx - n\int\left(-\frac{g\log(c+}{h^2}\right.$$

$$= \frac{gx\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h^2} - \frac{x^2\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\right.}{2h}$$

$$= \frac{nx^2\log(a+bx)}{2h} - \frac{nx^2\log(c+dx)}{2h} + \frac{gx\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h^2} - \frac{x^2}{}$$

$$= \frac{nx^2\log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2} - \frac{nx^2\log(c+dx)}{2h} + \frac{gn(c+dx)\log(c+dx)}{dh^2} + \frac{gx\left(n\right.}{}$$

$$= \frac{anx}{2bh} - \frac{cnx}{2dh} - \frac{a^2n\log(a+bx)}{2b^2h} + \frac{nx^2\log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2} + \frac{c^2n\log(c+dx)}{2d^2h} -$$

$$= \frac{anx}{2bh} - \frac{cnx}{2dh} - \frac{a^2n\log(a+bx)}{2b^2h} + \frac{nx^2\log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2} + \frac{c^2n\log(c+dx)}{2d^2h} -$$

$$= \frac{anx}{2bh} - \frac{cnx}{2dh} - \frac{a^2n\log(a+bx)}{2b^2h} + \frac{nx^2\log(a+bx)}{2h} - \frac{gn(a+bx)\log(a+bx)}{bh^2} + \frac{c^2n\log(c+dx)}{2d^2h} -$$

Mathematica [A] time = 1.37733, size = 1240, normalized size = 1.19

$$\frac{x^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)h^2 + \frac{n\left(b\left(b\log(c+dx)c^2+d(ad-bc)x\right)-a^2d^2\log(a+bx)\right)h^2}{b^2d^2} - \frac{2g(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)h}{b} + \frac{2(bc-ad)gn\log(c+dx)h}{bd} + \frac{2fg\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] (h^2*x^2*Log[e*((a + b*x)/(c + d*x))^n] - (2*g*h*(a + b*x)*Log[e*((a + b*x)
/(c + d*x))^n])/b + (2*(b*c - a*d)*g*h*n*Log[c + d*x])/(b*d) + (h^2*n*(-(a^
2*d^2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])))/(b^2*d^
2) + (2*f*g*h*Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*
h*x])/Sqrt[g^2 - 4*f*h] + (g^2 - f*h)*(1 - g/Sqrt[g^2 - 4*f*h])*Log[e*((a +
 b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] - (2*f*g*h*Log[e*((a
 + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x])/Sqrt[g^2 - 4*f*h]
 + (g^2 - f*h)*(1 + g/Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log
[g + Sqrt[g^2 - 4*f*h] + 2*h*x] - (2*f*g*h*n*((Log[(2*h*(a + b*x))/(-(b*g)
+ 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*S
qrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(-g
```

```
+ Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Pol
yLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g + Sqrt[g^2 - 4*
f*h]))]))/Sqrt[g^2 - 4*f*h] - ((g^2 - f*h)*(-g + Sqrt[g^2 - 4*f*h])*n*((Log
[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*
x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*
h*x] + PolyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*
Sqrt[g^2 - 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*
h + d*(-g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] + (2*f*g*h*n*((Log[(2*
h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*(c + d*x))/(2*
c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] + Pol
yLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g^2 - 4*f
*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(g + Sq
rt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] - ((g^2 - f*h)*(g + Sqrt[g^2 - 4*f*h
])*n*((Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*
(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h]
+ 2*h*x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g +
Sqrt[g^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c
*h + d*(g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h])/(2*h^3)
```

Maple [F] time = 1.511, size = 0, normalized size = 0.

$$\int \frac{x^3}{hx^2+gx+f}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

```
[Out] int(x^3*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2+gx+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas
")
```

```
[Out] integral(x^3*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2 + gx + f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] integrate(x^3*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

3.83 $\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2} \, dx$

Optimal. Leaf size=831

$$\frac{n(a+bx)\log(a+bx)}{bh} - \frac{\left(g - \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right) n \log\left(-\frac{b\left(g+2hx-\sqrt{g^2-4fh}\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)\log(a+bx)}{2h^2} - \frac{\left(g + \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right) n \log\left(-\frac{b\left(g+2hx+\sqrt{g^2-4fh}\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right.}{2h^2}$$

```
[Out] (n*(a + b*x)*Log[a + b*x])/(b*h) - (n*(c + d*x)*Log[c + d*x])/(d*h) - (x*(n
*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/h + ((g^2
 - 2*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((
a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h^2*Sqrt[g^2 - 4*f*h]) - ((g - (
g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f
*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*h^2) + ((g - (g^2 -
 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] +
 2*h*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))])/(2*h^2) - ((g + (g^2 - 2*f*
h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*
x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^2) + ((g + (g^2 - 2*f*h)/Sq
rt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(
2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^2) + (g*(n*Log[a + b*x] - Log[e*
((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2])/(2*h^2) -
((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h
- b*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^2) - ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f
*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*
h^2) + ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/
(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^2) + ((g + (g^2 - 2*f*h)/Sqrt[g^
2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])
)])/(2*h^2)
```

Rubi [A] time = 1.06817, antiderivative size = 831, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 12, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2513, 2418, 2389, 2295, 2394, 2393, 2391, 703, 634, 618, 206, 628}

$$\frac{n(a+bx)\log(a+bx)}{bh} - \frac{\left(g - \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right) n \log\left(-\frac{b\left(g+2hx-\sqrt{g^2-4fh}\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)\log(a+bx)}{2h^2} - \frac{\left(g + \frac{g^2-2fh}{\sqrt{g^2-4fh}}\right) n \log\left(-\frac{b\left(g+2hx+\sqrt{g^2-4fh}\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right.}{2h^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] (n*(a + b*x)*Log[a + b*x])/(b*h) - (n*(c + d*x)*Log[c + d*x])/(d*h) - (x*(n
*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/h + ((g^2
 - 2*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((
a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h^2*Sqrt[g^2 - 4*f*h]) - ((g - (
g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f
*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*h^2) + ((g - (g^2 -
 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] +
 2*h*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))])/(2*h^2) - ((g + (g^2 - 2*f*
h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*
x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^2) + ((g + (g^2 - 2*f*h)/Sq
rt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(
2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*h^2) + (g*(n*Log[a + b*x] - Log[e*
((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2])/(2*h^2) -
```

```
((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h
- b*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^2) - ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f
*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*
h^2) + ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/
(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))])/(2*h^2) + ((g + (g^2 - 2*f*h)/Sqrt[g^
2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])
)])/(2*h^2)
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
```

```
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = n\int \frac{x^2\log(a+bx)}{f+gx+hx^2}\,dx - n\int \frac{x^2\log(c+dx)}{f+gx+hx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c\right.$$

$$= -\frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h} + n\int\left(\frac{\log(a+bx)}{h} - \frac{(f+gx)\log(a+b}{h\left(f+gx+hx^2\right)}\right.$$

$$= -\frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h} + \frac{n\int\log(a+bx)\,dx}{h} - \frac{n\int\frac{(f+gx)\log(a+bx}{f+gx+hx^2}}{h}$$

$$= -\frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h} + \frac{g\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\,l\right.}{2h^2}$$

$$= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+\right.}{h}$$

$$= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+\right.}{h}$$

$$= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+\right.}{h}$$

$$= \frac{n(a+bx)\log(a+bx)}{bh} - \frac{n(c+dx)\log(c+dx)}{dh} - \frac{x\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+\right.}{h}$$

Mathematica [A] time = 5.47177, size = 1105, normalized size = 1.33

$$2dh\sqrt{g^2-4fh}(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - 2bdfh\log\left(g+2hx-\sqrt{g^2-4fh}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + bdg\left(g-\sqrt{g^2-4fh}\right)\log\left(g\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] (2*d*h*Sqrt[g^2 - 4*f*h]*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*(b*c
- a*d)*h*Sqrt[g^2 - 4*f*h]*n*Log[c + d*x] - 2*b*d*f*h*Log[e*((a + b*x)/(c +
 d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + b*d*g*(g - Sqrt[g^2 - 4*f*h]
)*Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + 2*b*d
*f*h*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] - b*
d*g*(g + Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2
 - 4*f*h] + 2*h*x] + 2*b*d*f*h*n*((Log[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*
Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*
f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(-g + Sqrt[g^2 -
 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - PolyLog[2, (d*(
-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g + Sqrt[g^2 - 4*f*h]))]) - b
```

```
*d*g*(g - Sqrt[g^2 - 4*f*h])*n*((Log[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sq
rt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*f*
h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(-g + Sqrt[g^2 - 4
*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - PolyLog[2, (d*(-g
 + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g + Sqrt[g^2 - 4*f*h]))]) - 2*b
*d*f*h*n*((Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(
2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f
*h] + 2*h*x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(
g + Sqrt[g^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(
-2*c*h + d*(g + Sqrt[g^2 - 4*f*h]))]) + b*d*g*(g + Sqrt[g^2 - 4*f*h])*n*((L
og[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*(c + d*x
))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x]
 + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g^2
 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(
g + Sqrt[g^2 - 4*f*h]))]))/(2*b*d*h^2*Sqrt[g^2 - 4*f*h])
```

Maple [F] time = 1.393, size = 0, normalized size = 0.

$$\int \frac{x^2}{hx^2+gx+f} \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

```
[Out] int(x^2*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2+gx+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas
")
```

```
[Out] integral(x^2*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2 + gx + f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] integrate(x^2*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

3.84 $\int \frac{x \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

Optimal. Leaf size=685

$$\frac{n\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\mathrm{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}+\frac{n\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)\mathrm{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(\sqrt{g^2-4fh}+g\right)}\right)}{2h}-\frac{n\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\mathrm{Pol}}{}$$

```
[Out] -((g*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((a + b
*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h*Sqrt[g^2 - 4*f*h])) + ((1 - g/Sqrt[
g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a
*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*h) - ((1 - g/Sqrt[g^2 - 4*f*h])*n*Log
[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g^
2 - 4*f*h])))])/(2*h) + ((1 + g/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*
(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*
h) - ((1 + g/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f
*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*h) - ((n*Log[a + b*
x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2])
/(2*h) + ((1 - g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b
*(g - Sqrt[g^2 - 4*f*h]))])/(2*h) + ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2,
 (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*h) - ((1 - g/Sqrt
[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*
h]))])/(2*h) - ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c
*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*h)
```

Rubi [A] time = 0.63553, antiderivative size = 685, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2513, 2418, 2394, 2393, 2391, 634, 618, 206, 628}

$$\frac{n\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\mathrm{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{2h}+\frac{n\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)\mathrm{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(\sqrt{g^2-4fh}+g\right)}\right)}{2h}-\frac{n\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\mathrm{Pol}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] -((g*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((a + b
*x)/(c + d*x))^n] - n*Log[c + d*x]))/(h*Sqrt[g^2 - 4*f*h])) + ((1 - g/Sqrt[
g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a
*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*h) - ((1 - g/Sqrt[g^2 - 4*f*h])*n*Log
[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g^
2 - 4*f*h])))])/(2*h) + ((1 + g/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*
(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*
h) - ((1 + g/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f
*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*h) - ((n*Log[a + b*
x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2])
/(2*h) + ((1 - g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b
*(g - Sqrt[g^2 - 4*f*h]))])/(2*h) + ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2,
 (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*h) - ((1 - g/Sqrt
[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*
h]))])/(2*h) - ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c
*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*h)
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = n\int \frac{x\log(a+bx)}{f+gx+hx^2}\,dx - n\int \frac{x\log(c+dx)}{f+gx+hx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c\right.$$

$$= n\int\left(\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\log(a+bx)}{g-\sqrt{g^2-4fh}+2hx} + \frac{\left(1+\frac{g}{\sqrt{g^2-4fh}}\right)\log(a+bx)}{g+\sqrt{g^2-4fh}+2hx}\right)dx - n\int\left(\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\log(}{g-\sqrt{g^2-4fh}+}\right.$$

$$= -\frac{\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)\log\left(f+gx+hx^2\right)}{2h} + \left(\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)\right.$$

$$= -\frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h\sqrt{g^2-4fh}} + \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log}{}$$

$$= -\frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h\sqrt{g^2-4fh}} + \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log}{}$$

$$= -\frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{h\sqrt{g^2-4fh}} + \frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log}{}$$

Mathematica [A] time = 0.684548, size = 539, normalized size = 0.79

$$\frac{n\left(g-\sqrt{g^2-4fh}\right)\left(\text{PolyLog}\left(2,\frac{b\left(\sqrt{g^2-4fh}-g-2hx\right)}{2ah+b\sqrt{g^2-4fh}+b(-g)}\right) - \text{PolyLog}\left(2,\frac{d\left(\sqrt{g^2-4fh}-g-2hx\right)}{2ch+d\left(\sqrt{g^2-4fh}-g\right)}\right) + \log\left(-\sqrt{g^2-4fh}+g+2hx\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x + h*x^2),x]
```

```
[Out] ((-g + Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 -
 4*f*h] + 2*h*x] + (g + Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*L
og[g + Sqrt[g^2 - 4*f*h] + 2*h*x] + (g - Sqrt[g^2 - 4*f*h])*n*((Log[(2*h*(a
 + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d
*g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + P
olyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2
 - 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-
g + Sqrt[g^2 - 4*f*h]))]) - (g + Sqrt[g^2 - 4*f*h])*n*((Log[(2*h*(a + b*x))
/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*(c + d*x))/(2*c*h - d*(g +
 Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(
g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g^2 - 4*f*h]))] - Pol
yLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(g + Sqrt[g^2 - 4*f
*h]))]))/(2*h*Sqrt[g^2 - 4*f*h])
```

Maple [F] time = 1.349, size = 0, normalized size = 0.

$$\int \frac{x}{h x^2 + g x + f} \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

```
[Out] int(x*ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2 + gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas")
```

```
[Out] integral(x*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2 + gx + f}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] integrate(x*log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

3.85 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx$

Optimal. Leaf size=401

$$-\frac{n\text{PolyLog}\left(2,\frac{2(a+bx)(c^2h-cdg+d^2f)}{(c+dx)\left(-\sqrt{g^2-4fh}(bc-ad)+2ach-adg-bcg+2bdf\right)}\right)}{\sqrt{g^2-4fh}}+\frac{n\text{PolyLog}\left(2,\frac{2(a+bx)(c^2h-cdg+d^2f)}{(c+dx)\left(\sqrt{g^2-4fh}(bc-ad)+2ach-adg-bcg+2bdf\right)}\right)}{\sqrt{g^2-4fh}}-\frac{\log\left(e\left(\right.\right.}{}$$

```
[Out] -((Log[e*((a + b*x)/(c + d*x))^n]*Log[1 - (2*(d^2*f - c*d*g + c^2*h)*(a + b
*x))/((2*b*d*f - b*c*g - a*d*g + 2*a*c*h - (b*c - a*d)*Sqrt[g^2 - 4*f*h])*(
c + d*x))]])/Sqrt[g^2 - 4*f*h]) + (Log[e*((a + b*x)/(c + d*x))^n]*Log[1 - (2
*(d^2*f - c*d*g + c^2*h)*(a + b*x))/((2*b*d*f - b*c*g - a*d*g + 2*a*c*h + (
b*c - a*d)*Sqrt[g^2 - 4*f*h])*(c + d*x))]])/Sqrt[g^2 - 4*f*h] - (n*PolyLog[2
, (2*(d^2*f - c*d*g + c^2*h)*(a + b*x))/((2*b*d*f - b*c*g - a*d*g + 2*a*c*h
 - (b*c - a*d)*Sqrt[g^2 - 4*f*h])*(c + d*x))]])/Sqrt[g^2 - 4*f*h] + (n*PolyL
og[2, (2*(d^2*f - c*d*g + c^2*h)*(a + b*x))/((2*b*d*f - b*c*g - a*d*g + 2*a
*c*h + (b*c - a*d)*Sqrt[g^2 - 4*f*h])*(c + d*x))]])/Sqrt[g^2 - 4*f*h]
```

Rubi [A] time = 0.51787, antiderivative size = 545, normalized size of antiderivative = 1.36, number of steps used = 19, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2513, 2418, 2394, 2393, 2391, 618, 206}

$$\frac{n\text{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)}{\sqrt{g^2-4fh}}-\frac{n\text{PolyLog}\left(2,\frac{2h(a+bx)}{2ah-b\left(\sqrt{g^2-4fh}+g\right)}\right)}{\sqrt{g^2-4fh}}-\frac{n\text{PolyLog}\left(2,\frac{2h(c+dx)}{2ch-d\left(g-\sqrt{g^2-4fh}\right)}\right)}{\sqrt{g^2-4fh}}+\frac{n\text{PolyLog}\left(2,\frac{}{2c}\right.}{\sqrt{g^2-}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(f + g*x + h*x^2),x]
```

```
[Out] (2*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((a + b*x
)/(c + d*x))^n] - n*Log[c + d*x]))/Sqrt[g^2 - 4*f*h] + (n*Log[a + b*x]*Log[
-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))]
)/Sqrt[g^2 - 4*f*h] - (n*Log[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h
*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))]])/Sqrt[g^2 - 4*f*h] - (n*Log[a +
b*x]*Log[-((b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4
*f*h])))]])/Sqrt[g^2 - 4*f*h] + (n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*f
*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))]])/Sqrt[g^2 - 4*f*h] + (n
*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))]])/Sqrt[g^2
- 4*f*h] - (n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])
)]])/Sqrt[g^2 - 4*f*h] - (n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[
g^2 - 4*f*h]))]])/Sqrt[g^2 - 4*f*h] + (n*PolyLog[2, (2*h*(c + d*x))/(2*c*h -
 d*(g + Sqrt[g^2 - 4*f*h]))]])/Sqrt[g^2 - 4*f*h]
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx+hx^2}\,dx = n\int \frac{\log(a+bx)}{f+gx+hx^2}\,dx - n\int \frac{\log(c+dx)}{f+gx+hx^2}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx\right.$$

$$= n\int\left(\frac{2h\log(a+bx)}{\sqrt{g^2-4fh}\left(g-\sqrt{g^2-4fh}+2hx\right)} - \frac{2h\log(a+bx)}{\sqrt{g^2-4fh}\left(g+\sqrt{g^2-4fh}+2hx\right)}\right)dx - n\int\left(\frac{}{\sqrt{g^2-}}\right.$$

$$= \frac{2\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{g^2-4fh}} + \frac{(2hn)\int \frac{\log(a+bx)}{g-\sqrt{g^2-4fh}+2hx}\,dx}{\sqrt{g^2-4fh}}$$

$$= \frac{2\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{g^2-4fh}} + \frac{n\log(a+bx)\log\left(-\frac{b(g-}{2ah-}\right.}{\sqrt{g^2-4fh}}$$

$$= \frac{2\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{g^2-4fh}} + \frac{n\log(a+bx)\log\left(-\frac{b(g-}{2ah-}\right.}{\sqrt{g^2-4fh}}$$

$$= \frac{2\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{\sqrt{g^2-4fh}} + \frac{n\log(a+bx)\log\left(-\frac{b(g-}{2ah-}\right.}{\sqrt{g^2-4fh}}$$

Mathematica [A] time = 0.307974, size = 515, normalized size = 1.28

$$-n\text{PolyLog}\left(2,\frac{b\left(\sqrt{g^2-4fh}-g-2hx\right)}{2ah+b\left(\sqrt{g^2-4fh}-g\right)}\right)+n\text{PolyLog}\left(2,\frac{b\left(\sqrt{g^2-4fh}+g+2hx\right)}{b\left(\sqrt{g^2-4fh}+g\right)-2ah}\right)+n\text{PolyLog}\left(2,\frac{d\left(\sqrt{g^2-4fh}-g-2hx\right)}{2ch+d\sqrt{g^2-4fh}+d(-g)}\right)-n\text{PolyLog}\left(2,\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(f + g*x + h*x^2),x]
```

```
[Out] (-(n*Log[(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])]*Log[g - Sq
rt[g^2 - 4*f*h] + 2*h*x]) + Log[e*((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2
 - 4*f*h] + 2*h*x] + n*Log[(2*h*(c + d*x))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4
*f*h])]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + n*Log[(2*h*(a + b*x))/(2*a*h -
 b*(g + Sqrt[g^2 - 4*f*h]))]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] - Log[e*((a
 + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] - n*Log[(2*h*(c +
d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*
x] + n*PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(d*g) + 2*c*h + d*
Sqrt[g^2 - 4*f*h])] - n*PolyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*
a*h + b*(-g + Sqrt[g^2 - 4*f*h]))] + n*PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h]
 + 2*h*x))/(-2*a*h + b*(g + Sqrt[g^2 - 4*f*h]))] - n*PolyLog[2, (d*(g + Sqr
t[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d*(g + Sqrt[g^2 - 4*f*h]))])/Sqrt[g^2 -
4*f*h]
```

Maple [F] time = 1.362, size = 0, normalized size = 0.

$$\int \frac{1}{hx^2+gx+f}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2+gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="fricas")
```

```
[Out] integral(log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^2+gx+f}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] integrate(log(e*((b*x + a)/(d*x + c))^n)/(h*x^2 + g*x + f), x)
```

3.86 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx$

Optimal. Leaf size=800

$$\frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f}-\frac{\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)n\log\left(-\frac{b\left(g+2hx-\sqrt{g^2-4fh}\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)\log(a+bx)}{2f}-\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log\left(-\frac{b\left(g+2hx+\sqrt{g^2-4fh}\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f}$$

```
[Out] (n*Log[-((b*x)/a)]*Log[a + b*x])/f - (n*Log[-((d*x)/c)]*Log[c + d*x])/f - (
g*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((a + b*x)
/(c + d*x))^n] - n*Log[c + d*x]))/(f*Sqrt[g^2 - 4*f*h]) - (Log[x]*(n*Log[a
+ b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/f - ((1 + g/Sqrt
[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*
a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*f) + ((1 + g/Sqrt[g^2 - 4*f*h])*n*Lo
g[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g
^2 - 4*f*h])))])/(2*f) - ((1 - g/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b
*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2
*f) + ((1 - g/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*
f*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*f) + ((n*Log[a + b
*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2]
)/(2*f) - ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h -
b*(g - Sqrt[g^2 - 4*f*h]))])/(2*f) - ((1 - g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2
, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*f) + (n*PolyLog[
2, 1 + (b*x)/a])/f + ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x)
)/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))])/(2*f) + ((1 - g/Sqrt[g^2 - 4*f*h])*
n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*f) -
(n*PolyLog[2, 1 + (d*x)/c])/f
```

Rubi [A] time = 0.980036, antiderivative size = 800, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 12, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2513, 2418, 2394, 2315, 2393, 2391, 705, 29, 634, 618, 206, 628}

$$\frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f}-\frac{\left(\frac{g}{\sqrt{g^2-4fh}}+1\right)n\log\left(-\frac{b\left(g+2hx-\sqrt{g^2-4fh}\right)}{2ah-b\left(g-\sqrt{g^2-4fh}\right)}\right)\log(a+bx)}{2f}-\frac{\left(1-\frac{g}{\sqrt{g^2-4fh}}\right)n\log\left(-\frac{b\left(g+2hx+\sqrt{g^2-4fh}\right)}{2ah-b\left(g+\sqrt{g^2-4fh}\right)}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f + g*x + h*x^2)),x]
```

```
[Out] (n*Log[-((b*x)/a)]*Log[a + b*x])/f - (n*Log[-((d*x)/c)]*Log[c + d*x])/f - (
g*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] - Log[e*((a + b*x)
/(c + d*x))^n] - n*Log[c + d*x]))/(f*Sqrt[g^2 - 4*f*h]) - (Log[x]*(n*Log[a
+ b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/f - ((1 + g/Sqrt
[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*
a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*f) + ((1 + g/Sqrt[g^2 - 4*f*h])*n*Lo
g[c + d*x]*Log[-((d*(g - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g
^2 - 4*f*h])))])/(2*f) - ((1 - g/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b
*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2
*f) + ((1 - g/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4*
f*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*f) + ((n*Log[a + b
*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h*x^2]
)/(2*f) - ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h -
b*(g - Sqrt[g^2 - 4*f*h]))])/(2*f) - ((1 - g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2
```

```
, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))])/(2*f) + (n*PolyLog[
2, 1 + (b*x)/a])/f + ((1 + g/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x)
)/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))])/(2*f) + ((1 - g/Sqrt[g^2 - 4*f*h])*
n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*f) -
(n*PolyLog[2, 1 + (d*x)/c])/f
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x\left(f+gx+hx^2\right)}\,dx = n\int \frac{\log(a+bx)}{x\left(f+gx+hx^2\right)}\,dx - n\int \frac{\log(c+dx)}{x\left(f+gx+hx^2\right)}\,dx - \left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) -$$

$$= n\int \left(\frac{\log(a+bx)}{fx} + \frac{(-g-hx)\log(a+bx)}{f\left(f+gx+hx^2\right)}\right)dx - n\int \left(\frac{\log(c+dx)}{fx} + \frac{(-g-hx)\log(c+dx)}{f\left(f+gx+hx^2\right)}\right.$$

$$= -\frac{\log(x)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - n\log(c+dx)\right)}{f} + \frac{n\int \frac{\log(a+bx)}{x}\,dx}{f} + \frac{n\int \frac{(-g-hx)\log}{f+gx+}}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{\log(x)\left(n\log(a+bx) - \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{f}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \mathrm{lo}\right.}{f\sqrt{g^2-4f}}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \mathrm{lo}\right.}{f\sqrt{g^2-4f}}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \mathrm{lo}\right.}{f\sqrt{g^2-4f}}$$

$$= \frac{n\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f} - \frac{n\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f} - \frac{g\tanh^{-1}\left(\frac{g+2hx}{\sqrt{g^2-4fh}}\right)\left(n\log(a+bx) - \mathrm{lo}\right.}{f\sqrt{g^2-4f}}$$

Mathematica [A] time = 0.940115, size = 625, normalized size = 0.78

$$\frac{n\left(\sqrt{g^2-4fh}+g\right)\left(\mathrm{PolyLog}\left(2,\frac{b\left(\sqrt{g^2-4fh}-g-2hx\right)}{2ah+b\sqrt{g^2-4fh}+b(-g)}\right)-\mathrm{PolyLog}\left(2,\frac{d\left(\sqrt{g^2-4fh}-g-2hx\right)}{2ch+d\left(\sqrt{g^2-4fh}-g\right)}\right)+\log\left(-\sqrt{g^2-4fh}+g+2hx\right)\left(\log\left(\frac{2h(a+bx)}{2ah+b\sqrt{g^2-4fh}+b(-g)}\right)-\log\left(\frac{2h(c+dx)}{2ch+d\sqrt{g^2-4fh}+d}\right.\right.\right.}{\sqrt{g^2-4fh}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x*(f + g*x + h*x^2)),x]
```

```
[Out] (2*Log[x]*Log[e*((a + b*x)/(c + d*x))^n] - (1 + g/Sqrt[g^2 - 4*f*h])*Log[e*
((a + b*x)/(c + d*x))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] - (1 - g/Sqrt[g
^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h
*x] - 2*n*(Log[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x
)/a)] - PolyLog[2, -((d*x)/c)]) + ((g + Sqrt[g^2 - 4*f*h])*n*((Log[(2*h*(a
+ b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x))/(-(d*
g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + Po
lyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2
- 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h + d*(-g
```

```
 + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] + ((-g + Sqrt[g^2 - 4*f*h])*n*(
(Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))] - Log[(2*h*(c + d
*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*
x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*a*h + b*(g + Sqrt[g
^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*c*h + d
*(g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h])/(2*f)
```

Maple [F] time = 1.414, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(hx^2+gx+f\right)}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^3+gx^2+fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="fricas")
```

```
[Out] integral(log(e*((b*x + a)/(d*x + c))^n)/(h*x^3 + g*x^2 + f*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/x/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(hx^2+gx+f)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x/(h*x^2+g*x+f),x, algorithm="giac")

[Out] integrate(log(e*((b*x + a)/(d*x + c))^n)/((h*x^2 + g*x + f)*x), x)
```

3.87 $\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2(f+gx+hx^2)}\,dx$

Optimal. Leaf size=995

result too large to display

```
[Out] (b*n*Log[x])/(a*f) - (d*n*Log[x])/(c*f) - (b*n*Log[a + b*x])/(a*f) - (n*Log
[a + b*x])/(f*x) - (g*n*Log[-((b*x)/a)]*Log[a + b*x])/f^2 + (d*n*Log[c + d*
x])/(c*f) + (n*Log[c + d*x])/(f*x) + (g*n*Log[-((d*x)/c)]*Log[c + d*x])/f^2
 + (n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])/(f*x)
 + ((g^2 - 2*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] -
Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(f^2*Sqrt[g^2 - 4*f*h]) +
 (g*Log[x]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x
]))/f^2 + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g
 - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*f^
2) - ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g - Sq
rt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h])))])/(2*f^2) +
((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^
2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h])))])/(2*f^2) - ((g -
 (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4
*f*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h])))])/(2*f^2) - (g*(n*Log[
a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h
*x^2])/(2*f^2) + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(
a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*f^2) + ((g - (g^2 - 2*f*
h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2
 - 4*f*h]))])/(2*f^2) - (g*n*PolyLog[2, 1 + (b*x)/a])/f^2 - ((g + (g^2 - 2*
f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g
^2 - 4*f*h]))])/(2*f^2) - ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[
2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*f^2) + (g*n*Pol
yLog[2, 1 + (d*x)/c])/f^2
```

Rubi [A] time = 1.29131, antiderivative size = 995, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 16, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {2513, 2418, 2395, 36, 29, 31, 2394, 2315, 2393, 2391, 709, 800, 634, 618, 206, 628}

$$\frac{bn\log(x)}{af}-\frac{dn\log(x)}{cf}+\frac{g\left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)\right)\log(x)}{f^2}-\frac{bn\log(a+bx)}{af}-\frac{gn\log\left(-\frac{bx}{a}\right)\log(a}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f + g*x + h*x^2)),x]
```

```
[Out] (b*n*Log[x])/(a*f) - (d*n*Log[x])/(c*f) - (b*n*Log[a + b*x])/(a*f) - (n*Log
[a + b*x])/(f*x) - (g*n*Log[-((b*x)/a)]*Log[a + b*x])/f^2 + (d*n*Log[c + d*
x])/(c*f) + (n*Log[c + d*x])/(f*x) + (g*n*Log[-((d*x)/c)]*Log[c + d*x])/f^2
 + (n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])/(f*x)
 + ((g^2 - 2*f*h)*ArcTanh[(g + 2*h*x)/Sqrt[g^2 - 4*f*h]]*(n*Log[a + b*x] -
Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x]))/(f^2*Sqrt[g^2 - 4*f*h]) +
 (g*Log[x]*(n*Log[a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x
]))/f^2 + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g
 - Sqrt[g^2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h])))])/(2*f^
2) - ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g - Sq
```

```
rt[g^2 - 4*f*h] + 2*h*x))/(2*c*h - d*(g - Sqrt[g^2 - 4*f*h]))))])/(2*f^2) +
((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[a + b*x]*Log[-((b*(g + Sqrt[g^
2 - 4*f*h] + 2*h*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))))])/(2*f^2) - ((g -
 (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*Log[c + d*x]*Log[-((d*(g + Sqrt[g^2 - 4
*f*h] + 2*h*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))))])/(2*f^2) - (g*(n*Log[
a + b*x] - Log[e*((a + b*x)/(c + d*x))^n] - n*Log[c + d*x])*Log[f + g*x + h
*x^2])/(2*f^2) + ((g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(
a + b*x))/(2*a*h - b*(g - Sqrt[g^2 - 4*f*h]))])/(2*f^2) + ((g - (g^2 - 2*f*
h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2
 - 4*f*h]))])/(2*f^2) - (g*n*PolyLog[2, 1 + (b*x)/a])/f^2 - ((g + (g^2 - 2*
f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[2, (2*h*(c + d*x))/(2*c*h - d*(g - Sqrt[g
^2 - 4*f*h]))])/(2*f^2) - ((g - (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*n*PolyLog[
2, (2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])/(2*f^2) + (g*n*Pol
yLog[2, 1 + (d*x)/c])/f^2
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x^2\left(f+gx+hx^2\right)}\,dx = n\int \frac{\log(a+bx)}{x^2\left(f+gx+hx^2\right)}\,dx - n\int \frac{\log(c+dx)}{x^2\left(f+gx+hx^2\right)}\,dx - \left(n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right.\right.$$

$$= \frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx} + n\int\left(\frac{\log(a+bx)}{fx^2}-\frac{g\log(a+bx)}{f^2x}+\frac{(g^2}{}\right.$$

$$= \frac{n\log(a+bx)-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log(c+dx)}{fx} + \frac{n\int\frac{\left(g^2-fh+ghx\right)\log(a+bx)}{f+gx+hx^2}\,dx}{f^2} - \frac{n\int\frac{\left(g^2-fh+g\right.}{f+g}}{}$$

$$= -\frac{n\log(a+bx)}{fx} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} + \frac{n\log(c+dx)}{fx} + \frac{gn\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f^2} + $$

$$= -\frac{n\log(a+bx)}{fx} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} + \frac{n\log(c+dx)}{fx} + \frac{gn\log\left(-\frac{dx}{c}\right)\log(c+dx)}{f^2} + $$

$$= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} + \frac{dn\,\mathrm{l}}{}$$

$$= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} + \frac{dn\,\mathrm{l}}{}$$

$$= \frac{bn\log(x)}{af} - \frac{dn\log(x)}{cf} - \frac{bn\log(a+bx)}{af} - \frac{n\log(a+bx)}{fx} - \frac{gn\log\left(-\frac{bx}{a}\right)\log(a+bx)}{f^2} + \frac{dn\,\mathrm{l}}{}$$

Mathematica [A] time = 0.8248, size = 721, normalized size = 0.72

$$-\frac{n\left(g\sqrt{g^2-4fh}-2fh+g^2\right)\left(\mathrm{PolyLog}\left(2,\frac{b\left(\sqrt{g^2-4fh}-g-2hx\right)}{2ah+b\sqrt{g^2-4fh}+b(-g)}\right)-\mathrm{PolyLog}\left(2,\frac{d\left(\sqrt{g^2-4fh}-g-2hx\right)}{2ch+d\left(\sqrt{g^2-4fh}-g\right)}\right)+\log\left(-\sqrt{g^2-4fh}+g+2hx\right)\left(\log\left(\frac{2h(a+bx)}{2ah+b\sqrt{g^2-4fh}+b(-g)}\right)-\log\left(\frac{}{2ch+d\sqrt{}}\right.\right.\right.}{\sqrt{g^2-4fh}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[e*((a + b*x)/(c + d*x))^n]/(x^2*(f + g*x + h*x^2)),x]
```

```
[Out] ((-2*f*Log[e*((a + b*x)/(c + d*x))^n])/x - 2*g*Log[x]*Log[e*((a + b*x)/(c +
 d*x))^n] + (2*f*n*((b*c - a*d)*Log[x] - b*c*Log[a + b*x] + a*d*Log[c + d*x
]))/(a*c) + (g + (g^2 - 2*f*h)/Sqrt[g^2 - 4*f*h])*Log[e*((a + b*x)/(c + d*x
))^n]*Log[g - Sqrt[g^2 - 4*f*h] + 2*h*x] + (g + (-g^2 + 2*f*h)/Sqrt[g^2 - 4
*f*h])*Log[e*((a + b*x)/(c + d*x))^n]*Log[g + Sqrt[g^2 - 4*f*h] + 2*h*x] +
2*g*n*(Log[x]*(Log[1 + (b*x)/a] - Log[1 + (d*x)/c]) + PolyLog[2, -((b*x)/a)
```

```
] - PolyLog[2, -((d*x)/c)]) - ((g^2 - 2*f*h + g*Sqrt[g^2 - 4*f*h])*n*((Log[
(2*h*(a + b*x))/(-(b*g) + 2*a*h + b*Sqrt[g^2 - 4*f*h])] - Log[(2*h*(c + d*x
))/(-(d*g) + 2*c*h + d*Sqrt[g^2 - 4*f*h])])*Log[g - Sqrt[g^2 - 4*f*h] + 2*h
*x] + PolyLog[2, (b*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(-(b*g) + 2*a*h + b*S
qrt[g^2 - 4*f*h])] - PolyLog[2, (d*(-g + Sqrt[g^2 - 4*f*h] - 2*h*x))/(2*c*h
 + d*(-g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h] + ((g^2 - 2*f*h - g*Sqr
t[g^2 - 4*f*h])*n*((Log[(2*h*(a + b*x))/(2*a*h - b*(g + Sqrt[g^2 - 4*f*h]))
] - Log[(2*h*(c + d*x))/(2*c*h - d*(g + Sqrt[g^2 - 4*f*h]))])*Log[g + Sqrt[
g^2 - 4*f*h] + 2*h*x] + PolyLog[2, (b*(g + Sqrt[g^2 - 4*f*h] + 2*h*x))/(-2*
a*h + b*(g + Sqrt[g^2 - 4*f*h]))] - PolyLog[2, (d*(g + Sqrt[g^2 - 4*f*h] +
2*h*x))/(-2*c*h + d*(g + Sqrt[g^2 - 4*f*h]))]))/Sqrt[g^2 - 4*f*h])/(2*f^2)
```

Maple [F] time = 1.398, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(hx^2+gx+f\right)}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{hx^4+gx^3+fx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="fricas
")
```

```
[Out] integral(log(e*((b*x + a)/(d*x + c))^n)/(h*x^4 + g*x^3 + f*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)/x**2/(h*x**2+g*x+f),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(hx^2+gx+f)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)/x^2/(h*x^2+g*x+f),x, algorithm="giac")
```

```
[Out] integrate(log(e*((b*x + a)/(d*x + c))^n)/((h*x^2 + g*x + f)*x^2), x)
```

3.88 $\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx$

Optimal. Leaf size=46

$$-\frac{\text{PolyLog}\left(2,1-\frac{a}{a+bx}\right)}{b}-\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}$$

```
[Out] -((Log[a/(a + b*x)]*Log[(c*x)/(a + b*x)])/b) - PolyLog[2, 1 - a/(a + b*x)]/
b
```

Rubi [A] time = 0.158601, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2488, 2411, 2343, 2333, 2315}

$$-\frac{\text{PolyLog}\left(2,1-\frac{a}{a+bx}\right)}{b}-\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*x)/(a + b*x)]/(a + b*x),x]
```

```
[Out] -((Log[a/(a + b*x)]*Log[(c*x)/(a + b*x)])/b) - PolyLog[2, 1 - a/(a + b*x)]/
b
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\,dx &= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}+\frac{a\int\frac{\log\left(\frac{a}{a+bx}\right)}{x(a+bx)}\,dx}{b}\\ &= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}+\frac{a\,\mathrm{Subst}\left(\int\frac{\log\left(\frac{a}{x}\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)}\,dx,x,a+bx\right)}{b^2}\\ &= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{a\,\mathrm{Subst}\left(\int\frac{\log(ax)}{\left(-\frac{a}{b}+\frac{1}{bx}\right)x}\,dx,x,\frac{1}{a+bx}\right)}{b^2}\\ &= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{a\,\mathrm{Subst}\left(\int\frac{\log(ax)}{\frac{1}{b}-\frac{ax}{b}}\,dx,x,\frac{1}{a+bx}\right)}{b^2}\\ &= -\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}-\frac{\mathrm{Li}_2\left(\frac{bx}{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0167248, size = 84, normalized size = 1.83

$$-\frac{\mathrm{PolyLog}\left(2,\frac{a+bx}{a}\right)}{b}-\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{b}+\frac{\log^2\left(\frac{a}{a+bx}\right)}{2b}+\frac{\log\left(-\frac{bx}{a}\right)\log\left(\frac{a}{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x)/(a + b*x)]/(a + b*x),x]
```

```
[Out] (Log[-((b*x)/a)]*Log[a/(a + b*x)])/b + Log[a/(a + b*x)]^2/(2*b) - (Log[a/(a
 + b*x)]*Log[(c*x)/(a + b*x)])/b - PolyLog[2, (a + b*x)/a]/b
```

Maple [B] time = 0.098, size = 97, normalized size = 2.1

$$-\frac{1}{b}\mathrm{dilog}\left(-\frac{1}{c}\left(b\left(\frac{c}{b}-\frac{ac}{b\,(bx+a)}\right)-c\right)\right)-\frac{1}{b}\ln\left(\frac{c}{b}-\frac{ac}{b\,(bx+a)}\right)\ln\left(-\frac{1}{c}\left(b\left(\frac{c}{b}-\frac{ac}{b\,(bx+a)}\right)-c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x/(b*x+a))/(b*x+a),x)
```

```
[Out] -dilog(-(b*(c/b-a*c/b/(b*x+a))-c)/c)/b-ln(c/b-a*c/b/(b*x+a))*ln(-(b*(c/b-a*
c/b/(b*x+a))-c)/c)/b
```

Maxima [B] time = 1.12936, size = 128, normalized size = 2.78

$$\frac{\log\left(bx+a\right)\log\left(\frac{cx}{bx+a}\right)}{b}-\frac{\frac{c\log(bx+a)^2}{b}-\frac{2\left(\log\left(\frac{bx}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)\right)c}{b}}{2\,c}+\frac{\left(c\log\left(bx+a\right)-c\log\left(x\right)\right)\log\left(bx+a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))/(b*x+a),x, algorithm="maxima")
```

```
[Out] log(b*x + a)*log(c*x/(b*x + a))/b - 1/2*(c*log(b*x + a)^2/b - 2*(log(b*x/a
+ 1)*log(x) + dilog(-b*x/a))*c/b)/c + (c*log(b*x + a) - c*log(x))*log(b*x +
 a)/(b*c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\frac{cx}{bx+a}\right)}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(log(c*x/(b*x + a))/(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{cx}{a+bx}\right)}{a+bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x/(b*x+a))/(b*x+a),x)
```

```
[Out] Integral(log(c*x/(a + b*x))/(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{cx}{bx+a}\right)}{bx+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(c*x/(b*x + a))/(b*x + a), x)
```

3.89 $\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)} dx$

Optimal. Leaf size=20

$$\frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

```
[Out] Log[(c*x)/(a + b*x)]^3/(3*a)
```

Rubi [A] time = 0.0579114, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2505}

$$\frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*x)/(a + b*x)]^2/(x*(a + b*x)),x]
```

```
[Out] Log[(c*x)/(a + b*x)]^3/(3*a)
```

Rule 2505

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{h = Simplify[u*(a + b*x)*(c + d*x)]}
, Simp[(h*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1))/(p*r*(s + 1)*(b*c -
 a*d)), x] /; FreeQ[h, x]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && N
eQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[s, -1]
```

Rubi steps

$$\int \frac{\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)} dx = \frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

Mathematica [A] time = 0.0961977, size = 20, normalized size = 1.

$$\frac{\log^3\left(\frac{cx}{a+bx}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x)/(a + b*x)]^2/(x*(a + b*x)),x]
```

```
[Out] Log[(c*x)/(a + b*x)]^3/(3*a)
```

Maple [A] time = 0.059, size = 29, normalized size = 1.5

$$\frac{1}{3a}\left(\ln\left(\frac{c}{b} - \frac{ac}{b(bx+a)}\right)\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x/(b*x+a))^2/x/(b*x+a),x)
```

```
[Out] 1/3/a*ln(c/b-a*c/b/(b*x+a))^3
```

Maxima [B] time = 1.19479, size = 190, normalized size = 9.5

$$-\left(\frac{\log(bx+a)}{a}-\frac{\log(x)}{a}\right)\log\left(\frac{cx}{bx+a}\right)^2-\frac{\left(c\log(bx+a)^2-2\,c\log(bx+a)\log(x)+c\log(x)^2\right)\log\left(\frac{cx}{bx+a}\right)}{ac}-\frac{c^2\log(bx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(log(b*x + a)/a - log(x)/a)*log(c*x/(b*x + a))^2 - (c*log(b*x + a)^2 - 2*c
*log(b*x + a)*log(x) + c*log(x)^2)*log(c*x/(b*x + a))/(a*c) - 1/3*(c^2*log(
b*x + a)^3 - 3*c^2*log(b*x + a)^2*log(x) + 3*c^2*log(b*x + a)*log(x)^2 - c^
2*log(x)^3)/(a*c^2)
```

Fricas [A] time = 1.98203, size = 38, normalized size = 1.9

$$\frac{\log\left(\frac{cx}{bx+a}\right)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x/(b*x + a))^3/a
```

Sympy [A] time = 0.390677, size = 14, normalized size = 0.7

$$\frac{\log\left(\frac{cx}{a+bx}\right)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x/(b*x+a))**2/x/(b*x+a),x)
```

```
[Out] log(c*x/(a + b*x))**3/(3*a)
```

Giac [A] time = 1.15435, size = 24, normalized size = 1.2

$$\frac{\log\left(\frac{cx}{bx+a}\right)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*log(c*x/(b*x + a))^3/a
```

3.90 $\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx$

Optimal. Leaf size=82

$$-\frac{\text{PolyLog}\left(2,1-\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{a}+\frac{2\text{PolyLog}\left(3,1-\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{a}-\frac{2\text{PolyLog}\left(4,1-\frac{a}{a+bx}\right)}{a}$$

```
[Out] -((Log[(c*x)/(a + b*x)]^2*PolyLog[2, 1 - a/(a + b*x)])/a) + (2*Log[(c*x)/(a
 + b*x)]*PolyLog[3, 1 - a/(a + b*x)])/a - (2*PolyLog[4, 1 - a/(a + b*x)])/a
```

Rubi [A] time = 0.168258, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2506, 2508, 6610}

$$-\frac{\text{PolyLog}\left(2,1-\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{a}+\frac{2\text{PolyLog}\left(3,1-\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{a}-\frac{2\text{PolyLog}\left(4,1-\frac{a}{a+bx}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[a/(a + b*x)]*Log[(c*x)/(a + b*x)]^2)/(x*(a + b*x)),x]
```

```
[Out] -((Log[(c*x)/(a + b*x)]^2*PolyLog[2, 1 - a/(a + b*x)])/a) + (2*Log[(c*x)/(a
 + b*x)]*PolyLog[3, 1 - a/(a + b*x)])/a - (2*PolyLog[4, 1 - a/(a + b*x)])/a
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2508

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[(v*(c +
 d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(h*PolyLog[n
+ 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] - Dist[h*p*
r*s, Int[(PolyLog[n + 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(
(a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(\frac{a}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{x(a+bx)}\,dx &= -\frac{\log^2\left(\frac{cx}{a+bx}\right)\operatorname{Li}_2\left(1-\frac{a}{a+bx}\right)}{a} + 2\int \frac{\log\left(\frac{cx}{a+bx}\right)\operatorname{Li}_2\left(1-\frac{a}{a+bx}\right)}{x(a+bx)}\,dx \\ &= -\frac{\log^2\left(\frac{cx}{a+bx}\right)\operatorname{Li}_2\left(1-\frac{a}{a+bx}\right)}{a} + \frac{2\log\left(\frac{cx}{a+bx}\right)\operatorname{Li}_3\left(1-\frac{a}{a+bx}\right)}{a} - 2\int \frac{\operatorname{Li}_3\left(1-\frac{a}{a+bx}\right)}{x(a+bx)}\,dx \\ &= -\frac{\log^2\left(\frac{cx}{a+bx}\right)\operatorname{Li}_2\left(1-\frac{a}{a+bx}\right)}{a} + \frac{2\log\left(\frac{cx}{a+bx}\right)\operatorname{Li}_3\left(1-\frac{a}{a+bx}\right)}{a} - \frac{2\operatorname{Li}_4\left(1-\frac{a}{a+bx}\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.013578, size = 76, normalized size = 0.93

$$-\frac{\text{PolyLog}\left(2,\frac{bx}{a+bx}\right)\log^2\left(\frac{cx}{a+bx}\right)}{a} + \frac{2\text{PolyLog}\left(3,\frac{bx}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)}{a} - \frac{2\text{PolyLog}\left(4,\frac{bx}{a+bx}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[a/(a + b*x)]*Log[(c*x)/(a + b*x)]^2)/(x*(a + b*x)),x]
```

```
[Out] -((Log[(c*x)/(a + b*x)]^2*PolyLog[2, (b*x)/(a + b*x)])/a) + (2*Log[(c*x)/(a
 + b*x)]*PolyLog[3, (b*x)/(a + b*x)])/a - (2*PolyLog[4, (b*x)/(a + b*x)])/a
```

Maple [F] time = 0.531, size = 0, normalized size = 0.

$$\int \frac{1}{x\,(bx+a)}\ln\left(\frac{a}{bx+a}\right)\left(\ln\left(\frac{cx}{bx+a}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a/(b*x+a))*ln(c*x/(b*x+a))^2/x/(b*x+a),x)
```

```
[Out] int(ln(a/(b*x+a))*ln(c*x/(b*x+a))^2/x/(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log(bx+a)^4 - 4\log(bx+a)^3\log(x)}{4a} + \int \frac{a\log(a)\log(c)^2 + 2a\log(a)\log(c)\log(x) + a\log(a)\log(x)^2 + (a(\log(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="maxima"
)
```

```
[Out] 1/4*(log(b*x + a)^4 - 4*log(b*x + a)^3*log(x))/a + integrate((a*log(a)*log(
c)^2 + 2*a*log(a)*log(c)*log(x) + a*log(a)*log(x)^2 + (a*(log(a) + 2*log(c)
) + (3*b*x + 2*a)*log(x))*log(b*x + a)^2 - (2*a*(log(a) + log(c))*log(x) +
a*log(x)^2 + (2*log(a)*log(c) + log(c)^2)*a)*log(b*x + a))/(a*b*x^2 + a^2*x
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\frac{cx}{bx+a}\right)^2\log\left(\frac{a}{bx+a}\right)}{bx^2+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="fricas"
)
```

```
[Out] integral(log(c*x/(b*x + a))^2*log(a/(b*x + a))/(b*x^2 + a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(\frac{a}{a+bx}\right)\log\left(\frac{cx}{a+bx}\right)^2}{x\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a/(b*x+a))*ln(c*x/(b*x+a))**2/x/(b*x+a),x)
```

```
[Out] Integral(log(a/(a + b*x))*log(c*x/(a + b*x))**2/(x*(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(\frac{cx}{bx+a}\right)^2\log\left(\frac{a}{bx+a}\right)}{(bx+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a/(b*x+a))*log(c*x/(b*x+a))^2/x/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(c*x/(b*x + a))^2*log(a/(b*x + a))/((b*x + a)*x), x)
```

3.91 $\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx$

Optimal. Leaf size=150

$$-\frac{\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)}+\frac{2\text{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)}-\frac{2\text{PolyLog}\left(4,1-\frac{bc-ad}{b(c+dx)}\right)}{g(bc-ad)}$$

```
[Out] -((Log[(e*(a + b*x))/(c + d*x)]^2*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))]
)/((b*c - a*d)*g)) + (2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[3, 1 - (b*c -
a*d)/(b*(c + d*x))])/((b*c - a*d)*g) - (2*PolyLog[4, 1 - (b*c - a*d)/(b*(c
+ d*x))])/((b*c - a*d)*g)
```

Rubi [A] time = 0.247639, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.055, Rules used = {2506, 2508, 6610}

$$-\frac{\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)}+\frac{2\text{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)}{g(bc-ad)}-\frac{2\text{PolyLog}\left(4,1-\frac{bc-ad}{b(c+dx)}\right)}{g(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x))/(c + d*x)]^2)/((c + d
*x)*(a*g + b*g*x)),x]
```

```
[Out] -((Log[(e*(a + b*x))/(c + d*x)]^2*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))]
)/((b*c - a*d)*g)) + (2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[3, 1 - (b*c -
a*d)/(b*(c + d*x))])/((b*c - a*d)*g) - (2*PolyLog[4, 1 - (b*c - a*d)/(b*(c
+ d*x))])/((b*c - a*d)*g)
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2508

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[(v*(c +
 d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(h*PolyLog[n
+ 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] - Dist[h*p*
r*s, Int[(PolyLog[n + 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(
(a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)}{(c+dx)(ag+bgx)}\,dx = -\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right)\text{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{2\int \frac{\log\left(\frac{e(a+bx)}{c+dx}\right)\text{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}\,dx}{g}$$

$$= -\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right)\text{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{2\log\left(\frac{e(a+bx)}{c+dx}\right)\text{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} - \frac{2\int \frac{\text{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}\,d}{g}$$

$$= -\frac{\log^2\left(\frac{e(a+bx)}{c+dx}\right)\text{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{2\log\left(\frac{e(a+bx)}{c+dx}\right)\text{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} - \frac{2\text{Li}_4\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g}$$

Mathematica [A] time = 0.0376798, size = 110, normalized size = 0.73

$$\frac{-\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\log^2\left(\frac{e(a+bx)}{c+dx}\right)+2\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)-2\text{PolyLog}\left(4,\frac{d(a+bx)}{b(c+dx)}\right)}{g(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x))/(c + d*x)]^2)/(
(c + d*x)*(a*g + b*g*x)),x]
```

```
[Out] (-(Log[(e*(a + b*x))/(c + d*x)]^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])
+ 2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))] -
2*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))])/((b*c - a*d)*g)
```

Maple [F] time = 1.102, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(bgx+ag\right)}\ln\left(\frac{-ad+bc}{b\,(dx+c)}\right)\left(\ln\left(\frac{e\,(bx+a)}{dx+c}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*d+b*c)/b/(d*x+c))*ln(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x+a*g),x)
```

```
[Out] int(ln((-a*d+b*c)/b/(d*x+c))*ln(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x+a*g),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\log(bx+a)\log(dx+c)^3-\log(dx+c)^4}{4\left(bcg-adg\right)} - \int \frac{((d\log(bc-ad)-d\log(b))a-(c\log(bc-ad)-c\log(b))b)\log(bx+a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*d+b*c)/b/(d*x+c))*log(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x
+a*g),x, algorithm="maxima")
```

```
[Out] -1/4*(4*log(b*x + a)*log(d*x + c)^3 - log(d*x + c)^4)/(b*c*g - a*d*g) - int
egrate((((d*log(b*c - a*d) - d*log(b))*a - (c*log(b*c - a*d) - c*log(b))*b)
```

```
*log(b*x + a)^2 + ((d*log(b*c - a*d) - d*log(b) + 2*d*log(e))*a - (c*(log(b
*c - a*d) + 2*log(e)) - c*log(b))*b - (3*b*d*x + 2*b*c + a*d)*log(b*x + a))
*log(d*x + c)^2 + (d*log(b*c - a*d)*log(e)^2 - d*log(b)*log(e)^2)*a - (c*lo
g(b*c - a*d)*log(e)^2 - c*log(b)*log(e)^2)*b + 2*((d*log(b*c - a*d)*log(e)
- d*log(b)*log(e))*a - (c*log(b*c - a*d)*log(e) - c*log(b)*log(e))*b)*log(b
*x + a) + ((b*c - a*d)*log(b*x + a)^2 - (2*d*log(b*c - a*d)*log(e) - 2*d*lo
g(b)*log(e) + d*log(e)^2)*a - (2*c*log(b)*log(e) - (2*log(b*c - a*d)*log(e)
 + log(e)^2)*c)*b - 2*((d*log(b*c - a*d) - d*log(b) + d*log(e))*a - (c*(log
(b*c - a*d) + log(e)) - c*log(b))*b)*log(b*x + a))*log(d*x + c))/(a*b*c^2*g
 - a^2*c*d*g + (b^2*c*d*g - a*b*d^2*g)*x^2 + (b^2*c^2*g - a^2*d^2*g)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\frac{bc-ad}{bdx+bc}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{bdgx^2+acg+(bc+ad)gx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*d+b*c)/b/(d*x+c))*log(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x
+a*g),x, algorithm="fricas")
```

```
[Out] integral(log((b*c - a*d)/(b*d*x + b*c))*log((b*e*x + a*e)/(d*x + c))^2/(b*d
*g*x^2 + a*c*g + (b*c + a*d)*g*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*d+b*c)/b/(d*x+c))*ln(e*(b*x+a)/(d*x+c))**2/(d*x+c)/(b*g*x+
a*g),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{(bx+a)e}{dx+c}\right)^2\log\left(\frac{bc-ad}{(dx+c)b}\right)}{(bgx+ag)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*d+b*c)/b/(d*x+c))*log(e*(b*x+a)/(d*x+c))^2/(d*x+c)/(b*g*x
+a*g),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)*e/(d*x + c))^2*log((b*c - a*d)/((d*x + c)*b))/((b*g
*x + a*g)*(d*x + c)), x)
```

3.92 $\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx$

Optimal. Leaf size=160

$$-\frac{\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)}+\frac{2n\text{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)}-\frac{2n^2\text{PolyLog}\left(4,1-\frac{bc-ad}{b(c+dx)}\right)}{g(bc-ad)}$$

```
[Out] -((Log[e*((a + b*x)/(c + d*x))^n]^2*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x)
)])/((b*c - a*d)*g)) + (2*n*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[3, 1 - (
b*c - a*d)/(b*(c + d*x))])/((b*c - a*d)*g) - (2*n^2*PolyLog[4, 1 - (b*c - a
*d)/(b*(c + d*x))])/((b*c - a*d)*g)
```

Rubi [A] time = 0.250151, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2506, 2508, 6610}

$$-\frac{\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)}+\frac{2n\text{PolyLog}\left(3,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(bc-ad)}-\frac{2n^2\text{PolyLog}\left(4,1-\frac{bc-ad}{b(c+dx)}\right)}{g(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*(c + d*x))])/((c +
 d*x)*(a*g + b*g*x)),x]
```

```
[Out] -((Log[e*((a + b*x)/(c + d*x))^n]^2*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x)
)])/((b*c - a*d)*g)) + (2*n*Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[3, 1 - (
b*c - a*d)/(b*(c + d*x))])/((b*c - a*d)*g) - (2*n^2*PolyLog[4, 1 - (b*c - a
*d)/(b*(c + d*x))])/((b*c - a*d)*g)
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 2508

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] :> With[{g = Simplify[(v*(c +
 d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(h*PolyLog[n
+ 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] - Dist[h*p*
r*s, Int[(PolyLog[n + 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/(
(a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(ag+bgx)}\,dx = -\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{(2n)\int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}\,dx}{g}$$

$$= -\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{2n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} - \frac{(2n^2}{}$$

$$= -\frac{\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_2\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} + \frac{2n\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\text{Li}_3\left(1-\frac{bc-ad}{b(c+dx)}\right)}{(bc-ad)g} - \frac{2n^2\text{I}}{}$$

Mathematica [B] time = 0.455601, size = 559, normalized size = 3.49

$$\frac{3n\left(-2\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)+2\log\left(\frac{a+bx}{c+dx}\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)+\log\left(\frac{bc-ad}{bc+bdx}\right)\log^2\left(\frac{a+bx}{c+dx}\right)\right)\left(n\log\left(\frac{a+bx}{c+dx}\right)-\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*(c + d*x))])
/((c + d*x)*(a*g + b*g*x)),x]
```

```
[Out] (Log[(a + b*x)/(c + d*x)]*(3*Log[e*((a + b*x)/(c + d*x))^n]^2 - 3*n*Log[e*(
(a + b*x)/(c + d*x))^n]*Log[(a + b*x)/(c + d*x)] + n^2*Log[(a + b*x)/(c + d
*x)]^2)*Log[(b*c - a*d)/(b*c + b*d*x)] + (3*(Log[e*((a + b*x)/(c + d*x))^n]
 - n*Log[(a + b*x)/(c + d*x)])^2*(-Log[c/d + x]^2 - 2*Log[a/b + x]*Log[c +
d*x] + 2*Log[c/d + x]*Log[c + d*x] + 2*Log[(a + b*x)/(c + d*x)]*Log[c + d*x
] + 2*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + 2*PolyLog[2, (d*(a + b*
x))/(-(b*c) + a*d)]))/2 + 3*n*(-Log[e*((a + b*x)/(c + d*x))^n] + n*Log[(a +
 b*x)/(c + d*x)])*(Log[(a + b*x)/(c + d*x)]^2*Log[(b*c - a*d)/(b*c + b*d*x)
] + 2*Log[(a + b*x)/(c + d*x)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*
PolyLog[3, (d*(a + b*x))/(b*(c + d*x))]) - n^2*(Log[(a + b*x)/(c + d*x)]^3*
Log[(b*c - a*d)/(b*c + b*d*x)] + 3*Log[(a + b*x)/(c + d*x)]^2*PolyLog[2, (d
*(a + b*x))/(b*(c + d*x))] - 6*Log[(a + b*x)/(c + d*x)]*PolyLog[3, (d*(a +
b*x))/(b*(c + d*x))] + 6*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))]))/(3*(b*c
- a*d)*g)
```

Maple [F] time = 5.443, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)\left(bgx+ag\right)}\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2\ln\left(\frac{-ad+bc}{b\,(dx+c)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b*g*x+a*g
),x)
```

```
[Out] int(ln(e*((b*x+a)/(d*x+c))^n)^2*ln((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b*g*x+a*g
),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2 \log\left(\frac{bc-ad}{(dx+c)b}\right)}{(bgx+ag)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b
*g*x+a*g),x, algorithm="maxima")
```

```
[Out] integrate(log(e*((b*x + a)/(d*x + c))^n)^2*log((b*c - a*d)/((d*x + c)*b))/(
(b*g*x + a*g)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2 \log\left(\frac{bc-ad}{bdx+bc}\right)}{bdgx^2+acg+(bc+ad)gx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b
*g*x+a*g),x, algorithm="fricas")
```

```
[Out] integral(log(e*((b*x + a)/(d*x + c))^n)^2*log((b*c - a*d)/(b*d*x + b*c))/(b
*d*g*x^2 + a*c*g + (b*c + a*d)*g*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*((b*x+a)/(d*x+c))**n)**2*ln((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b
*g*x+a*g),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2 \log\left(\frac{bc-ad}{(dx+c)b}\right)}{(bgx+ag)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*((b*x+a)/(d*x+c))^n)^2*log((-a*d+b*c)/b/(d*x+c))/(d*x+c)/(b
*g*x+a*g),x, algorithm="giac")
```

```
[Out] integrate(log(e*((b*x + a)/(d*x + c))^n)^2*log((b*c - a*d)/((d*x + c)*b))/(
(b*g*x + a*g)*(d*x + c)), x)
```

3.93 $\int \log \left(\frac{c(b+ax)}{x}\right) dx$

Optimal. Leaf size=25

$$x\log \left(ac+\frac{bc}{x}\right)+\frac{b\log (ax+b)}{a}$$

```
[Out] x*Log[a*c + (b*c)/x] + (b*Log[b + a*x])/a
```

Rubi [A] time = 0.0115837, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2453, 2448, 263, 31}

$$x\log \left(ac+\frac{bc}{x}\right)+\frac{b\log (ax+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*(b + a*x))/x],x]
```

```
[Out] x*Log[a*c + (b*c)/x] + (b*Log[b + a*x])/a
```

Rule 2453

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \log\left(\frac{c(b+ax)}{x}\right)dx = \int \log\left(ac+\frac{bc}{x}\right)dx$$
$$= x\log\left(ac+\frac{bc}{x}\right) + (bc)\int \frac{1}{\left(ac+\frac{bc}{x}\right)x}dx$$
$$= x\log\left(ac+\frac{bc}{x}\right) + (bc)\int \frac{1}{bc+acx}dx$$
$$= x\log\left(ac+\frac{bc}{x}\right) + \frac{b\log(b+ax)}{a}$$

Mathematica [A] time = 0.0048467, size = 28, normalized size = 1.12

$$\frac{(ax+b)\log\left(\frac{c(ax+b)}{x}\right)}{a} + \frac{b\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x))/x],x]
```

```
[Out] (b*Log[x])/a + ((b + a*x)*Log[(c*(b + a*x))/x])/a
```

Maple [A] time = 0.188, size = 44, normalized size = 1.8

$$-\frac{b}{a}\ln\left(\frac{bc}{x}\right) + x\ln\left(ac+\frac{bc}{x}\right) + \frac{b}{a}\ln\left(ac+\frac{bc}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a*x+b)/x),x)
```

```
[Out] -b/a*ln(b*c/x)+x*ln(a*c+b*c/x)+b*ln(a*c+b*c/x)/a
```

Maxima [A] time = 1.17715, size = 34, normalized size = 1.36

$$x\log\left(\frac{(ax+b)c}{x}\right) + \frac{b\log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x),x, algorithm="maxima")
```

```
[Out] x*log((a*x + b)*c/x) + b*log(a*x + b)/a
```

Fricas [A] time = 1.93737, size = 63, normalized size = 2.52

$$\frac{ax\log\left(\frac{acx+bc}{x}\right) + b\log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x),x, algorithm="fricas")
```

```
[Out] (a*x*log((a*c*x + b*c)/x) + b*log(a*x + b))/a
```

Sympy [A] time = 0.341546, size = 20, normalized size = 0.8

$$x\log\left(\frac{c(ax+b)}{x}\right)+\frac{b\log(ax+b)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)/x),x)
```

```
[Out] x*log(c*(a*x + b)/x) + b*log(a*x + b)/a
```

Giac [A] time = 1.25673, size = 35, normalized size = 1.4

$$x\log\left(\frac{(ax+b)c}{x}\right)+\frac{b\log(|ax+b|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x),x, algorithm="giac")
```

```
[Out] x*log((a*x + b)*c/x) + b*log(abs(a*x + b))/a
```

3.94 $\int \log^2\left(\frac{c(b+ax)}{x}\right) dx$

Optimal. Leaf size=67

$$-\frac{2b\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{a}+\frac{(ax+b)\log^2\left(ac+\frac{bc}{x}\right)}{a}-\frac{2b\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)\right)}{a}$$

```
[Out] ((b + a*x)*Log[a*c + (b*c)/x]^2)/a - (2*b*Log[c*(a + b/x)]*Log[-(b/(a*x))])
/a - (2*b*PolyLog[2, 1 + b/(a*x)])/a
```

Rubi [A] time = 0.0737622, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2453, 2449, 2454, 2394, 2315}

$$-\frac{2b\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{a}+\frac{(ax+b)\log^2\left(ac+\frac{bc}{x}\right)}{a}-\frac{2b\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*(b + a*x))/x]^2,x]
```

```
[Out] ((b + a*x)*Log[a*c + (b*c)/x]^2)/a - (2*b*Log[c*(a + b/x)]*Log[-(b/(a*x))])
/a - (2*b*PolyLog[2, 1 + b/(a*x)])/a
```

Rule 2453

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

Rule 2449

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[((e + d*x)*(a + b*Log[c*(d + e/x)^p])^q)/d, x] + Dist[(b*e*p*q)/d, In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \log^2\left(\frac{c(b+ax)}{x}\right) dx = \int \log^2\left(ac+\frac{bc}{x}\right) dx$$
$$= \frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a} + \frac{(2b)\int \frac{\log\left(ac+\frac{bc}{x}\right)}{x} dx}{a}$$
$$= \frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{\log(ac+bcx)}{x} dx, x, \frac{1}{x}\right)}{a}$$
$$= \frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a} - \frac{2b\log\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} + \frac{(2b^2c)\,\mathrm{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{ac+bcx} dx, x, \frac{1}{x}\right)}{a}$$
$$= \frac{(b+ax)\log^2\left(ac+\frac{bc}{x}\right)}{a} - \frac{2b\log\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} - \frac{2b\mathrm{Li}_2\left(1+\frac{b}{ax}\right)}{a}$$

Mathematica [A] time = 0.0135973, size = 63, normalized size = 0.94

$$\frac{\log\left(\frac{c(ax+b)}{x}\right)\left((ax+b)\log\left(\frac{c(ax+b)}{x}\right)-2b\log\left(-\frac{b}{ax}\right)\right)-2b\mathrm{PolyLog}\left(2,\frac{b}{ax}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x))/x]^2,x]
```

```
[Out] (Log[(c*(b + a*x))/x]*(-2*b*Log[-(b/(a*x))] + (b + a*x)*Log[(c*(b + a*x))/x
]) - 2*b*PolyLog[2, 1 + b/(a*x)])/a
```

Maple [F] time = 0.655, size = 0, normalized size = 0.

$$\int \left(\ln\left(\frac{c(ax+b)}{x}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a*x+b)/x)^2,x)
```

```
[Out] int(ln(c*(a*x+b)/x)^2,x)
```

Maxima [A] time = 1.17904, size = 153, normalized size = 2.28

$$x\log\left(\frac{(ax+b)c}{x}\right)^2 + \frac{2b\log(ax+b)\log\left(\frac{(ax+b)c}{x}\right)}{a} + \frac{\left(\frac{c\log(ax+b)^2}{a} - \frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)c}{a}\right)b - \frac{2(c\log(ax+b)-c\log(x))b\log(ax}{a}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^2,x, algorithm="maxima")
```

```
[Out] x*log((a*x + b)*c/x)^2 + 2*b*log(a*x + b)*log((a*x + b)*c/x)/a + ((c*log(a*
x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b - 2*(c*log(a*
x + b) - c*log(x))*b*log(a*x + b)/a)/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\log\left(\frac{acx+bc}{x}\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^2,x, algorithm="fricas")
```

```
[Out] integral(log((a*c*x + b*c)/x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$2b\int\frac{\log\left(ac+\frac{bc}{x}\right)}{ax+b}\,dx + x\log\left(\frac{c\,(ax+b)}{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)/x)**2,x)
```

```
[Out] 2*b*Integral(log(a*c + b*c/x)/(a*x + b), x) + x*log(c*(a*x + b)/x)**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\log\left(\frac{(ax+b)c}{x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^2,x, algorithm="giac")
```

```
[Out] integrate(log((a*x + b)*c/x)^2, x)
```

3.95 $\int \log^3\left(\frac{c(b+ax)}{x}\right) dx$

Optimal. Leaf size=97

$$-\frac{6b\text{PolyLog}\left(2,\frac{b}{ax}+1\right)\log\left(c\left(a+\frac{b}{x}\right)\right)}{a}+\frac{6b\text{PolyLog}\left(3,\frac{b}{ax}+1\right)}{a}+\frac{(ax+b)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{3b\log\left(-\frac{b}{ax}\right)\log^2\left(c\left(a+\frac{b}{x}\right)\right)}{a}$$

```
[Out] ((b + a*x)*Log[a*c + (b*c)/x]^3)/a - (3*b*Log[c*(a + b/x)]^2*Log[-(b/(a*x))
])/a - (6*b*Log[c*(a + b/x)]*PolyLog[2, 1 + b/(a*x)])/a + (6*b*PolyLog[3, 1
 + b/(a*x)])/a
```

Rubi [A] time = 0.110328, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2453, 2449, 2454, 2396, 2433, 2374, 6589}

$$-\frac{6b\text{PolyLog}\left(2,\frac{b}{ax}+1\right)\log\left(c\left(a+\frac{b}{x}\right)\right)}{a}+\frac{6b\text{PolyLog}\left(3,\frac{b}{ax}+1\right)}{a}+\frac{(ax+b)\log^3\left(ac+\frac{bc}{x}\right)}{a}-\frac{3b\log\left(-\frac{b}{ax}\right)\log^2\left(c\left(a+\frac{b}{x}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*(b + a*x))/x]^3,x]
```

```
[Out] ((b + a*x)*Log[a*c + (b*c)/x]^3)/a - (3*b*Log[c*(a + b/x)]^2*Log[-(b/(a*x))
])/a - (6*b*Log[c*(a + b/x)]*PolyLog[2, 1 + b/(a*x)])/a + (6*b*PolyLog[3, 1
 + b/(a*x)])/a
```

Rule 2453

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

Rule 2449

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[((e + d*x)*(a + b*Log[c*(d + e/x)^p])^q)/d, x] + Dist[(b*e*p*q)/d, In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \log^3\left(\frac{c(b+ax)}{x}\right) dx = \int \log^3\left(ac+\frac{bc}{x}\right) dx$$
$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} + \frac{(3b)\int \frac{\log^2\left(ac+\frac{bc}{x}\right)}{x} dx}{a}$$
$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{(3b)\,\text{Subst}\left(\int \frac{\log^2(ac+bcx)}{x} dx, x, \frac{1}{x}\right)}{a}$$
$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} + \frac{(6b^2c)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)\log(ac+bcx)}{ac+bcx}\right.}{a}$$
$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} + \frac{(6b)\,\text{Subst}\left(\int \frac{\log(x)\log\left(-\frac{b\left(-\frac{a}{b}+\frac{x}{bc}\right)}{a}\right)}{x}\right.}{a}$$
$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} - \frac{6b\log\left(c\left(a+\frac{b}{x}\right)\right)\text{Li}_2\left(1+\frac{b}{ax}\right)}{a} + $$
$$= \frac{(b+ax)\log^3\left(ac+\frac{bc}{x}\right)}{a} - \frac{3b\log^2\left(c\left(a+\frac{b}{x}\right)\right)\log\left(-\frac{b}{ax}\right)}{a} - \frac{6b\log\left(c\left(a+\frac{b}{x}\right)\right)\text{Li}_2\left(1+\frac{b}{ax}\right)}{a} + $$

Mathematica [A] time = 0.0253013, size = 91, normalized size = 0.94

$$\frac{-6b\text{PolyLog}\left(2,\frac{b}{ax}+1\right)\log\left(\frac{c(ax+b)}{x}\right)+6b\text{PolyLog}\left(3,\frac{b}{ax}+1\right)+\left((ax+b)\log\left(\frac{c(ax+b)}{x}\right)-3b\log\left(-\frac{b}{ax}\right)\right)\log^2\left(\frac{c(ax+b)}{x}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x))/x]^3,x]
```

```
[Out] (Log[(c*(b + a*x))/x]^2*(-3*b*Log[-(b/(a*x))] + (b + a*x)*Log[(c*(b + a*x))
/x]) - 6*b*Log[(c*(b + a*x))/x]*PolyLog[2, 1 + b/(a*x)] + 6*b*PolyLog[3, 1
+ b/(a*x)])/a
```

Maple [F] time = 0.549, size = 0, normalized size = 0.

$$\int \left(\ln \left(\frac{c\,(ax+b)}{x}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a*x+b)/x)^3,x)
```

```
[Out] int(ln(c*(a*x+b)/x)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(ax+b)\log{(ax+b)}^{3}+3\,(ax\log{(c)}-ax\log{(x)})\log{(ax+b)}^{2}}{a}+\int \frac{ax\log{(c)}^{3}+b\log{(c)}^{3}-(ax+b)\log{(x)}^{3}+3\,(ax\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^3,x, algorithm="maxima")
```

```
[Out] ((a*x + b)*log(a*x + b)^3 + 3*(a*x*log(c) - a*x*log(x))*log(a*x + b)^2)/a +
 integrate((a*x*log(c)^3 + b*log(c)^3 - (a*x + b)*log(x)^3 + 3*(a*x*log(c)
+ b*log(c))*log(x)^2 + 3*((log(c)^2 - 2*log(c))*a*x + b*log(c)^2 + (a*x + b
)*log(x)^2 - 2*(a*x*(log(c) - 1) + b*log(c))*log(x))*log(a*x + b) - 3*(a*x*
log(c)^2 + b*log(c)^2)*log(x))/(a*x + b), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\log\left(\frac{acx+bc}{x}\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^3,x, algorithm="fricas")
```

```
[Out] integral(log((a*c*x + b*c)/x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$3b\int \frac{\log\left(ac+\frac{bc}{x}\right)^{2}}{ax+b}\,dx+x\log\left(\frac{c\,(ax+b)}{x}\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)/x)**3,x)
```

```
[Out] 3*b*Integral(log(a*c + b*c/x)**2/(a*x + b), x) + x*log(c*(a*x + b)/x)**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\frac{(ax+b)c}{x}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)/x)^3,x, algorithm="giac")
```

```
[Out] integrate(log((a*x + b)*c/x)^3, x)
```

3.96 $\int \log \left(\frac{c(b+ax)^2}{x^2}\right) dx$

Optimal. Leaf size=28

$$x\log\left(\frac{c(ax+b)^2}{x^2}\right)+\frac{2b\log(ax+b)}{a}$$

```
[Out] (2*b*Log[b + a*x])/a + x*Log[(c*(b + a*x)^2)/x^2]
```

Rubi [A] time = 0.0068001, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2486, 31}

$$x\log\left(\frac{c(ax+b)^2}{x^2}\right)+\frac{2b\log(ax+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*(b + a*x)^2)/x^2],x]
```

```
[Out] (2*b*Log[b + a*x])/a + x*Log[(c*(b + a*x)^2)/x^2]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(\frac{c(b+ax)^2}{x^2}\right)dx &= x\log\left(\frac{c(b+ax)^2}{x^2}\right)+(2b)\int\frac{1}{b+ax}\,dx\\ &= \frac{2b\log(b+ax)}{a}+x\log\left(\frac{c(b+ax)^2}{x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0025771, size = 28, normalized size = 1.

$$x\log\left(\frac{c(ax+b)^2}{x^2}\right)+\frac{2b\log(ax+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x)^2)/x^2],x]
```

```
[Out] (2*b*Log[b + a*x])/a + x*Log[(c*(b + a*x)^2)/x^2]
```

Maple [A] time = 0.125, size = 40, normalized size = 1.4

$$x\ln\left(c\left(a+\frac{b}{x}\right)^{2}\right)-2\,\frac{b\ln\left(x^{-1}\right)}{a}+2\,\frac{b}{a}\ln\left(a+\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a*x+b)^2/x^2),x)
```

```
[Out] x*ln(c*(a+b/x)^2)-2*b/a*ln(1/x)+2*b/a*ln(a+b/x)
```

Maxima [A] time = 1.19178, size = 38, normalized size = 1.36

$$x\log\left(\frac{(ax+b)^{2}c}{x^{2}}\right)+\frac{2\,b\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2),x, algorithm="maxima")
```

```
[Out] x*log((a*x + b)^2*c/x^2) + 2*b*log(a*x + b)/a
```

Fricas [A] time = 1.87655, size = 93, normalized size = 3.32

$$\frac{ax\log\left(\frac{a^{2}cx^{2}+2\,abcx+b^{2}c}{x^{2}}\right)+2\,b\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2),x, algorithm="fricas")
```

```
[Out] (a*x*log((a^2*c*x^2 + 2*a*b*c*x + b^2*c)/x^2) + 2*b*log(a*x + b))/a
```

Sympy [A] time = 0.311584, size = 26, normalized size = 0.93

$$x\log\left(\frac{c\,(ax+b)^{2}}{x^{2}}\right)+\frac{2b\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)**2/x**2),x)
```

```
[Out] x*log(c*(a*x + b)**2/x**2) + 2*b*log(a*x + b)/a
```

Giac [A] time = 1.14719, size = 39, normalized size = 1.39

$$x\log\left(\frac{(ax+b)^2c}{x^2}\right)+\frac{2b\log(|ax+b|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2),x, algorithm="giac")
```

```
[Out] x*log((a*x + b)^2*c/x^2) + 2*b*log(abs(a*x + b))/a
```

3.97 $\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right) dx$

Optimal. Leaf size=67

$$\frac{8b\text{PolyLog}\left(2,1-\frac{b}{ax+b}\right)}{a}+x\log^2\left(\frac{c(ax+b)^2}{x^2}\right)-\frac{4b\log\left(\frac{b}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}$$

```
[Out] (-4*b*Log[b/(b + a*x)]*Log[(c*(b + a*x)^2)/x^2])/a + x*Log[(c*(b + a*x)^2)/
x^2]^2 + (8*b*PolyLog[2, 1 - b/(b + a*x)])/a
```

Rubi [A] time = 0.156801, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2486, 2488, 2411, 2343, 2333, 2315}

$$\frac{8b\text{PolyLog}\left(2,1-\frac{b}{ax+b}\right)}{a}+x\log^2\left(\frac{c(ax+b)^2}{x^2}\right)-\frac{4b\log\left(\frac{b}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*(b + a*x)^2)/x^2]^2,x]
```

```
[Out] (-4*b*Log[b/(b + a*x)]*Log[(c*(b + a*x)^2)/x^2])/a + x*Log[(c*(b + a*x)^2)/
x^2]^2 + (8*b*PolyLog[2, 1 - b/(b + a*x)])/a
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
```

```
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \log^2\left(\frac{c(b+ax)^2}{x^2}\right) dx &= x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + (4b)\int \frac{\log\left(\frac{c(b+ax)^2}{x^2}\right)}{b+ax} dx \\
&= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) - \frac{(8b^2)\int \frac{\log\left(\frac{b}{b+ax}\right)}{x(b+ax)} dx}{a} \\
&= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) - \frac{(8b^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{b}{x}\right)}{x\left(-\frac{b}{a}+\frac{x}{a}\right)} dx, x, b+ax\right)}{a^2} \\
&= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{(8b^2)\,\text{Subst}\left(\int \frac{\log(bx)}{\left(-\frac{b}{a}+\frac{1}{ax}\right)x} dx, x, \frac{1}{b+ax}\right)}{a^2} \\
&= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{(8b^2)\,\text{Subst}\left(\int \frac{\log(bx)}{\frac{1}{a}-\frac{bx}{a}} dx, x, \frac{1}{b+ax}\right)}{a^2} \\
&= -\frac{4b\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{a} + x\log^2\left(\frac{c(b+ax)^2}{x^2}\right) + \frac{8b\text{Li}_2\left(\frac{ax}{b+ax}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0156204, size = 106, normalized size = 1.58

$$\frac{8b\text{PolyLog}\left(2,\frac{ax+b}{b}\right)}{a} + x\log^2\left(\frac{c(ax+b)^2}{x^2}\right) - \frac{4b\log\left(\frac{b}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a} - \frac{4b\log^2\left(\frac{b}{ax+b}\right)}{a} - \frac{8b\log\left(-\frac{ax}{b}\right)\log\left(\frac{b}{ax+b}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x)^2)/x^2]^2,x]
```

```
[Out] (-8*b*Log[-((a*x)/b)]*Log[b/(b + a*x)])/a - (4*b*Log[b/(b + a*x)]^2)/a - (4
*b*Log[b/(b + a*x)]*Log[(c*(b + a*x)^2)/x^2])/a + x*Log[(c*(b + a*x)^2)/x^2
]^2 + (8*b*PolyLog[2, (b + a*x)/b])/a
```

Maple [F] time = 0.526, size = 0, normalized size = 0.

$$\int \left(\ln\left(\frac{c\left(ax+b\right)^2}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a*x+b)^2/x^2)^2,x)
```

```
[Out] int(ln(c*(a*x+b)^2/x^2)^2,x)
```

Maxima [A] time = 1.22382, size = 159, normalized size = 2.37

$$x\log\left(\frac{(ax+b)^2c}{x^2}\right)^2+\frac{4\,b\log(ax+b)\log\left(\frac{(ax+b)^2c}{x^2}\right)}{a}+\frac{4\left(\left(\frac{c\log(ax+b)^2}{a}-\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)c}{a}\right)b-\frac{2\,(c\log(ax+b)-c\log(x}{a}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] x*log((a*x + b)^2*c/x^2)^2 + 4*b*log(a*x + b)*log((a*x + b)^2*c/x^2)/a + 4*
((c*log(a*x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b - 2
*(c*log(a*x + b) - c*log(x))*b*log(a*x + b)/a)/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\log\left(\frac{a^2cx^2+2\,abcx+b^2c}{x^2}\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] integral(log((a^2*c*x^2 + 2*a*b*c*x + b^2*c)/x^2)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$4b\int\frac{\log\left(a^2c+\frac{2abc}{x}+\frac{b^2c}{x^2}\right)}{ax+b}\,dx+x\log\left(\frac{c\,(ax+b)^2}{x^2}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)**2/x**2)**2,x)
```

```
[Out] 4*b*Integral(log(a**2*c + 2*a*b*c/x + b**2*c/x**2)/(a*x + b), x) + x*log(c*
(a*x + b)**2/x**2)**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\log\left(\frac{(ax+b)^2c}{x^2}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate(log((a*x + b)^2*c/x^2)^2, x)
```

3.98 $\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right) dx$

Optimal. Leaf size=102

$$\frac{24b\text{PolyLog}\left(2,\frac{ax}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}+\frac{48b\text{PolyLog}\left(3,\frac{ax}{ax+b}\right)}{a}+x\log^3\left(\frac{c(ax+b)^2}{x^2}\right)-\frac{6b\log\left(1-\frac{ax}{ax+b}\right)\log^2\left(\frac{c(ax+b)^2}{x^2}\right)}{a}$$

```
[Out] x*Log[(c*(b + a*x)^2)/x^2]^3 - (6*b*Log[(c*(b + a*x)^2)/x^2]^2*Log[1 - (a*x
)/(b + a*x)])/a + (24*b*Log[(c*(b + a*x)^2)/x^2]*PolyLog[2, (a*x)/(b + a*x)
])/a + (48*b*PolyLog[3, (a*x)/(b + a*x)])/a
```

Rubi [A] time = 0.131848, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2486, 2488, 2506, 6610}

$$\frac{24b\text{PolyLog}\left(2,1-\frac{b}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}+\frac{48b\text{PolyLog}\left(3,1-\frac{b}{ax+b}\right)}{a}+x\log^3\left(\frac{c(ax+b)^2}{x^2}\right)-\frac{6b\log\left(\frac{b}{ax+b}\right)\log^2\left(\frac{c(ax+b)^2}{x^2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*(b + a*x)^2)/x^2]^3,x]
```

```
[Out] (-6*b*Log[b/(b + a*x)]*Log[(c*(b + a*x)^2)/x^2]^2)/a + x*Log[(c*(b + a*x)^2
)/x^2]^3 + (24*b*Log[(c*(b + a*x)^2)/x^2]*PolyLog[2, 1 - b/(b + a*x)])/a +
(48*b*PolyLog[3, 1 - b/(b + a*x)])/a
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \log^3\left(\frac{c(b+ax)^2}{x^2}\right)dx = x\log^3\left(\frac{c(b+ax)^2}{x^2}\right)+(6b)\int\frac{\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{b+ax}dx$$

$$= -\frac{6b\log\left(\frac{b}{b+ax}\right)\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{a}+x\log^3\left(\frac{c(b+ax)^2}{x^2}\right)-\frac{(24b^2)\int\frac{\log\left(\frac{b}{b+ax}\right)\log\left(\frac{c(b+ax)^2}{x^2}\right)}{x(b+ax)}dx}{a}$$

$$= -\frac{6b\log\left(\frac{b}{b+ax}\right)\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{a}+x\log^3\left(\frac{c(b+ax)^2}{x^2}\right)+\frac{24b\log\left(\frac{c(b+ax)^2}{x^2}\right)\text{Li}_2\left(1-\frac{b}{b+ax}\right)}{a}+\frac{(48b$$

$$= -\frac{6b\log\left(\frac{b}{b+ax}\right)\log^2\left(\frac{c(b+ax)^2}{x^2}\right)}{a}+x\log^3\left(\frac{c(b+ax)^2}{x^2}\right)+\frac{24b\log\left(\frac{c(b+ax)^2}{x^2}\right)\text{Li}_2\left(1-\frac{b}{b+ax}\right)}{a}+\frac{48b\text{I}$$

Mathematica [A] time = 0.0245463, size = 98, normalized size = 0.96

$$\frac{24b\text{PolyLog}\left(2,\frac{ax}{ax+b}\right)\log\left(\frac{c(ax+b)^2}{x^2}\right)}{a}+\frac{48b\text{PolyLog}\left(3,\frac{ax}{ax+b}\right)}{a}+x\log^3\left(\frac{c(ax+b)^2}{x^2}\right)-\frac{6b\log\left(\frac{b}{ax+b}\right)\log^2\left(\frac{c(ax+b)^2}{x^2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*(b + a*x)^2)/x^2]^3,x]
```

```
[Out] (-6*b*Log[b/(b + a*x)]*Log[(c*(b + a*x)^2)/x^2]^2)/a + x*Log[(c*(b + a*x)^2
)/x^2]^3 + (24*b*Log[(c*(b + a*x)^2)/x^2]*PolyLog[2, (a*x)/(b + a*x)])/a +
(48*b*PolyLog[3, (a*x)/(b + a*x)])/a
```

Maple [F] time = 0.504, size = 0, normalized size = 0.

$$\int\left(\ln\left(\frac{c(ax+b)^2}{x^2}\right)\right)^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(a*x+b)^2/x^2)^3,x)
```

```
[Out] int(ln(c*(a*x+b)^2/x^2)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{4\left(2(ax+b)\log(ax+b)^3+3(ax\log(c)-2ax\log(x))\log(ax+b)^2\right)}{a}+\int\frac{ax\log(c)^3+b\log(c)^3-8(ax+b)\log(x)^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 4*(2*(a*x + b)*log(a*x + b)^3 + 3*(a*x*log(c) - 2*a*x*log(x))*log(a*x + b)^
2)/a + integrate((a*x*log(c)^3 + b*log(c)^3 - 8*(a*x + b)*log(x)^3 + 12*(a*
x*log(c) + b*log(c))*log(x)^2 + 6*((log(c)^2 - 4*log(c))*a*x + b*log(c)^2 +
 4*(a*x + b)*log(x)^2 - 4*(a*x*(log(c) - 2) + b*log(c))*log(x))*log(a*x + b
) - 6*(a*x*log(c)^2 + b*log(c)^2)*log(x))/(a*x + b), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\log\left(\frac{a^2cx^2 + 2\,abcx + b^2c}{x^2}\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] integral(log((a^2*c*x^2 + 2*a*b*c*x + b^2*c)/x^2)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$6b\int \frac{\log\left(a^2c + \frac{2abc}{x} + \frac{b^2c}{x^2}\right)^2}{ax + b}\,dx + x\log\left(\frac{c\,(ax+b)^2}{x^2}\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(a*x+b)**2/x**2)**3,x)
```

```
[Out] 6*b*Integral(log(a**2*c + 2*a*b*c/x + b**2*c/x**2)**2/(a*x + b), x) + x*log
(c*(a*x + b)**2/x**2)**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\frac{(ax+b)^2c}{x^2}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(a*x+b)^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate(log((a*x + b)^2*c/x^2)^3, x)
```

3.99 $\int \log\left(\frac{cx^2}{(b+ax)^2}\right) dx$

Optimal. Leaf size=28

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2b\log(ax+b)}{a}$$

```
[Out] x*Log[(c*x^2)/(b + a*x)^2] - (2*b*Log[b + a*x])/a
```

Rubi [A] time = 0.0066862, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2486, 31}

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2b\log(ax+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*x^2)/(b + a*x)^2],x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2] - (2*b*Log[b + a*x])/a
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(\frac{cx^2}{(b+ax)^2}\right) dx &= x\log\left(\frac{cx^2}{(b+ax)^2}\right)-(2b)\int \frac{1}{b+ax}\,dx\\ &= x\log\left(\frac{cx^2}{(b+ax)^2}\right)-\frac{2b\log(b+ax)}{a}\end{aligned}$$

Mathematica [A] time = 0.0028988, size = 28, normalized size = 1.

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2b\log(ax+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x^2)/(b + a*x)^2],x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2] - (2*b*Log[b + a*x])/a
```

Maple [B] time = 0.224, size = 79, normalized size = 2.8

$$\ln\left(\frac{c}{a^2}\left(\frac{b}{ax+b}-1\right)^2\right)x+2\,\frac{b\ln\left((ax+b)^{-1}\right)}{a}-2\,\frac{b}{a}\ln\left(\frac{b}{ax+b}-1\right)+\frac{b}{a}\ln\left(\frac{c}{a^2}\left(\frac{b}{ax+b}-1\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^2/(a*x+b)^2),x)
```

```
[Out] ln(c*(b/(a*x+b)-1)^2/a^2)*x+2/a*b*ln(1/(a*x+b))-2/a*b*ln(b/(a*x+b)-1)+1/a*l
n(c*(b/(a*x+b)-1)^2/a^2)*b
```

Maxima [A] time = 1.24105, size = 38, normalized size = 1.36

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2\,b\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2),x, algorithm="maxima")
```

```
[Out] x*log(c*x^2/(a*x + b)^2) - 2*b*log(a*x + b)/a
```

Fricas [A] time = 1.96246, size = 88, normalized size = 3.14

$$\frac{ax\log\left(\frac{cx^2}{a^2x^2+2\,abx+b^2}\right)-2\,b\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2),x, algorithm="fricas")
```

```
[Out] (a*x*log(c*x^2/(a^2*x^2 + 2*a*b*x + b^2)) - 2*b*log(a*x + b))/a
```

Sympy [A] time = 0.363537, size = 26, normalized size = 0.93

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)-\frac{2b\log\left(ax+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**2/(a*x+b)**2),x)
```

```
[Out] x*log(c*x**2/(a*x + b)**2) - 2*b*log(a*x + b)/a
```

Giac [A] time = 1.26297, size = 39, normalized size = 1.39

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right) - \frac{2\,b\log\left(|ax+b|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2),x, algorithm="giac")
```

```
[Out] x*log(c*x^2/(a*x + b)^2) - 2*b*log(abs(a*x + b))/a
```

3.100 $\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right) dx$

Optimal. Leaf size=67

$$\frac{8b\text{PolyLog}\left(2,1-\frac{b}{ax+b}\right)}{a}+x\log^2\left(\frac{cx^2}{(ax+b)^2}\right)+\frac{4b\log\left(\frac{b}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}$$

```
[Out] x*Log[(c*x^2)/(b + a*x)^2]^2 + (4*b*Log[(c*x^2)/(b + a*x)^2]*Log[b/(b + a*x
)])/a + (8*b*PolyLog[2, 1 - b/(b + a*x)])/a
```

Rubi [A] time = 0.154865, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2486, 2488, 2411, 2343, 2333, 2315}

$$\frac{8b\text{PolyLog}\left(2,1-\frac{b}{ax+b}\right)}{a}+x\log^2\left(\frac{cx^2}{(ax+b)^2}\right)+\frac{4b\log\left(\frac{b}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*x^2)/(b + a*x)^2]^2,x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2]^2 + (4*b*Log[(c*x^2)/(b + a*x)^2]*Log[b/(b + a*x
)])/a + (8*b*PolyLog[2, 1 - b/(b + a*x)])/a
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
```

```
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \log^2\left(\frac{cx^2}{(b+ax)^2}\right)dx &= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right)-(4b)\int\frac{\log\left(\frac{cx^2}{(b+ax)^2}\right)}{b+ax}\,dx\\
&= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}-\frac{\left(8b^2\right)\int\frac{\log\left(\frac{b}{b+ax}\right)}{x(b+ax)}\,dx}{a}\\
&= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}-\frac{\left(8b^2\right)\text{Subst}\left(\int\frac{\log\left(\frac{b}{x}\right)}{x\left(-\frac{b}{a}+\frac{x}{a}\right)}\,dx,x,b+ax\right)}{a^2}\\
&= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}+\frac{\left(8b^2\right)\text{Subst}\left(\int\frac{\log(bx)}{\left(-\frac{b}{a}+\frac{1}{ax}\right)x}\,dx,x,\frac{1}{b+ax}\right)}{a^2}\\
&= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}+\frac{\left(8b^2\right)\text{Subst}\left(\int\frac{\log(bx)}{\frac{1}{a}-\frac{bx}{a}}\,dx,x,\frac{1}{b+ax}\right)}{a^2}\\
&= x\log^2\left(\frac{cx^2}{(b+ax)^2}\right)+\frac{4b\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a}+\frac{8b\text{Li}_2\left(\frac{ax}{b+ax}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0141174, size = 106, normalized size = 1.58

$$\frac{8b\text{PolyLog}\left(2,\frac{ax+b}{b}\right)}{a}+x\log^2\left(\frac{cx^2}{(ax+b)^2}\right)+\frac{4b\log\left(\frac{b}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}-\frac{4b\log^2\left(\frac{b}{ax+b}\right)}{a}-\frac{8b\log\left(-\frac{ax}{b}\right)\log\left(\frac{b}{ax+b}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x^2)/(b + a*x)^2]^2,x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2]^2 - (8*b*Log[-((a*x)/b)]*Log[b/(b + a*x)])/a + (
4*b*Log[(c*x^2)/(b + a*x)^2]*Log[b/(b + a*x)])/a - (4*b*Log[b/(b + a*x)]^2)
/a + (8*b*PolyLog[2, (b + a*x)/b])/a
```

Maple [F] time = 0.94, size = 0, normalized size = 0.

$$\int\left(\ln\left(\frac{cx^2}{(ax+b)^2}\right)\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^2/(a*x+b)^2)^2,x)
```

```
[Out] int(ln(c*x^2/(a*x+b)^2)^2,x)
```

Maxima [A] time = 1.1449, size = 159, normalized size = 2.37

$$x\log\left(\frac{cx^2}{(ax+b)^2}\right)^2 - \frac{4\,b\log(ax+b)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a} + \frac{4\left(\left(\frac{c\log(ax+b)^2}{a} - \frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)c}{a}\right)b - \frac{2\,(c\log(ax+b)-c\log(x))b}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^2,x, algorithm="maxima")
```

```
[Out] x*log(c*x^2/(a*x + b)^2)^2 - 4*b*log(a*x + b)*log(c*x^2/(a*x + b)^2)/a + 4*
((c*log(a*x + b)^2/a - 2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*c/a)*b - 2
*(c*log(a*x + b) - c*log(x))*b*log(a*x + b)/a)/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\log\left(\frac{cx^2}{a^2x^2+2\,abx+b^2}\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^2,x, algorithm="fricas")
```

```
[Out] integral(log(c*x^2/(a^2*x^2 + 2*a*b*x + b^2))^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-4b\int\frac{\log\left(\frac{cx^2}{a^2x^2+2abx+b^2}\right)}{ax+b}\,dx + x\log\left(\frac{cx^2}{(ax+b)^2}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**2/(a*x+b)**2)**2,x)
```

```
[Out] -4*b*Integral(log(c*x**2/(a**2*x**2 + 2*a*b*x + b**2))/(a*x + b), x) + x*lo
g(c*x**2/(a*x + b)**2)**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\log\left(\frac{cx^2}{(ax+b)^2}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^2,x, algorithm="giac")
```

```
[Out] integrate(log(c*x^2/(a*x + b)^2)^2, x)
```

3.101 $\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right)\,dx$

Optimal. Leaf size=98

$$\frac{24b\text{PolyLog}\left(2,\frac{ax}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}-\frac{48b\text{PolyLog}\left(3,\frac{ax}{ax+b}\right)}{a}+x\log^3\left(\frac{cx^2}{(ax+b)^2}\right)+\frac{6b\log\left(\frac{b}{ax+b}\right)\log^2\left(\frac{cx^2}{(ax+b)^2}\right)}{a}$$

```
[Out] x*Log[(c*x^2)/(b + a*x)^2]^3 + (6*b*Log[(c*x^2)/(b + a*x)^2]^2*Log[b/(b + a
*x)])/a + (24*b*Log[(c*x^2)/(b + a*x)^2]*PolyLog[2, (a*x)/(b + a*x)])/a - (
48*b*PolyLog[3, (a*x)/(b + a*x)])/a
```

Rubi [A] time = 0.127085, antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2486, 2488, 2506, 6610}

$$\frac{24b\text{PolyLog}\left(2,1-\frac{b}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a}-\frac{48b\text{PolyLog}\left(3,1-\frac{b}{ax+b}\right)}{a}+x\log^3\left(\frac{cx^2}{(ax+b)^2}\right)+\frac{6b\log\left(\frac{b}{ax+b}\right)\log^2\left(\frac{cx^2}{(ax+b)^2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*x^2)/(b + a*x)^2]^3,x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2]^3 + (6*b*Log[(c*x^2)/(b + a*x)^2]^2*Log[b/(b + a
*x)])/a + (24*b*Log[(c*x^2)/(b + a*x)^2]*PolyLog[2, 1 - b/(b + a*x)])/a - (
48*b*PolyLog[3, 1 - b/(b + a*x)])/a
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \log^3\left(\frac{cx^2}{(b+ax)^2}\right) dx = x\log^3\left(\frac{cx^2}{(b+ax)^2}\right) - (6b)\int \frac{\log^2\left(\frac{cx^2}{(b+ax)^2}\right)}{b+ax} dx$$

$$= x\log^3\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} - \frac{(24b^2)\int \frac{\log\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{x(b+ax)} dx}{a}$$

$$= x\log^3\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} + \frac{24b\log\left(\frac{cx^2}{(b+ax)^2}\right)\text{Li}_2\left(1-\frac{b}{b+ax}\right)}{a} - \frac{(48b^2)\int}{}$$

$$= x\log^3\left(\frac{cx^2}{(b+ax)^2}\right) + \frac{6b\log^2\left(\frac{cx^2}{(b+ax)^2}\right)\log\left(\frac{b}{b+ax}\right)}{a} + \frac{24b\log\left(\frac{cx^2}{(b+ax)^2}\right)\text{Li}_2\left(1-\frac{b}{b+ax}\right)}{a} - \frac{48b\text{Li}_3\left(1\right.}{a}$$

Mathematica [A] time = 0.0246982, size = 98, normalized size = 1.

$$\frac{24b\text{PolyLog}\left(2,\frac{ax}{ax+b}\right)\log\left(\frac{cx^2}{(ax+b)^2}\right)}{a} - \frac{48b\text{PolyLog}\left(3,\frac{ax}{ax+b}\right)}{a} + x\log^3\left(\frac{cx^2}{(ax+b)^2}\right) + \frac{6b\log\left(\frac{b}{ax+b}\right)\log^2\left(\frac{cx^2}{(ax+b)^2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x^2)/(b + a*x)^2]^3,x]
```

```
[Out] x*Log[(c*x^2)/(b + a*x)^2]^3 + (6*b*Log[(c*x^2)/(b + a*x)^2]^2*Log[b/(b + a
*x)])/a + (24*b*Log[(c*x^2)/(b + a*x)^2]*PolyLog[2, (a*x)/(b + a*x)])/a - (
48*b*PolyLog[3, (a*x)/(b + a*x)])/a
```

Maple [F] time = 0.786, size = 0, normalized size = 0.

$$\int \left(\ln\left(\frac{cx^2}{(ax+b)^2}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^2/(a*x+b)^2)^3,x)
```

```
[Out] int(ln(c*x^2/(a*x+b)^2)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\left(2(ax+b)\log(ax+b)^3 - 3(ax\log(c) + 2ax\log(x))\log\left((ax+b)^2\right)\right)}{a} - \int -\frac{ax\log(c)^3 + b\log(c)^3 + 8(ax+b)\log(x)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^3,x, algorithm="maxima")
```

```
[Out] -4*(2*(a*x + b)*log(a*x + b)^3 - 3*(a*x*log(c) + 2*a*x*log(x))*log(a*x + b)
^2)/a - integrate(-(a*x*log(c)^3 + b*log(c)^3 + 8*(a*x + b)*log(x)^3 + 12*(
a*x*log(c) + b*log(c))*log(x)^2 - 6*((log(c)^2 + 4*log(c))*a*x + b*log(c)^2
 + 4*(a*x + b)*log(x)^2 + 4*(a*x*(log(c) + 2) + b*log(c))*log(x))*log(a*x +
 b) + 6*(a*x*log(c)^2 + b*log(c)^2)*log(x))/(a*x + b), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\log\left(\frac{cx^2}{a^2x^2 + 2\,abx + b^2}\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^3,x, algorithm="fricas")
```

```
[Out] integral(log(c*x^2/(a^2*x^2 + 2*a*b*x + b^2))^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-6b\int\frac{\log\left(\frac{cx^2}{a^2x^2+2abx+b^2}\right)^2}{ax+b}\,dx + x\log\left(\frac{cx^2}{(ax+b)^2}\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**2/(a*x+b)**2)**3,x)
```

```
[Out] -6*b*Integral(log(c*x**2/(a**2*x**2 + 2*a*b*x + b**2))**2/(a*x + b), x) + x
*log(c*x**2/(a*x + b)**2)**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\log\left(\frac{cx^2}{(ax+b)^2}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(a*x+b)^2)^3,x, algorithm="giac")
```

```
[Out] integrate(log(c*x^2/(a*x + b)^2)^3, x)
```

3.102 $\int \frac{\text{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)} dx$

Optimal. Leaf size=35

$$-\frac{\text{PolyLog}\left(3,\frac{bc-ad}{d(a+bx)}+1\right)}{bc-ad}$$

```
[Out] -(PolyLog[3, 1 + (b*c - a*d)/(d*(a + b*x))]/(b*c - a*d))
```

Rubi [A] time = 0.0638609, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {6610}

$$-\frac{\text{PolyLog}\left(3,\frac{bc-ad}{d(a+bx)}+1\right)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)),x]
```

```
[Out] -(PolyLog[3, 1 + (b*c - a*d)/(d*(a + b*x))]/(b*c - a*d))
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\text{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)} dx = -\frac{\text{Li}_3\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}$$

Mathematica [A] time = 0.0115641, size = 30, normalized size = 0.86

$$\frac{\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{ad-bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)),x
]
```

```
[Out] PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]/(-(b*c) + a*d)
```

Maple [A] time = 0.062, size = 36, normalized size = 1.

$$\frac{1}{ad-bc}\text{polylog}\left(3,1-\frac{ad-bc}{d(bx+a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x)
```

```
[Out] 1/(a*d-b*c)*polylog(3,1-(a*d-b*c)/d/(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2\left(\frac{bc-ad}{(bx+a)d}+1\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="m
axima")
```

```
[Out] integrate(dilog((b*c - a*d)/((b*x + a)*d) + 1)/((b*x + a)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{Li}_2\left(\frac{bc-ad}{bdx+ad}+1\right)}{bdx^2+ac+(bc+ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="f
ricas")
```

```
[Out] integral(dilog((b*c - a*d)/(b*d*x + a*d) + 1)/(b*d*x^2 + a*c + (b*c + a*d)*
x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2\left(\frac{bc-ad}{(bx+a)d}+1\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,1+(-a*d+b*c)/d/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="g
iac")

[Out] integrate(dilog((b*c - a*d)/((b*x + a)*d) + 1)/((b*x + a)*(d*x + c)), x)
```

3.103 $\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{bc-ad}-\frac{\text{PolyLog}\left(3,\frac{bc-ad}{d(a+bx)}+1\right)}{bc-ad}$$

```
[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b
*c - a*d) - PolyLog[3, 1 + (b*c - a*d)/(d*(a + b*x))]/(b*c - a*d)
```

Rubi [A] time = 0.134357, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2506, 6610}

$$\frac{\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{bc-ad}-\frac{\text{PolyLog}\left(3,\frac{bc-ad}{d(a+bx)}+1\right)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)])/((a +
b*x)*(c + d*x)),x]
```

```
[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b
*c - a*d) - PolyLog[3, 1 + (b*c - a*d)/(d*(a + b*x))]/(b*c - a*d)
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx &= \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}+\int \frac{\text{Li}_2\left(1-\frac{-bc+ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}\,dx\\ &= \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}-\frac{\text{Li}_3\left(1+\frac{bc-ad}{d(a+bx)}\right)}{bc-ad}\end{aligned}$$

Mathematica [A] time = 0.0237251, size = 68, normalized size = 0.8

$$\frac{\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{e(c+dx)}{a+bx}\right)-\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(e*(c + d*x))/(a + b*x)])/
((a + b*x)*(c + d*x)),x]
```

```
[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] - Pol
yLog[3, (b*(c + d*x))/(d*(a + b*x))])/(b*c - a*d)
```

Maple [F] time = 1.318, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)} \ln\left(\frac{ad-bc}{d(bx+a)}\right) \ln\left(\frac{e(dx+c)}{bx+a}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((a*d-b*c)/d/(b*x+a))*ln(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x)
```

```
[Out] int(ln((a*d-b*c)/d/(b*x+a))*ln(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{bc-ad}{(bx+a)d}\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x
, algorithm="maxima")
```

```
[Out] integrate(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)/((b*x + a)*d))/((b*x
+ a)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(-\frac{bc-ad}{bdx+ad}\right)\log\left(\frac{dex+ce}{bx+a}\right)}{bdx^2+ac+(bc+ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x
, algorithm="fricas")
```

```
[Out] integral(log(-(b*c - a*d)/(b*d*x + a*d))*log((d*e*x + c*e)/(b*x + a))/(b*d*
x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((a*d-b*c)/d/(b*x+a))*ln(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{bc-ad}{(bx+a)d}\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((a*d-b*c)/d/(b*x+a))*log(e*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x
, algorithm="giac")
```

```
[Out] integrate(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)/((b*x + a)*d))/((b*x
+ a)*(d*x + c)), x)
```

3.104 $\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx$

Optimal. Leaf size=140

$$-\frac{2\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b}+\frac{2\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(\frac{ad-bc}{d(a+bx)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b}$$

```
[Out] -((Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2)/b) - (2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x
))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b + (2*PolyLog[3, (b*(c + d*x)
)/(d*(a + b*x))])/b
```

Rubi [A] time = 0.180268, antiderivative size = 149, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2488, 2506, 6610}

$$-\frac{2\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b}+\frac{2\text{PolyLog}\left(3,\frac{bc-ad}{d(a+bx)}+1\right)}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(a + b*x),x]
```

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2)/b) - (2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x
))]*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/b + (2*PolyLog[3, 1 + (b*c -
 a*d)/(d*(a + b*x))])/b
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{a+bx}\,dx = -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b} - \frac{(2(bc-ad))\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}dx}{b}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\mathrm{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{b} - \frac{(2(bc-ad))\int}{}$$

$$= -\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{b} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\mathrm{Li}_2\left(1+\frac{bc-ad}{d(a+bx)}\right)}{b} + \frac{2\mathrm{Li}_3\left(1+\frac{bc-}{d(a+}\right.}{b}$$

Mathematica [A] time = 0.0421648, size = 135, normalized size = 0.96

$$\frac{-2\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)+2\mathrm{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)-\log\left(\frac{ad-bc}{d(a+bx)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(a + b*x),
x]
```

```
[Out] (-(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2) - 2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*
PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*PolyLog[3, (b*(c + d*x))/(d*(a
+ b*x))])/b
```

Maple [B] time = 0.065, size = 879, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x)
```

```
[Out] -1/(b*c*f-b*d*e)*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a
*f-b*e)/b)^2*ln(1-(b*c*f-b*d*e)/(a*d*f-b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-
d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*c*f+1/(b*c*f-b*d*e)*ln(-(a*f-b*e)*(a
*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*ln(1-(b*c*f-b*d*e)/(
a*d*f-b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e
)/b))*d*e-2/(b*c*f-b*d*e)*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*
f-d*e)*(a*f-b*e)/b)*polylog(2,(b*c*f-b*d*e)/(a*d*f-b*d*e)*(-(a*f-b*e)*(a*d-
b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*c*f+2/(b*c*f-b*d*e)*ln(-
(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*polylog(2,
(b*c*f-b*d*e)/(a*d*f-b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*
f-d*e)*(a*f-b*e)/b))*d*e+2/(b*c*f-b*d*e)*polylog(3,(b*c*f-b*d*e)/(a*d*f-b*d
*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*c*f
-2/(b*c*f-b*d*e)*polylog(3,(b*c*f-b*d*e)/(a*d*f-b*d*e)*(-(a*f-b*e)*(a*d-b*c
)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*d*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log(dx+c)^3}{a} - \int -\frac{\left(\log\left(-be+af\right)^2 - 2\log\left(-be+af\right)\log\left(-de+cf\right)+\log\left(-de+cf\right)^2\right)bdx + \left(\log\left(-be+af\right)^2 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x, algorithm
="maxima")
```

```
[Out] log(d*x + c)^3/a - integrate(-((log(-b*e + a*f)^2 - 2*log(-b*e + a*f)*log(-
d*e + c*f) + log(-d*e + c*f)^2)*b*d*x + (log(-b*e + a*f)^2 - 2*log(-b*e + a
*f)*log(-d*e + c*f) + log(-d*e + c*f)^2)*b*c + (b*d*x + b*c)*log(b*x + a)^2
 - 2*(b*d*x*(log(-b*e + a*f) - log(-d*e + c*f)) + b*c*(log(-b*e + a*f) - lo
g(-d*e + c*f)))*log(b*x + a) + 2*(b*d*x*(log(-b*e + a*f) - log(-d*e + c*f))
 + b*c*(log(-b*e + a*f) - log(-d*e + c*f)) - (2*b*d*x + b*c + a*d)*log(b*x
+ a))*log(d*x + c))/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\frac{bce-acf+(bde-adf)x}{ade-acf+(bde-bcf)x}\right)^2}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x, algorithm
="fricas")
```

```
[Out] integral(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e -
b*c*f)*x))^2/(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))**2/(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)^2}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a),x, algorithm
="giac")
```

```
[Out] integrate(log((b*e - a*f)*(d*x + c)/((d*e - c*f)*(b*x + a)))^2/(b*x + a), x
)
```

3.105 $\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=109

$$\frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{PolyLog}\left(2,\frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)}+1\right)}{bc-ad}-\frac{\text{PolyLog}\left(3,\frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)}+1\right)}{bc-ad}$$

```
[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, 1 + ((b*c - a*d)*(e + f*x))/((d*e
- c*f)*(a + b*x))])/(b*c - a*d) - PolyLog[3, 1 + ((b*c - a*d)*(e + f*x))/((
d*e - c*f)*(a + b*x))]/(b*c - a*d)
```

Rubi [A] time = 0.163821, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2506, 6610}

$$\frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{PolyLog}\left(2,\frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)}+1\right)}{bc-ad}-\frac{\text{PolyLog}\left(3,\frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)}+1\right)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[(e*(c + d*x))/(a + b*x)]*Log[((-(b*c) + a*d)*(e + f*x))/((d*e - c*
f)*(a + b*x))])/((a + b*x)*(c + d*x)),x]
```

```
[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, 1 + ((b*c - a*d)*(e + f*x))/((d*e
- c*f)*(a + b*x))])/(b*c - a*d) - PolyLog[3, 1 + ((b*c - a*d)*(e + f*x))/((
d*e - c*f)*(a + b*x))]/(b*c - a*d)
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\log\left(\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx &= \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{Li}_2\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad}+\int \frac{\text{Li}_2\left(1-\frac{(-bc+ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}\,dx\\ &= \frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{Li}_2\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad}-\frac{\text{Li}_3\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{bc-ad}\end{aligned}$$

Mathematica [A] time = 0.0266982, size = 96, normalized size = 0.88

$$\frac{\log\left(\frac{e(c+dx)}{a+bx}\right)\text{PolyLog}\left(2,\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)-\text{PolyLog}\left(3,\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[(e*(c + d*x))/(a + b*x)]*Log[((-(b*c) + a*d)*(e + f*x))/((d*
e - c*f)*(a + b*x))])/((a + b*x)*(c + d*x)),x]
```

```
[Out] (Log[(e*(c + d*x))/(a + b*x)]*PolyLog[2, ((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))] - PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))
])/(b*c - a*d)
```

Maple [F] time = 2.268, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)}\ln\left(\frac{e(dx+c)}{bx+a}\right)\ln\left(\frac{(ad-bc)(fx+e)}{(-cf+de)(bx+a)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(e*(d*x+c)/(b*x+a))*ln((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(b*x+a)/
(d*x+c),x)
```

```
[Out] int(ln(e*(d*x+c)/(b*x+a))*ln((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(b*x+a)/
(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/
(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(log(b*x + a)^2 - 2*(log(b*x + a) - log(e))*log(d*x + c) + log(d*x + c
)^2 - 2*log(b*x + a)*log(e))*log(f*x + e)/(b*c - a*d) + integrate(1/2*(2*(e
*log(-b*c + a*d)*log(e) - e*log(d*e - c*f)*log(e))*b*c + (b*d*f*x^2 + 2*b*c
*e - (2*d*e - c*f)*a + (3*b*c*f - a*d*f)*x)*log(b*x + a)^2 - 2*(d*e*log(-b*
c + a*d)*log(e) - d*e*log(d*e - c*f)*log(e))*a + 2*((f*log(-b*c + a*d)*log(
e) - f*log(d*e - c*f)*log(e))*b*c - (d*f*log(-b*c + a*d)*log(e) - d*f*log(d
*e - c*f)*log(e))*a)*x - 2*(b*d*f*x^2*log(e) - (e*(log(d*e - c*f) - log(e))
 - e*log(-b*c + a*d))*b*c + (d*e*(log(d*e - c*f) - log(e)) - d*e*log(-b*c +
 a*d) + c*f*log(e))*a + ((f*log(-b*c + a*d) - f*log(d*e - c*f) + 2*f*log(e)
)*b*c - (d*f*log(-b*c + a*d) - d*f*log(d*e - c*f))*a)*x)*log(b*x + a) + 2*(
b*d*f*x^2*log(e) + (e*log(-b*c + a*d) - e*log(d*e - c*f))*b*c - (d*e*log(-b
*c + a*d) - d*e*log(d*e - c*f) - c*f*log(e))*a + ((f*log(-b*c + a*d) - f*lo
g(d*e - c*f) + f*log(e))*b*c - (d*f*log(-b*c + a*d) - (f*log(d*e - c*f) + f
*log(e))*d)*a)*x - (b*d*f*x^2 + 2*b*c*f*x + b*c*e - (d*e - c*f)*a)*log(b*x
+ a))*log(d*x + c))/(a*b*c^2*e - a^2*c*d*e + (b^2*c*d*f - a*b*d^2*f)*x^3 -
(a*b*d^2*e + a^2*d^2*f - (c*d*e + c^2*f)*b^2)*x^2 + (b^2*c^2*e + a*b*c^2*f
```

```
- (d^2*e + c*d*f)*a^2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(-\frac{(bc-ad)fx+(bc-ad)e}{ade-acf+(bde-bcf)x}\right)\log\left(\frac{dex+ce}{bx+a}\right)}{bdx^2+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/
(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log(-((b*c - a*d)*f*x + (b*c - a*d)*e)/(a*d*e - a*c*f + (b*d*e - b
*c*f)*x))*log((d*e*x + c*e)/(b*x + a))/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(e*(d*x+c)/(b*x+a))*ln((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/(b
*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{(dx+c)e}{bx+a}\right)\log\left(-\frac{(bc-ad)(fx+e)}{(de-cf)(bx+a)}\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(e*(d*x+c)/(b*x+a))*log((a*d-b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))/
(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log((d*x + c)*e/(b*x + a))*log(-(b*c - a*d)*(f*x + e)/((d*e - c*f
)*(b*x + a)))/((b*x + a)*(d*x + c)), x)
```

3.106 $\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}\,dx$

Optimal. Leaf size=204

$$\frac{2\text{PolyLog}\left(3,\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{be-af}-\frac{2\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\text{PolyLog}\left(2,\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{be-af}-\frac{\log\left(1-\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\log^2\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{be-af}$$

```
[Out] -((Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[1 - ((b*e - a
*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/(b*e - a*f)) - (2*Log[((b*e - a*f)
*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b*e - a*f)*(c + d*x))/((d
*e - c*f)*(a + b*x))])/(b*e - a*f) + (2*PolyLog[3, ((b*e - a*f)*(c + d*x))/
((d*e - c*f)*(a + b*x))])/(b*e - a*f)
```

Rubi [A] time = 0.253539, antiderivative size = 206, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2503, 2506, 6610}

$$\frac{2\text{PolyLog}\left(3,\frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)}+1\right)}{be-af}-\frac{2\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\text{PolyLog}\left(2,\frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)}+1\right)}{be-af}-\frac{\log\left(-\frac{(e+fx)(bc-ad)}{(a+bx)(de-cf)}\right)\log^2\left(\frac{(c+dx)(be-af}{(a+bx)(de-cf}\right.}{be-af}$$

Antiderivative was successfully verified.

```
[In] Int[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/((a + b*x)*(e +
f*x)),x]
```

```
[Out] -((Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[-(((b*c - a*d
)*(e + f*x))/((d*e - c*f)*(a + b*x)))])/(b*e - a*f)) - (2*Log[((b*e - a*f)*
(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, 1 + ((b*c - a*d)*(e + f*x))/
((d*e - c*f)*(a + b*x))])/(b*e - a*f) + (2*PolyLog[3, 1 + ((b*c - a*d)*(e +
 f*x))/((d*e - c*f)*(a + b*x))])/(b*e - a*f)
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Coeff[Simplify[1/(u*(a + b*x))],
 x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
)))])/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))])/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
 x]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(a+bx)(e+fx)}dx = -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af} - \frac{(2(bc-ad))\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{(a+bx)(c+dx)}dx}{be-af}$$

$$= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af} - \frac{(2(b \cdots}{\cdots}$$

$$= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af} - \frac{2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\text{Li}_2\left(1+\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\right)}{be-af} + \frac{2\text{Li} \cdots}{\cdots}$$

Mathematica [B] time = 0.469379, size = 1636, normalized size = 8.02

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/((a + b*x)
*(e + f*x)),x]
```

```
[Out] (-2*Log[a/b + x]^3 + 3*Log[a/b + x]^2*Log[a + b*x] - 6*Log[a/b + x]*Log[c/d
 + x]*Log[a + b*x] + 3*Log[c/d + x]^2*Log[a + b*x] + 6*Log[a/b + x]*Log[c/d
 + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] - 3*Log[c/d + x]^2*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] + 3*Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] - 3*Log
[a/b + x]^2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] + 6*Log[a/
b + x]*Log[a + b*x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] -
6*Log[c/d + x]*Log[a + b*x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b
*x))] + 6*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[((b*e - a*f)*(
c + d*x))/((d*e - c*f)*(a + b*x))] + 3*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Lo
g[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2 + 3*Log[a + b*x]*Log[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2 - 3*Log[a/b + x]^2*Log[e
+ f*x] + 6*Log[a/b + x]*Log[c/d + x]*Log[e + f*x] - 3*Log[c/d + x]^2*Log[e
+ f*x] - 6*Log[a/b + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))
]*Log[e + f*x] + 6*Log[c/d + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a
 + b*x))]*Log[e + f*x] - 3*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*
x))]^2*Log[e + f*x] + 3*Log[a/b + x]^2*Log[(b*(e + f*x))/(b*e - a*f)] - 6*L
og[a/b + x]*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[(b*(e + f*x))/(b*e - a*f)
] + 3*Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*Log[(b*(e + f*x))/(b*e - a*f)] +
6*Log[a/b + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*
(e + f*x))/(b*e - a*f)] - 6*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e - a
*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*f)] + 3*
Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[(b*(e + f*x))/(b
*e - a*f)] - 6*Log[a/b + x]*Log[c/d + x]*Log[(d*(e + f*x))/(d*e - c*f)] + 3
*Log[c/d + x]^2*Log[(d*(e + f*x))/(d*e - c*f)] + 6*Log[a/b + x]*Log[(f*(c +
 d*x))/(-(d*e) + c*f)]*Log[(d*(e + f*x))/(d*e - c*f)] - 3*Log[(f*(c + d*x))
/(-(d*e) + c*f)]^2*Log[(d*(e + f*x))/(d*e - c*f)] - 6*Log[c/d + x]*Log[((b*
e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)]
 + 6*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e -
c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)] - 3*Log[((b*e - a*f)*(c + d
*x))/((d*e - c*f)*(a + b*x))]^2*Log[((-(b*c) + a*d)*(e + f*x))/((d*e - c*f)
```

```
*(a + b*x))] + 6*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 6*
(Log[a/b + x] + Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)] + 6*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] - 6*Log[((b*e - a*f)
*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b*e - a*f)*(c + d*x))/((d
*e - c*f)*(a + b*x))] - 6*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*Poly
Log[3, (b*(c + d*x))/(b*c - a*d)] - 6*PolyLog[3, (b*(c + d*x))/(d*(a + b*x)
)] + 6*PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/(3*b*e
- 3*a*f)
```

Maple [A] time = 0.063, size = 357, normalized size = 1.8

$$\frac{1}{af-be}\left(\ln\left(-\frac{\left(af-be\right)(ad-bc)}{b\left(cf-de\right)(bx+a)}+\frac{\left(af-be\right)d}{b\left(cf-de\right)}\right)\right)^2\ln\left(1+\frac{\left(af-be\right)(ad-bc)}{b\left(cf-de\right)(bx+a)}-\frac{\left(af-be\right)d}{b\left(cf-de\right)}\right)+2\frac{1}{af-be}\ln\left(-\frac{\left(af-b\right.}{b\left(cf-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x)
```

```
[Out] 1/(a*f-b*e)*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*
e)/b)^2*ln(1+(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)-d/(c*f-d*e)*(a*f-b*e)/
b)+2/(a*f-b*e)*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f
-b*e)/b)*polylog(2,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*
f-b*e)/b)-2/(a*f-b*e)*polylog(3,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/
(c*f-d*e)*(a*f-b*e)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x, a
lgorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\frac{bce-acf+\left(bde-adf\right)x}{ade-acf+\left(bde-bcf\right)x}\right)^2}{bfx^2+ae+\left(be+af\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x, a
lgorithm="fricas")
```

```
[Out] integral(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e -
b*c*f)*x))^2/(b*f*x^2 + a*e + (b*e + a*f)*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))**2/(b*x+a)/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)^2}{(bx+a)(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(b*x+a)/(f*x+e),x, a
lgorithm="giac")
```

```
[Out] integrate(log((b*e - a*f)*(d*x + c)/((d*e - c*f)*(b*x + a)))^2/((b*x + a)*(
f*x + e)), x)
```

3.107 $\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx$

Optimal. Leaf size=322

$$-\frac{2\text{PolyLog}\left(3,\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f}-\frac{2\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f}+\frac{2\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\text{PolyLog}\left(2,\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f}+$$

```
[Out] -((Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2)/f) + (Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))
]^2*Log[1 - ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/f - (2*Log[((
b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*
(a + b*x))])/f + (2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Po
lyLog[2, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/f + (2*PolyLog[3
, (b*(c + d*x))/(d*(a + b*x))])/f - (2*PolyLog[3, ((b*e - a*f)*(c + d*x))/(
(d*e - c*f)*(a + b*x))])/f
```

Rubi [A] time = 0.507152, antiderivative size = 334, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2489, 2488, 2506, 6610, 2503}

$$-\frac{2\text{PolyLog}\left(3,1-\frac{(e+fx)(bc-ad)}{(c+dx)(be-af)}\right)}{f}+\frac{2\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{f}-\frac{2\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\text{PolyLog}\left(2,1-\frac{(e+}{(c+}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(e + f*x),x]
```

```
[Out] -((Log[(b*c - a*d)/(b*(c + d*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*
(a + b*x))]^2)/f) + (Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2
*Log[((b*c - a*d)*(e + f*x))/((b*e - a*f)*(c + d*x))])/f + (2*Log[((b*e - a
*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, 1 - (b*c - a*d)/(b*(c +
d*x))])/f - (2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog
[2, 1 - ((b*c - a*d)*(e + f*x))/((b*e - a*f)*(c + d*x))])/f + (2*PolyLog[3,
 1 - (b*c - a*d)/(b*(c + d*x))])/f - (2*PolyLog[3, 1 - ((b*c - a*d)*(e + f*
x))/((b*e - a*f)*(c + d*x))])/f
```

Rule 2489

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[d/h, Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^s/(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{a
, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0
] && NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 1]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
```

```
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Coeff[Simplify[1/(u*(a + b*x))],
 x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
)))])/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))])/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
 x]
```

Rubi steps

$$\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}\,dx = \frac{d\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{c+dx}\,dx}{f} - \frac{(de-cf)\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{(c+dx)(e+fx)}\,dx}{f}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right)}{f} - \frac{(2(bc-ad))\int \cdots}{\cdots}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right)}{f} + \frac{2\log\left(\frac{(be-af)(c+\cdots}{(de-cf)(a+\cdots}\right)\cdots}{\cdots}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f} + \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}\right)}{f} + \frac{2\log\left(\frac{(be-af)(c+\cdots}{(de-cf)(a+\cdots}\right)\cdots}{\cdots}$$

Mathematica [B] time = 0.234267, size = 1080, normalized size = 3.35

$$\frac{\log(e+fx)\log^2\left(\frac{a}{b}+x\right) - \log\left(\frac{b(e+fx)}{be-af}\right)\log^2\left(\frac{a}{b}+x\right) - 2\log\left(\frac{c}{d}+x\right)\log(e+fx)\log\left(\frac{a}{b}+x\right) + 2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log(e\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2/(e + f*x),
x]
```

```
[Out] (-(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2) + Log[a/b + x]^2*Log[e + f*x] - 2*Log[a/b + x]*Log[c/d +
x]*Log[e + f*x] + Log[c/d + x]^2*Log[e + f*x] + 2*Log[a/b + x]*Log[((b*e -
a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[e + f*x] - 2*Log[c/d + x]*Log[
((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[e + f*x] + Log[((b*e -
 a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[e + f*x] - Log[a/b + x]^2*L
og[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a/b + x]*Log[(f*(c + d*x))/(-(d*e) +
c*f)]*Log[(b*(e + f*x))/(b*e - a*f)] - Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*
Log[(b*(e + f*x))/(b*e - a*f)] - 2*Log[a/b + x]*Log[((b*e - a*f)*(c + d*x))
/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[(f*(c + d*
x))/(-(d*e) + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Lo
g[(b*(e + f*x))/(b*e - a*f)] - Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a
+ b*x))]^2*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a/b + x]*Log[c/d + x]*Log
[(d*(e + f*x))/(d*e - c*f)] - Log[c/d + x]^2*Log[(d*(e + f*x))/(d*e - c*f)]
 - 2*Log[a/b + x]*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[(d*(e + f*x))/(d*e
- c*f)] + Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*Log[(d*(e + f*x))/(d*e - c*f)
] + 2*Log[c/d + x]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log
[(d*(e + f*x))/(d*e - c*f)] - 2*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e
 - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)]
+ Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[((-(b*c) + a*d
)*(e + f*x))/((d*e - c*f)*(a + b*x))] - 2*Log[((b*e - a*f)*(c + d*x))/((d*e
 - c*f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + 2*Log[((b*e -
 a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b*e - a*f)*(c + d*x)
)/((d*e - c*f)*(a + b*x))] + 2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))] - 2*
PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/f
```

Maple [B] time = 0.072, size = 4733, normalized size = 14.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x)
```

```
[Out] -2*b^2/(b*c*f-b*d*e)/(a*f-b*e)/f/(a*d-b*c)*polylog(2,-(b*c*f-b*d*e)/(-a*d*f
+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))
*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*d^2*e
^2*a+2*b^3/(b*c*f-b*d*e)/(a*f-b*e)/f/(a*d-b*c)*polylog(2,-(b*c*f-b*d*e)/(-a
*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)
/b))*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*d
*e^2*c+b^3/(b*c*f-b*d*e)/(a*f-b*e)/f/(a*d-b*c)*ln(1+(b*c*f-b*d*e)/(-a*d*f+b
*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*l
n(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*d*e^2
*c-2*b/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*polylog(2,-(b*c*f-b*d*e)/(-a*d*f+b
*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*l
n(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*c*a^2*d
*f-b^3/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*ln(1+(b*c*f-b*d*e)/(-a*d*f+b*d*e)*
(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*ln(-(a*
f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*c^2*e-2*b^3
/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*polylog(2,-(b*c*f-b*d*e)/(-a*d*f+b*d*e)*
(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*ln(-(a*
f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*c^2*e+2/(a*f-
b*e)/f/(a*d-b*c)*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a
*f-b*e)/b)*polylog(2,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(
a*f-b*e)/b)*b^2*c*e+2/(a*f-b*e)/f/(a*d-b*c)*polylog(3,-(a*f-b*e)*(a*d-b*c)/
b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*a*b*d*e+1/(a*f-b*e)/f/(a*d-b*c
)*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*ln
```

```
(1+(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)-d/(c*f-d*e)*(a*f-b*e)/b)*b^2*c*e
-2*b^2/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*polylog(3,-(b*c*f-b*d*e)/(-a*d*f+b
*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*c
^2*a*f-2*b/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*polylog(3,-(b*c*f-b*d*e)/(-a*d
*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b
))*d^2*e*a^2-1/(a*f-b*e)/(a*d-b*c)*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x
+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*ln(1+(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)
-d/(c*f-d*e)*(a*f-b*e)/b)*a*b*c-2/(a*f-b*e)/(a*d-b*c)*ln(-(a*f-b*e)*(a*d-b*
c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*polylog(2,-(a*f-b*e)*(a*d-b
*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*a*b*c-2/(a*f-b*e)/f/(a*d-b
*c)*polylog(3,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e
)/b)*b^2*c*e+2*b^3/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*polylog(3,-(b*c*f-b*d*
e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*
f-b*e)/b))*c^2*e+1/(a*f-b*e)/(a*d-b*c)*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/
(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*ln(1+(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*
x+a)-d/(c*f-d*e)*(a*f-b*e)/b)*a^2*d+2/(a*f-b*e)/(a*d-b*c)*ln(-(a*f-b*e)*(a*
d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*polylog(2,-(a*f-b*e)*(a
*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*a^2*d+2/(a*f-b*e)/(a*d
-b*c)*polylog(3,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b
*e)/b)*a*b*c-2/(a*f-b*e)/(a*d-b*c)*polylog(3,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*
e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*a^2*d+2*b/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-
b*c)*polylog(2,-(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d
*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(
b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*d^2*e*a^2+b/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*
c)*ln(1+(b*c*f-b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x
+a)+d/(c*f-d*e)*(a*f-b*e)/b))*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d
/(c*f-d*e)*(a*f-b*e)/b)^2*d^2*e*a^2-1/(a*f-b*e)/f/(a*d-b*c)*ln(-(a*f-b*e)*(
a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*ln(1+(a*f-b*e)*(a*d
-b*c)/b/(c*f-d*e)/(b*x+a)-d/(c*f-d*e)*(a*f-b*e)/b)*a*b*d*e-2/(a*f-b*e)/f/(a
*d-b*c)*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b
)*polylog(2,-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/
b)*a*b*d*e+b^2/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*ln(1+(b*c*f-b*d*e)/(-a*d*f
+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b))
*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)^2*c^2
*a*f+2*b^2/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*polylog(2,-(b*c*f-b*d*e)/(-a*d
*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b
))*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)/b)*c^2
*a*f-2*b^3/(b*c*f-b*d*e)/(a*f-b*e)/f/(a*d-b*c)*polylog(3,-(b*c*f-b*d*e)/(-a
*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-b*e)
/b))*d*e^2*c+2*b/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*polylog(3,-(b*c*f-b*d*e)
/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f-
b*e)/b))*c*a^2*d*f+2*b^2/(b*c*f-b*d*e)/(a*f-b*e)/f/(a*d-b*c)*polylog(3,-(b*
c*f-b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-
d*e)*(a*f-b*e)/b))*d^2*e^2*a-b/(b*c*f-b*d*e)/(a*f-b*e)/(a*d-b*c)*ln(1+(b*c*
f-b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*
e)*(a*f-b*e)/b))*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a
*f-b*e)/b)^2*c*a^2*d*f-b^2/(b*c*f-b*d*e)/(a*f-b*e)/f/(a*d-b*c)*ln(1+(b*c*f-
b*d*e)/(-a*d*f+b*d*e)*(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)
*(a*f-b*e)/b))*ln(-(a*f-b*e)*(a*d-b*c)/b/(c*f-d*e)/(b*x+a)+d/(c*f-d*e)*(a*f
-b*e)/b)^2*d^2*e^2*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x, algorithm
```

```
="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\frac{bce-acf+(bde-adf)x}{ade-acf+(bde-bcf)x}\right)^2}{fx+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x, algorithm
="fricas")
```

```
[Out] integral(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e -
b*c*f)*x))^2/(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))**2/(f*x+e),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)^2}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^2/(f*x+e),x, algorithm
="giac")
```

```
[Out] integrate(log((b*e - a*f)*(d*x + c)/((d*e - c*f)*(b*x + a)))^2/(f*x + e), x
)
```

3.108 $\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=433

$$-\frac{\text{PolyLog}\left(3,\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{bc-ad}-\frac{\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{bc-ad}+\frac{\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\text{PolyLog}\left(2,\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{bc-ad}+$$

```
[Out] -(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f
)*(a + b*x))]^2)/(2*(b*c - a*d)) - (Log[((b*e - a*f)*(c + d*x))/((d*e - c*f
)*(a + b*x))]^2*Log[(b*(e + f*x))/(b*e - a*f)])/(2*(b*c - a*d)) + (Log[((b*
e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[1 - ((b*e - a*f)*(c + d*
x))/((d*e - c*f)*(a + b*x))])/(2*(b*c - a*d)) - (Log[((b*e - a*f)*(c + d*x)
)/((d*e - c*f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b*c -
a*d) + (Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, ((b
*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/(b*c - a*d) + PolyLog[3, (b*
(c + d*x))/(d*(a + b*x))]/(b*c - a*d) - PolyLog[3, ((b*e - a*f)*(c + d*x))/
((d*e - c*f)*(a + b*x))]/(b*c - a*d)
```

Rubi [A] time = 0.594495, antiderivative size = 445, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 6, integrand size = 65, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.092, Rules used = {2507, 2489, 2488, 2506, 6610, 2503}

$$-\frac{\text{PolyLog}\left(3,1-\frac{(e+fx)(bc-ad)}{(c+dx)(be-af)}\right)}{bc-ad}+\frac{\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)}{bc-ad}-\frac{\log\left(\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}\right)\text{PolyLog}\left(2,1-\frac{(e+f}{(c+d}\right.}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))
/(b*e - a*f)])/((a + b*x)*(c + d*x)),x]
```

```
[Out] -(Log[(b*c - a*d)/(b*(c + d*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(
a + b*x))]^2)/(2*(b*c - a*d)) - (Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(
a + b*x))]^2*Log[(b*(e + f*x))/(b*e - a*f)])/(2*(b*c - a*d)) + (Log[((b*e -
 a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]^2*Log[((b*c - a*d)*(e + f*x))/((b
*e - a*f)*(c + d*x))])/(2*(b*c - a*d)) + (Log[((b*e - a*f)*(c + d*x))/((d*e
 - c*f)*(a + b*x))]*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*c - a*d)
- (Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, 1 - ((b*
c - a*d)*(e + f*x))/((b*e - a*f)*(c + d*x))])/(b*c - a*d) + PolyLog[3, 1 -
(b*c - a*d)/(b*(c + d*x))]/(b*c - a*d) - PolyLog[3, 1 - ((b*c - a*d)*(e + f
*x))/((b*e - a*f)*(c + d*x))]/(b*c - a*d)
```

Rule 2507

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*Log[(i_.)*((j_.)*((g_.) + (h_.)*(x_))^(t_.))^(u_.)]*(v_), x_S
ymbol] :> With[{k = Simplify[v*(a + b*x)*(c + d*x)]}, Simp[(k*Log[i*(j*(g +
 h*x)^t)^u]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s + 1))/(p*r*(s + 1)*(b*c
 - a*d)), x] - Dist[(k*h*t*u)/(p*r*(s + 1)*(b*c - a*d)), Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^(s + 1)/(g + h*x), x], x] /; FreeQ[k, x]] /; FreeQ[{
a, b, c, d, e, f, g, h, i, j, p, q, r, s, t, u}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[p + q, 0] && NeQ[s, -1]
```

Rule 2489

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[d/h, Int[Log[e*(f*(a +
 b*x)^p*(c + d*x)^q)^r]^s/(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{a
, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0
] && NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 1]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q)^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_)
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x]] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Coeff[Simplify[1/(u*(a + b*x))],
 x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q)^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
)))])/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))])/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
 x]
```

Rubi steps

$$\int \frac{\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{(a+bx)(c+dx)}\,dx = -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{f\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{e+fx}dx}{2(bc-ad)}$$

$$= -\frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{d\int \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{c+dx}dx}{2(bc-ad)} - \frac{(de-cf)\int \frac{\log^2(\ldots}{(c+d\ldots}}{2(bc-a\ldots}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{\log^2\left(\ldots\right.}{\ldots}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{\log^2\left(\ldots\right.}{\ldots}$$

$$= -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{2(bc-ad)} - \frac{\log^2\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\left(\frac{b(e+fx)}{be-af}\right)}{2(bc-ad)} + \frac{\log^2\left(\ldots\right.}{\ldots}$$

Mathematica [B] time = 0.798781, size = 908, normalized size = 2.1

$$\frac{\log\left(\frac{b(e+fx)}{be-af}\right)\log^2(c+dx) - \log\left(\frac{d(e+fx)}{de-cf}\right)\log^2(c+dx) - 2\log(a+bx)\log\left(\frac{b(e+fx)}{be-af}\right)\log(c+dx) - 2\log\left(\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)\log\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e +
 f*x))/(b*e - a*f)])/((a + b*x)*(c + d*x)),x]
```

```
[Out] (-(Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*
f)*(a + b*x))]^2) - 2*Log[a + b*x]*Log[c + d*x]*Log[(b*(e + f*x))/(b*e - a*
f)] + Log[c + d*x]^2*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a + b*x]*Log[(f
*(c + d*x))/(-(d*e) + c*f)]*Log[(b*(e + f*x))/(b*e - a*f)] - Log[(f*(c + d*
x))/(-(d*e) + c*f)]^2*Log[(b*(e + f*x))/(b*e - a*f)] - 2*Log[c + d*x]*Log[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*
f)] + 2*Log[(f*(c + d*x))/(-(d*e) + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e
 - c*f)*(a + b*x))]*Log[(b*(e + f*x))/(b*e - a*f)] - Log[((b*e - a*f)*(c +
d*x))/((d*e - c*f)*(a + b*x))]^2*Log[(b*(e + f*x))/(b*e - a*f)] + 2*Log[a +
 b*x]*Log[c + d*x]*Log[(d*(e + f*x))/(d*e - c*f)] - Log[c + d*x]^2*Log[(d*(
e + f*x))/(d*e - c*f)] - 2*Log[a + b*x]*Log[(f*(c + d*x))/(-(d*e) + c*f)]*L
og[(d*(e + f*x))/(d*e - c*f)] + Log[(f*(c + d*x))/(-(d*e) + c*f)]^2*Log[(d*
(e + f*x))/(d*e - c*f)] + 2*Log[c + d*x]*Log[((b*e - a*f)*(c + d*x))/((d*e
- c*f)*(a + b*x))]*Log[(d*(e + f*x))/(d*e - c*f)] - 2*Log[(f*(c + d*x))/(-(
d*e) + c*f)]*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Log[(d*(e
 + f*x))/(d*e - c*f)] + Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))
]^2*Log[((-(b*c) + a*d)*(e + f*x))/((d*e - c*f)*(a + b*x))] - 2*Log[((b*e -
 a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2, (b*(c + d*x))/(d*(a +
b*x))] + 2*Log[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*PolyLog[2,
((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))] + 2*PolyLog[3, (b*(c + d*x
))/(d*(a + b*x))] - 2*PolyLog[3, ((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a +
b*x))])/(2*b*c - 2*a*d)
```

Maple [F] time = 2.336, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)}\ln\left(\frac{(-af+be)(dx+c)}{(-cf+de)(bx+a)}\right)\ln\left(\frac{b(fx+e)}{-af+be}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e))/(b*x
+a)/(d*x+c),x)
```

```
[Out] int(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e))/(b*x
+a)/(d*x+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b*
e))/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\frac{bce-acf+(bde-adf)x}{ade-acf+(bde-bcf)x}\right)\log\left(\frac{bfx+be}{be-af}\right)}{bdx^2+ac+(bc+ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b*
e))/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((b*c*e - a*c*f + (b*d*e - a*d*f)*x)/(a*d*e - a*c*f + (b*d*e -
b*c*f)*x))*log((b*f*x + b*e)/(b*e - a*f))/(b*d*x^2 + a*c + (b*c + a*d)*x),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*ln(b*(f*x+e)/(-a*f+b*e)
)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{(fx+e)b}{be-af}\right)\log\left(\frac{(be-af)(dx+c)}{(de-cf)(bx+a)}\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))*log(b*(f*x+e)/(-a*f+b*
e))/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(log((f*x + e)*b/(b*e - a*f))*log((b*e - a*f)*(d*x + c)/((d*e - c*
f)*(b*x + a)))/((b*x + a)*(d*x + c)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
             fi;
             #both result and optimal complex
             if leaf_count_result<=2*leaf_count_optimal then
                return "A";
             else
                return "B";
             end if
          else #result contains complex but optimal is not
             if debug then
                print("result contains complex but optimal is not");
             fi;
             return "C";
          end if
       else # result do not contain complex
            # this assumes optimal do not as well
            if debug then
                print("result do not contain complex, this assumes optimal do not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
               if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
               fi;
               return "A";
            else
               if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
               fi;
               return "B";
            end if
       end if
   else #ExpnType(result) > ExpnType(optimal)
       if debug then
          print("ExpnType(result) > ExpnType(optimal)");
       fi;
       return "C";
   end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```